LEC. 29 11/05/99

GRAPHS

GRAPH ≈ SET + BINARY RELATION

VERTICES EDGES

def A SIMPLE GRAPH $G=(V,E)$

$V \neq \emptyset$ SET OF VERTICES

E SET OF UNORDERED PAIRS OF DISTINCT VERTICES

EXAMPLE COMPUTER NET CONNECTED BY TELEPHONE LINES. DATA FLOWS BOTH WAYS. NO COMPUTER IS CONNECTED TO ITSELF BY TELEPHONE

MULTIGRAPH ≈ GRAPH WITH MULTIPLE EDGES

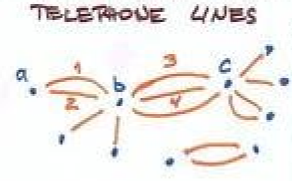
def A MULTIGRAPH $G=(V,E,f)$

$V \neq \emptyset$ SET OF VERTICES

E SET OF EDGES

f FUNCTION FROM E TO THE SET OF UNORDERED PAIRS OF VERTICES

EXAMPLE. COMPUTER NET WITH MULTIPLE TELEPHONE LINES



FUNCTION f READS THE PAIR OF VERTICES CONNECTED BY A GIVEN EDGE

$f(1)=f(2)=\{a,b\}$. $f(3)=f(4)=\{b,c\}$

PARALLEL EDGES $f(e_1)=f(e_2)$

STILL NO LOOPS, DIRECTION DOES NOT MATTER

AUTO, AIRPLANE CONNECTION MAPS

PSEUDOGRAPH ≈ MULTIGRAPH WITH LOOPS

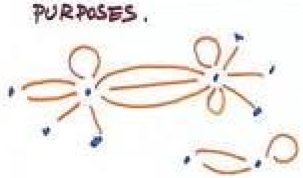
def A PSEUDOGRAPH $G=(V,E,f)$

$V \neq \emptyset$ SET OF VERTICES

E SET OF EDGES

f VERTICES READING FUNCTION FROM E TO SET OF UNORDERED PAIRS OF VERTICES NOT NECESSARILY DISTINCT.

EXAMPLE. A COMPUTER NET WITH MULTIPLE TELEPHONE LINES, INCLUDING SELF CONNECTED FOR DIAGNOSTIC PURPOSES.



DIRECTED GRAPH ≈ GRAPH WITH DIRECTED EDGES

def. A DIRECTED GRAPH $G=(V,E)$

$V \neq \emptyset$ SET OF VERTICES

$E \subseteq V \times V$. EDGES = ORDERED PAIRS OF VERTICES

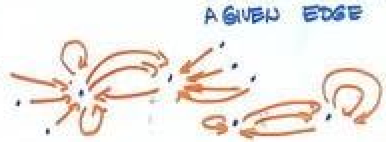
EXAMPLE. COMPUTER NET WITH ASSYMMETRIC LINES, NO MULTIPLE LINES IN THE SAME DIRECTION.

DIRECTED MULTIGRAPH $G=(V,E,f)$

$V \neq \emptyset$ SET OF VERTICES

E SET OF EDGES

$f: E \longrightarrow V^2$ READS INITIAL AND TERMINAL VERTICES OF A GIVEN EDGE



def <u>ADJACENT VERTICES</u> $u, v \in V$ SUCH THAT $\{u,v\} \in E$. $e = \{u,v\}$ <u>CONNECTS</u> u, v. e <u>IS INCIDENT</u> WITH u, v. u, v ARE <u>ENDPOINTS</u> OF e.

def <u>DEGREE OF A VERTEX</u> = # OF EDGES INCIDENT WITH IT
(VALID FOR ALL PSEUDOGRAPHS)
(A LOOP CONTRIBUTES TWICE !)

AN ISOLATED VERTEX

$\deg(a) = 2$
$\deg(b) = 6$
$\deg(c) = 3$
$\deg(d) = 3$
$\deg(e) = 0$

Th. (THE HANDSHAKING THEOREM) $G = (V, E)$ PSEUDOGRAPH (UNDIRECTED)

$$2e = \sum_{v \in V} \deg(v) \qquad e = |E|$$

<u>PROOF</u>. EVERY EDGE IS COUNTED TWICE IN Σ.

Th. AN UNDIRECTED (PSEUDO)GRAPH HAS AN EVEN NUMBER OF VERTICES OF ODD DEGREE.

PROOF. LET $V = V_1 + V_2$ WHERE V_1 = VERTICES WITH EVEN DEGREES, V_2 - ODD

$$\underbrace{2e}_{\text{EVEN}} = \sum_{v \in V} \deg(v) = \underbrace{\sum_{v \in V_1} \deg(v)}_{\text{EVEN}} + \sum_{v \in V_2} \deg(v)$$

$\Rightarrow \sum_{v \in V_2} \deg(v)$ IS EVEN $\Rightarrow |V_2|$ IS EVEN.