

LEC.26 10/29/99

RELATIONS

A BINARY RELATION FROM A TO B (A, B SETS)

IS $R \subseteq A \times B$

NOTATION: $(a, b) \in R$, $a R b$

EXAMPLES: a IS A PARENT OF b ; $a|b$, $a \leq b$

NOTE, SUBSET $S \subseteq A \sim$ A BIT STRING OF LENGTH $|A|$

RELATION $R \subseteq A \times B \sim$ A BIT MATRIX $|A| \times |B|$

$a \backslash b$	$b_1 \dots b_m$
a_1	
$\vdots$	
a_n	

$$\delta_{ij} = \begin{cases} 1, & \text{if } (a_i, b_j) \in R \\ 0, & \text{o.w.} \end{cases}$$

OF RELATIONS FROM A TO B IS $2^{|A| \cdot |B|}$

EXAMPLE. CITY - STATE 50 COLUMNS,
VERY MANY ROWS
CAPITAL CITY - STATE 50 x 50
MATRIX

NOTE. FUNCTIONS FROM A TO B ARE SPECIAL SORT
OF RELATIONS (NOT NECESSARILY TOTAL, DETERMINISTIC)

RELATION ON A: $R \subseteq A \times A$

REFLEXIVE: $(a, a) \in R$ FOR ALL $a \in A$

EXAMPLES: $x \leq y$, $n \mid m$, $n \equiv m \pmod{5}$, ...

"a IS A PARENT OF b" IS NOT REFLEXIVE

SYMMETRIC: $(a, b) \in R \Rightarrow (b, a) \in R$ / $a R b \Rightarrow b R a$

EXAMPLES: $n \equiv m \pmod{5}$, "a HAS A JOINT PAPER WITH b", $\emptyset$, ...

$x \leq y$, $n \mid m$, "a IS A PARENT OF b" - NOT SYMMETRIC

ANTISYMMETRIC: $a R b \text{ \& } b R a \Rightarrow a = b$

EXAMPLES: $x \leq y$, $n \mid m$, $x < y$, "a IS A PARENT OF b", ...
 $\text{identity}(A) = \{(a, a) \mid a \in A\}$ IS BOTH SYMMETRIC AND ANTISYMMETRIC.

TRANSITIVE: $a R b \text{ \& } b R c \Rightarrow a R c$

EXAMPLES: $x \leq y$, $x < y$, $n \mid m$, $n \equiv m \pmod{5}$, $\text{id}(A)$, $\emptyset$, ...

"a IS A PARENT OF b", "a HAS A JOINT PAPER WITH b" NOT TRANSITIVE.

EQUIVALENCE RELATION ON A: REFLEXIVE, SYMMETRIC, TRANSITIVE

$n \equiv m \pmod{5}$, $\text{id}(A)$

OPERATIONS ON RELATIONS:

$R_1 \cup R_2, R_1 \cap R_2, R_1 - R_2$ - SET THEORETICAL



$$S \circ R = \{(a, c) \mid \exists b \in B: aRb \wedge bSc\}$$

misleading notation.

$R \circ S$ would be better.

EXAMPLE: $R = S =$ "a IS A PARENT OF b"

$R \circ S =$ "a IS A GRANDPARENT OF b"

$$R^0 = id, R^1 = R, R^{n+1} = R^n \circ R$$

N-ARY RELATIONS

$$R \subseteq A_1 \times A_2 \times \dots \times A_n$$

↑
DOMAINS

↑
degree

EXAMPLES: $(n, q, r) =$ "r IS THE REMAINDER OF n DIVIDED BY q"
TERNARY
(degree 3)

DATA BASE PERSON $\times$ SS# $\times$ COURSE TAKEN $\times$ GRADE
RELATION OF DEGREE 4.

A PRIMARY KEY A DOMAIN A_i WHICH VALUE DETERMINES THE WHOLE n -TUPLE

EXAMPLE. PERSON, SS#, DATE OF BIRTH, INCOME
NAME

SS# IS A PRIMARY KEY, DOB, INCOME USUALLY
PERSON'S NAME - USUALLY A PRIMARY KEY ^{NOT}
SOMETIMES NOT (FOR BIG DATABASES).

A COMPOSITE KEY: SEVERAL DOMAINS WHEN COMPOSITE VALUE DETERMINES THE WHOLE RECORD

PROJECTION: $P_{i_1 \dots i_m} : (a_1 \dots a_n) \rightarrow (a_{i_1} \dots a_{i_m})$
 $i_1 < i_2 < \dots < i_m$

JOIN: $J_p(R, S) = \left. \begin{array}{l} R \text{ degree } m \\ S \text{ degree } n \\ p \leq n, m \end{array} \right\}$
 $= (a_1 \dots a_{m-p} \quad c_1 \dots c_p \quad b_1 \dots b_{n-p})$

SUCH THAT $(a_1 \dots a_{m-p} \quad c_1 \dots c_p) \in R$
 $(c_1 \dots c_p \quad b_1 \dots b_{n-p}) \in S$