

LEC.26 10/29/99

RELATIONS

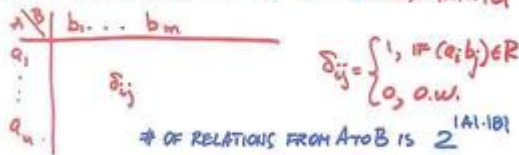
A BINARY RELATION FROM A TO B (A, B SETS)
IS $R \subseteq A \times B$

NOTATION: $(a, b) \in R, aRb$

EXAMPLES: a IS A PARENT OF b: $a/b, a \leq b$

NOTE, SUBSET $S \subseteq A \sim$ A BIT STRING OF LENGTH |A|

RELATION $R \subseteq A \times B \sim$ A BIT MATRIX |A| x |B|

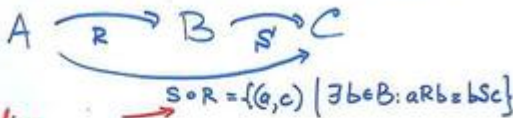


EXAMPLE. CITY - STATE 50 COLUMNS,
VERY MANY ROWS
CAPITAL CITY - STATE 50 x 50
MATRIX

NOTE. FUNCTIONS FROM A TO B ARE SPECIAL SORT
OF RELATIONS (NOT NECESSARILY TOTAL, DETERMINISTIC)

OPERATIONS ON RELATIONS:

$R, R_1, R_2, R_1 \cap R_2, R_1 - R_2$ - SET THEORETICAL



misleading notation. $R \circ S$ would be better.

EXAMPLE: $R = S =$ "a IS A PARENT OF b"

$R \circ S =$ "a IS A GRANDPARENT OF b"

$R^0 = id, R^1 = R, R^{n+1} = R^n \circ R$

N-ARY RELATIONS $R \subseteq A_1 \times A_2 \times \dots \times A_n$
↑ domains ↑ degree

EXAMPLES: $(n, q, r) =$ "r IS THE REMAINDER OF
n DIVIDED BY q"
(degree 3)

DATA BASE PERSON x SS# x COURSE TAKEN x GRADE
RELATION OF DEGREE 4.

RELATION ON A: $R \subseteq A \times A$

REFLEXIVE: $(a, a) \in R$ FOR ALL $a \in A$

EXAMPLES: $x \leq y, n \mid m, n \equiv m \pmod{5}, \dots$
"a IS A PARENT OF b" IS NOT REFLEXIVE

SYMMETRIC: $(a, b) \in R \Rightarrow (b, a) \in R / aRb \Rightarrow bRa$

EXAMPLES: $n \equiv m \pmod{5},$ "a HAS A JOINT PAPER
WITH b", $\emptyset, \dots$

$x \leq y, n \mid m,$ "a IS A PARENT OF b" - NOT SYMMETRIC

ANTISYMMETRIC: $aRb \wedge bRa \Rightarrow a = b$

EXAMPLES: $x \leq y, n \mid m, x < y,$ "a IS A PARENT OF b",
 $identity(A) = \{(a, a) \mid a \in A\}$ IS BOTH SYMMETRIC
AND ANTISYMMETRIC.

TRANSITIVE: $aRb \wedge bRc \Rightarrow aRc$

EXAMPLES: $x \leq y, x < y, n \mid m, n \equiv m \pmod{5}, id(A), \emptyset, \dots$
"a IS A PARENT OF b", "a HAS A JOINT PAPER WITH b"
NOT TRANSITIVE.

EQUIVALENCE RELATION ON A: REFLEXIVE, SYMMETRIC,
TRANSITIVE

$n \equiv m \pmod{5}, id(A)$

A PRIMARY KEY A DOMAIN A; WHICH VALUE
DETERMINES THE WHOLE n-TUPLE

EXAMPLE. PERSON, SS#, DATE OF BIRTH, INCOME
NAME

SS# IS A PRIMARY KEY, DOB, INCOME USUALLY
NOT
PERSON'S NAME - USUALLY A PRIMARY KEY
SOMETIMES NOT (FOR BIG "DATA BASES").

A COMPOSITE KEY: SEVERAL DOMAINS WHEN
COMPOSITE VALUE DETERMINES
THE WHOLE RECORD

PROJECTION: $P_{i_1, \dots, i_m} : (a_1, \dots, a_n) \rightarrow (a_{i_1}, \dots, a_{i_m})$
 $i_1 < i_2 < \dots < i_m$

JOIN: $J_P(R, S) = \begin{cases} R & \text{degree } m \\ S & \text{degree } n \end{cases}$
 $= (a_1, \dots, a_{m-p}, c_1, \dots, c_p, b_1, \dots, b_{n-p})$
 $p \in \{1, \dots, m\}$

SUCH THAT $(a_1, \dots, a_{m-p}, c_1, \dots, c_p) \in R$
 $(c_1, \dots, c_p, b_1, \dots, b_{n-p}) \in S$