4.7: 4, 6

LEC 23. 10/22/99

HOW TO GENERATE PERMUTATIONS

SOME METHODS OF OPTIMIZATION REQUIRE GENERATING ALL PERMUTATIONS

$$\boxed{G} \xrightarrow{\dots P_3 \; P_2 \; P_1} \boxed{\min f} \qquad f(P_i) \to \min$$

THERE IS AN INTELLIGENT WAY OF RUNNING $\boxed{G}$ BASED ON THE LEXICOGRAPHIC ORDERING

1. $A = \{\sigma_1, \dots, \sigma_n\}$. WITHOUT LOSS OF GENERALITY WE ASSUME $A = \{1, \dots, n\}$

2. $a_1 a_2 \dots a_k \prec b_1 b_2 \dots b_k \iff$ $a_1 = b_1$, ..., $a_i = b_i$, $a_{i+1} < b_{i+1}$

"PRECEDS LEXICOGRAPHICALLY"

AT THE FIRST POSITION WHERE a AND b DIFFER $a_{i+1} < b_{i+1}$

EXAMPLE. $123 < 132 < 213 < 231 < 312 < 321$

EXAMPLE. FIND IMMEDIATE SUCCESSORS

$12345 \prec 12354$	$12453 \prec 12534$
$12354 \prec 12435$	$21345 \prec 21354$
$12435 \prec 12453$	54321 NONE

GENERAL METHOD OF GENERATING "THE NEXT" PERMUTATION, GIVEN

1. $a_1 a_2 \ldots a_{i-1} a_i a_{i+1} \ldots a_n$ FIND THE GROUP

$$a_i < a_{i+1} > a_{i+2} > \ldots > a_n$$

(IF THERE IS NO SUCH i TERMINATE WITH FAILURE

2. CALCULATE $b_i = \min$ OF $a_{i+1}, \ldots, a_n$ WHICH IS $> a_i$

$b_{i+1}, b_{i+2}, \ldots, b_n$ = REMAINING OF $\{a_i, \ldots a_n\}$ IN INCREASING ORDER

3. THE NEXT $= a_1 \ldots a_{i-1} b_i \ldots b_n$

EXAMPLE $n=4$

$1234 < 1243 < 1324 < 1342 < 1423 < 1432 < 2134 < \ldots$

$\ldots < 4213 < 4231 < 4312 < 4321$

EXAMPLE THE NEXT LARGEST OF

78312654 IS 78314256

GENERATING COMBINATIONS

COMBINATION = SUBSET OF $\{1, \dots, n\}$ ~ BIT STRING

$S = \{1, \dots, n\}$ $\quad X \subseteq S$ $\quad \sigma_1 \dots \sigma_n$

$$\sigma_i = \begin{cases} 1, & \text{IF } i \in X \\ 0, & \text{IF } i \notin X \end{cases}$$

$S = \{1,2,3\}$

σ (IN BINARY ORDER):

000	001	010	011	100	101	110	111
$\emptyset$	$\{3\}$	$\{2\}$	$\{2,3\}$	$\{1\}$	$\{1,3\}$	$\{1,2\}$	$\{1,2,3\}$

ORDER ON STRINGS = INCREASING BINARY EXTENSIONS

EXAMPLE: NEXT LARGEST STRING 1001101011 IS 1001101100

NEXT LARGEST SET (OUT OF $\{1,2,3,4,5\}$) FOR $\{\underline{2,4,5}\}$ IS: $\{2,3\}$

$X \cdot \sigma_X = 01011 < 01100$