

4.7: 4, 6

LEC 23. 10/22/99

HOW TO GENERATE PERMUTATIONS

SOME METHODS OF OPTIMIZATION REQUIRE GENERATING ALL PERMUTATIONS



THERE IS AN INTELLIGENT WAY OF RUNNING G BASED ON THE LEXICOGRAPHIC ORDERING

1. $A = \{a_1, \dots, a_n\}$. WITHOUT LOSS OF GENERALITY WE ASSUME $A = \{1, \dots, n\}$

2. $a_1 a_2 \dots a_k \prec b_1 b_2 \dots b_n \iff$

"PRECEDES LEXICOGRAPHICALLY"

AT THE FIRST POSITION WHERE a AND b DIFFER $a_{i+1} < b_{i+1}$

EXAMPLE. $123 < 132 < 213 < 231 < 312 < 321$

EXAMPLE. FIND IMMEDIATE SUCCESSORS

12345 $\prec$ 12354 12453 $\prec$ 12534
12354 $\prec$ 12435 21345 $\prec$ 21354
12435 $\prec$ 12453 54321 NONE

-138

GENERATING COMBINATIONS

COMBINATION = SUBSET OF $\{1, \dots, n\} \sim$ BIT STRING

$S = \{1, \dots, n\}$ $X \subseteq S$ $G_1, \dots, G_n$
 $G_i = \begin{cases} 1, & \text{if } i \in X \\ 0, & \text{if } i \notin X \end{cases}$

$S' = \{1, 2, 3\}$

G (IN BINARY ORDER):

000 001 010 011 100 101 110 111
 $\emptyset$ {3} {2} {2,3} {1} {1,3} {1,2} {1,2,3}

ORDER ON STRINGS = INCREASING BINARY EXTENSIONS

EXAMPLE: NEXT LARGEST STRING

1001101011 IS 1001101100

NEXT LARGEST SET (OUT OF $\{1, 2, 3, 4, 5\}$) FOR $\{2, 4, 5\}$ IS:

$X \cdot G_X = 01011 \prec 01100$ (with $\{2, 3\}$ above the second 1)

-140-

GENERAL METHOD OF GENERATING THE NEXT PERMUTATION, GIVEN

1. $a_1 a_2 \dots a_{i-1} a_i a_{i+1} \dots a_n$ FIND THE GROUP

$a_i < a_{i+1} > a_{i+2} > \dots > a_n$

(IF THERE IS NO SUCH i TERMINATE WITH FAILURE)

2. CALCULATE $b_i = \min$ OF $a_{i+1}, \dots, a_n$ WHICH IS $> a_i$

$b_{i+1}, b_{i+2}, \dots, b_n =$ REMAINING OF $\{a_i, \dots, a_n\}$ IN INCREASING ORDER

3. THE NEXT = $a_1 \dots a_{i-1} b_i \dots b_n$

EXAMPLE $n=4$

1234 $<$ 1243 $<$ 1324 $<$ 1342 $<$ 1423 $<$ 1432 $<$ 2134 $<$...
... $<$ 4213 $<$ 4231 $<$ 4312 $<$ 4321

EXAMPLE THE NEXT LARGEST OF

78312654 IS 78314256

-139-