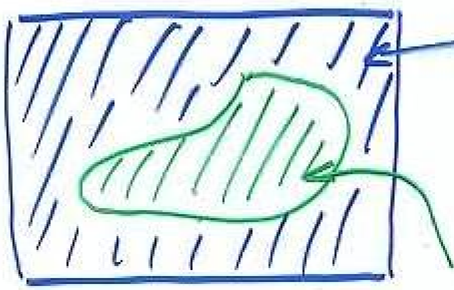
LEC 19 10/13/99

CLASSICAL PROBABILITY



SAMPLE SPACE = SET OF POSSIBLE OUTCOMES OF AN EXPERIMENT

EVENT = SUBSET OF THE SAMPLE SPACE

EXAMPLE. AN EXPERIMENT: TWO DICE ARE ROLLED

THE SAMPLE SPACE = $\{1,2,3,4,5,6\}^2 = \{(1,1)\ (1,2) \ldots (1,6)\ (2,1)\ (2,2) \ldots (2,6)\ (3,1)\ (3,2) \ldots (3,6) \ldots (6,1) \ldots (6,6)\}$

36 PAIRS TOTAL

AN EVENT: THE SUM ON THE TWO DICE IS five

$\{(1,4)\ (2,3)\ (3,2)\ (4,1)\}$ 4 PAIRS

THE MAIN ASSUMPTIONS UNDER WHICH THE CLASSICAL DEFINITION OF PROBABILITY OPERATES

1. SAMPLE SPACE S IS FINITE
2. ALL OUTCOMES IN S ARE EQUALLY LIKELY

def (LAPLACE). THE PROBABILITY OF AN EVENT E

$$p(E) = \frac{|E|}{|S|}$$

EXAMPLE THE PROBABILITY THAT WHEN TWO DICE ARE ROLLED THE SUM IS 5

$$p = \frac{4}{36} = \frac{1}{9}$$

EXAMPLE AN URN CONTAINS TREE BLUE BALLS AND FIVE RED BALLS. WHAT IS THE PROBABILITY THAT A BALL RANDOMLY CHOSEN IS BLUE?

$$|S| = 8 \;;\; |E|_B = 3 \;,\; P_B = \frac{|E|}{|S|} = \frac{3}{8}$$

EXAMPLE (LOTTERY) WHAT IS THE PROBABILITY TO PICK THE CORRECT SIX NUMBERS OUT OF 40

SAMPLE SPACE = ALL POSSIBLE 6-COMBINATIONS OUT OF 40. $|S| = \frac{40!}{6!\,34!} = 3838380$

EVENT = ONE WINNING COMBINATION $|E| = 1$

$$P = \frac{|E|}{|S|} = \frac{1}{3\,838\,380}$$

EXAMPLE (POKER) FIND THE PROBABILITY THAT A HAND OF FIVE CARDS CONTAINS FOUR CARDS OF ONE KIND.

S = SET OF ALL HANDS; $|S| = C_{52}^{5}$

$|E|$ = (BY THE PRODUCT RULE) # OF WAYS TO PICK A KIND $\cdot$ # OF WAYS TO PICK THE FIFTH CARD

$= C_{13}^{1} \cdot C_{48}^{1}$

$$p = \frac{C_{13}^{1} \cdot C_{48}^{1}}{C_{52}^{5}} = \frac{13 \cdot 48}{2\,598\,960} \sim 0.00024$$

COMPLEMENTARY EVENT $\overline{E} = S - E$

$|\overline{E}| = |S| - |E|$

Th. $p(\overline{E}) = 1 - p(E)$

PROOF: $p(\overline{E}) = \frac{|\overline{E}|}{|S|} = \frac{|S| - |E|}{|S|} = \frac{|S|}{|S|} - \frac{|E|}{|S|} = 1 - p(E)$

EXAMPLE FIND THE PROBABILITY THAT AN INTEGER x ($0 \le x \le 999999$) HAS AT LEAST ONE 9 IN ITS DECIMAL EXPANSION.

E = "x HAS NO 9"; BY THE PRODUCT RULE: SIX DECIMAL POSITIONS, DIGITS $\{0, 1, \dots, 8\}$ FOR EACH POSITION

$|E| = 9 \cdot 9 \cdot 9 \cdot 9 \cdot 9 \cdot 9 = 9^6$.

$|S|$ = TOTAL # OF x'S FROM 0 TO $999999 = 10^6$

$p(E) = \frac{9^6}{10^6} = (0.9)^6 = 0.53$; $p(\overline{E}) = 1 - 0.53 = 0.47$

EXAMPLE (TOSSING A COIN) A SEQUENCE OF 10 BITS IS RANDOMLY GENERATED. WHAT IS THE PROBABILITY THAT AT LEAST ONE OF THEM IS 1?

E = "ALL BITS ARE 0'S"; $|E| = 1$; $|S| = 2^{10} = 1024$

$P(\overline{E}) = 1 - P(E) = 1 - \frac{1}{1024} = \frac{1023}{1024}$

Th. $P(E_1 \cup E_2) = P(E_1) + P(E_2) - P(E_1 \cap E_2)$

PROOF. $|E_1 \cup E_2| = |E_1| + |E_2| - |E_1 \cap E_2|$

$$P(E_1 \cup E_2) = \frac{|E_1 \cup E_2|}{|S|} = \frac{|E_1|}{|S|} + \frac{|E_2|}{|S|} - \frac{|E_1 \cap E_2|}{S} =$$

$$= P(E_1) + P(E_2) - P(E_1 \cap E_2)$$

EXAMPLE. POSITIVE INTEGERS ≤ 100 ARE RANDOMLY SELECTED. WHAT IS THE PROBABILITY THAT A NUMBER IS DIVISIBLE BY 2 OR 3?

E_1 X IS DIVISIBLE BY 2 $\quad P_1 = \frac{50}{100}$

E_2 X IS DIVISIBLE BY 3 $\quad P_2 = \frac{33}{100}$

$E_1 \cap E_2$ X IS DIVISIBLE BY BOTH 2 AND 3 $\quad P_3 = \frac{16}{100}$

$$P = P_1 + P_2 - P_3 = \frac{50}{100} + \frac{33}{100} - \frac{16}{100} = \frac{50+33-16}{100} = \frac{67}{100}$$

HW. 4.4: 8 14 24a,d 32