HW 4.3: 8 16b,d 24b 32

LEC.18 10/06/99

PERMUTATIONS AND COMBINATIONS

A PERMUTATION IS AN ORDERED K-TUPLE OF DISTINCT OBJECTS. (K-PERMUTATION)

EXAMPLE $S = \{a,b,c\}$ PERMUTATIONS OF S:
$(a,b,c), (a,c,b) (b,a,c), (b,c,a), (c,a,b), (c,b,a)$

Th. THE NUMBER OF r-PERMUTATIONS OF A SET WITH n DISTINCT ELEMENTS IS

$$P(n,r) = n\cdot(n-1)\cdot(n-2)\cdots(n-r+1) = \frac{n!}{(n-r)!}$$

PROOF n WAYS TO CHOOSE THE FIRST ELEMENT
n-1 WAYS TO CHOOSE THE SECOND ELEMENT FROM n-1 ELEMENTS REMAINING
n-2 WAYS TO CHOOSE THE THIRD ELEMENT
. . . .
n-r+1 WAYS TO CHOOSE THE rth ELEMENT

BY THE PRODUCT RULE $P(n,r) = n(n-1)\cdot(n-2)\cdots(n-r+1)$

COR. THE TOTAL # OF PERMUTATIONS OF A SET WITH n DISTINCT ELEMENT IS $n!$

EXAMPLE THE # OF WAYS TO AWARD 3 MEDALS TO 10 PLAYERS IS $P(10,3) = 10\cdot 9\cdot 8 = 720$

r-COMBINATION OF S IS A SUBSET OF S WITH r ELEMENTS

$S = \{a,b,c\}$. 2-COMBINATIONS OF S ARE
$\{a,b\}, \{a,c\}, \{b,c\}$ || 2 PERMUTATIONS OF S ARE (ab)(ba) (ac)(ca) (bc)(cb)

Th. THE NUMBER OF r-COMBINATIONS OF A SET WITH n ELEMENTS EQUALS

$$C(n,r) = \frac{n!}{r!\,(n-r)!}$$

|| CALLED BINOMIAL COEFFICIENTS
ALTERNATIVE NOTATIONS C_n^k, $\binom{k}{n}$

PROOF $\underbrace{P(n,r)}_{\text{CHOOSING r-PERMUTATION}} = \underbrace{C(n,r)}_{\text{CHOOSING r-COMBINATION}} \cdot \underbrace{P(r,r)}_{\text{AND THEN ORDERING THE SUBSET OF r ELEMENTS CHOSEN.}}$

$$C(n,r) = \frac{P(n,r)}{P(r,r)} = \frac{n!}{(n-r)!\,r!}$$

EXAMPLE THE # OF WAYS TO PICK A TEAM OF THREE HACKERS OUT OF 20 TO REPRESENT CORNELL IN A COMPETITION IN EUROPE IS $C(20,3) = \frac{20!}{3!(20-3)!} = \frac{20!}{3!\,17!} =$

$$= \frac{20\cdot 19\cdot 18\cdot \cancel{17!}}{3!\cdot \cancel{17!}} = \frac{20\cdot 19\cdot 18}{6} = 20\cdot 19\cdot 3 =$$
$$= 60\cdot 19 = 1140$$

Th. (PASCAL'S IDENTITY)

$$C(n+1,k) = C(n,k-1) + C(n,k)$$

PROOF. $|T| = n+1$, $a \in T$, $S := T - \{a\}$

ANY K-SUBSET X OF T

1° $a \in X$ THEN $X' = X - \{a\}$ IS A (k-1)-SUBSET OF S — $C(n,k-1)$ COMBINATIONS

OR 2° $a \notin X$ THE X IS A k-SUBSET OF S — $C(n,k)$ COMBINATIONS

BY THE SUM RULE THE TOTAL IS $C(n,k-1) + C(n,k)$

PASCAL TRIANGLE

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