LEC. 16 10/01/99

PROGRAM CORRECTNESS

PROGRAM IS CODE + CORRECTNESS PROOF
(VERIFICATION)

SYNTACTICAL CORRECTNESS = THE FORMAT IS OK
EASY TO VERIFY, EVERY COMPILER DOES IT

SEMANTICAL CORRECTNESS = THE PROGRAM TERMINATES AND GIVES THE CORRECT OUTPUT

UNIVERSAL AUTOMATED VERIFICATION IS IMPOSSIBLE. DOES NOT FOLLOW FROM THE SYNTACTICAL CORRECTNESS

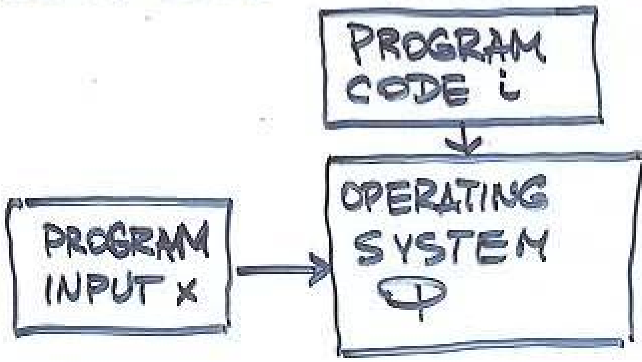
CONSTRUCTIVE REASONING: CORRECTNESS PROOF YIELDS A CODE OF P
(R. CONSTABLE, et al.)

CORRECT PROGRAM $\equiv$ CONSTRUCTIVE PROOF OF IS CORRECTNESS

ITERATIVE PROGRAM = PROOF BY MATHEMATICAL INDUCTION

Th. (RICE). NO NONTRIVIAL PROPERTY OF A COMPUTABLE FUNCTION CAN BE AUTOMATICALLY VERIFIED FROM ITS PROGRAM.

SPECIAL CASE – HALTING PROBLEM



$\to$ OUTPUT $\Phi(i,x) = P_i(x)$

P_i = THE PROGRAM WITH THE CODE i

Th. THERE IS NO ALGORITHM $\mathfrak{A}$ SUCH THAT

$$\mathfrak{A}(i,x) = \begin{cases} 1, & \text{IF } P_i(x) \text{ TERMINATES} \\ 0, & \text{O.W.} \end{cases}$$

PROOF. SUPPOSE SUCH AN ALGORITHM EXISTS. DEFINE AN AUXILIARY FUNCTION

$$B(m) = \begin{cases} 0, & \text{IF } \mathfrak{A}(m,m) = 1 \wedge \Phi(m,m) \neq 0 \\ 1, & \text{IF } \mathfrak{A}(m,m) = 0 \vee \\ & (\mathfrak{A}(m,m) = 1 \wedge \Phi(m,m) = 0) \end{cases}$$

TOTAL, COMPUTABLE

LET n BE A CODE OF $B(y)$, i.e. $B(k) = \Phi(n,k)$ FOR ALL k'S

NOTE THAT $\mathfrak{A}(n,k) = 1$, SINCE B IS TOTAL.

$B(n) = 0 \Rightarrow \Phi(n,n) \neq 0 \Rightarrow B(n) \neq 0$

$B(n) \neq 0 \Rightarrow B(n) = 1 \Rightarrow \Phi(n,n) = 0 \Rightarrow B(n) = 0$

CONTRADICTION!

A PROGRAM P IS SAID TO BE <u>PARTIALLY CORRECT</u> IF THE CORRECT OUTPUT IS OBTAINED WHENEVER P TERMINATES

PROVING PARTIAL CORRECTNESS

HOARE IMPLICATION $\quad p\,\{S\}\,q$

INITIAL ASSERTION $\quad$ PROGRAM SEGMENT $\quad$ FINAL ASSERTION

$p\{S\}q \iff$ IF p IS TRUE FOR THE INPUT VALUE(S) AND S TERMINATES THEN q IS TRUE FOR THE OUTPUT VALUE(S).

<u>EXAMPLE</u> $\quad S: \begin{cases} y := 2 \\ z := x+y \end{cases} \qquad \begin{array}{l} p:\ x=1 \\ q:\ z=3 \end{array}$

$p\{S\}q$ HOLDS. INDEED, ASSUMING $x=1$ WE HAVE $y:=2$ AND $z:=x+y=1+2=3$

TERMINOLOGY: S IS (PARTIALLY) CORRECT WITH RESPECT TO THE INITIAL ASSERTION p AND THE FINAL ASSERTION q

RULES OF INFERENCE FOR HOARE IMPLICATION

THE COMPOSITION RULE

$$\frac{\begin{array}{c} p\{S_1\}q \\ q\{S_2\}r \end{array}}{p\{S_1;S_2\}r}$$

$S = S_1;S_2$ IS S_1 FOLLOWED BY S_2

THE CONDITIONAL RULE COVERS A PROGRAM SEGMENT

$$\frac{\begin{array}{c} (p \wedge \text{CONDITION})\{S\}q \\ (p \wedge \neg\text{CONDITION}) \rightarrow q \end{array}}{p\{\text{IF CONDITION THEN } S\}q}$$

IF CONDITION THEN
  S

IF CONDITION IS TRUE THEN EXECUTE S. O.W. DON'T.

$$\frac{\begin{array}{c} (p \wedge \text{CONDITION})\{S_1\}q \\ (p \wedge \neg\text{CONDITION})\{S_2\}q \end{array}}{p\{\text{IF CONDITION THEN } S_1 \text{ ELSE } S_2\}q}$$

IF CONDITION THEN
  S_1
ELSE
  S_2

EXAMPLE VERIFY THAT THE SEGMENT IS CORRECT W.R.T. THE INITIAL ASSERTION 'T' AND THE FINAL ASSERTION $abs = |x|$

```
IF x<0 THEN
      abs:=-x
ELSE
      abs:=x
```

(T IS A DEFAULT INITIAL ASSERTION "ASSUME NOTHING PARTICULAR")

1° $x < 0 \Rightarrow abs = -x = |x|$

2° $x \geq 0 \Rightarrow abs = x = |x|$

$$\underbrace{(x<0) \vee (x \geq 0)}_{T} \Rightarrow abs = |x|$$

LOOP INVARIANTS. CONSIDER A SEGMENT

```
WHILE CONDITION
   S
```

S IS REPEATEDLY EXECUTED UNTILL CONDITION BECOMES FALSE

p IS A LOOP INVARIANT IF $(p \wedge \text{CONDITION})\{S\}p$

RULE OF INFERENCE:

$$\frac{(p \wedge \text{CONDITION})\{S\}p}{p\{\text{WHILE CONDITION } S\}(\neg\text{CONDITION} \wedge p)}$$

EXAMPLE VERIFY THAT THE SEGMENT PRODUCES $factorial = n!$

```
i := 1
factorial := 1
WHILE i < n
BEGIN
    i := i+1
    factorial := factorial * i
END
```

SOLUTION. 1) CHECK THAT $p(i)$ IS A LOOP INVARIANT

$p(i)$: " $factorial = i! \wedge i \leq n$ "

INDUCTION ON i. $(p(i) \wedge \text{CONDITION}(i))\{S\}\, p(i+1) = A(i)$

$i = 1$

$A(1) \Leftarrow \begin{cases} p(1) \wedge \text{condition}(1): & factorial = 1 \wedge \cancel{1 \leq n} \wedge 1 < n \\ p(2): & factorial = 2 \wedge 2 \leq n \end{cases}$

$i = k$

$A(k) \Rightarrow A(k+1) \begin{cases} factorial = k! \wedge \cancel{k \leq n} \wedge k < n & p(k) \wedge \text{condition} \\ factorial = (k+1)! \wedge k \leq n & p(k+1) \end{cases}$

2) CONCLUDE: $p\,\{\text{WHILE } i < n\ S\}\,[i \geq n \wedge p]$

$i \geq n \Rightarrow i = n$, $p \Rightarrow p(n) \Rightarrow factorial = n!$

HW 3.5: 2, 4, 10