LEC. 15 09/29/99

RECURSIVE ALGORITHMS

BY THE EUCLIDEAN ALGORITHM

$gcd(a,b) = gcd(b, \underbrace{r_2}_{})$ CIRCULAR PROCEDURE? No!

THIS GUARANTEES THE CONVERGENCY ⟸ A SMALLER INPUT $\min(a,b) \gneq \min(b,r_2)$

def AN ALGORITHM IS CALLED <u>RECURSIVE</u> IF IT WORKS BY REDUCING TO ITS OWN VALUE ON SMALLER INPUTS (USUALLY PERFORMED BACKWARD)

<u>EXAMPLE</u> $gcd(a,b) = gcd(b, r_2)$ WHERE $a \geq b > 0$

$$a = b \cdot q_1 + r_2 \quad 0 \leq r_2 < b$$

$$\begin{cases} gcd(a,b) = gcd(b, a \bmod b) \\ gcd(d,0) = d \text{ when } d > 0 \end{cases}$$

SPECIFIES A SEQUENCE OF REDUCTIONS THAT ENDS by $gcd(d,0) = d$

RECURSIVE COMPUTING OF a^n

$$\begin{cases} a^0 = 1 \\ a^n = a\, a^{n-1} \end{cases}$$

THE SAME PROCEDURE, SMALLER INPUT

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PROCEDURE POWER(a: NONZERO REAL, n > 0 INTEGER)
IF n=0 THEN POWER(a,n)=1
ELSE POWER(a,n) := a * power(a, n-1)
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RECURSIVE SEQUENTIAL SEARCH ALGORITHM: FIND x IN $a_1, \ldots, a_n$

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PROCEDURE SEARCH(i,n,x)
IF a_i = x THEN
    location := i
ELSE IF i=n THEN
    location := 0
ELSE
    SEARCH(i+1,n,x)
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SEARCHES FOR x IN $a_i, a_{i+1}, \ldots a_n$

DECREASING PARAMETER $(n-i)$ IS ONE LESS AFTER EACH ITERATION

$\Rightarrow$ CONVERGES

Th. IF $f(n)$ IS DEFINED RECURSIVELY

$$\begin{cases} f(0) = a \\ f(n) = \text{SOME FUNCTION } G \text{ OF } f(0), \ldots, f(n-1) \end{cases}$$

AND G IS COMPUTABLE, THEN f IS COMPUTABLE.

PROOF. COMPUTE $f(0), f(1), f(2), \ldots, f(n)$

def ITERATIVE PROCEDURE P:

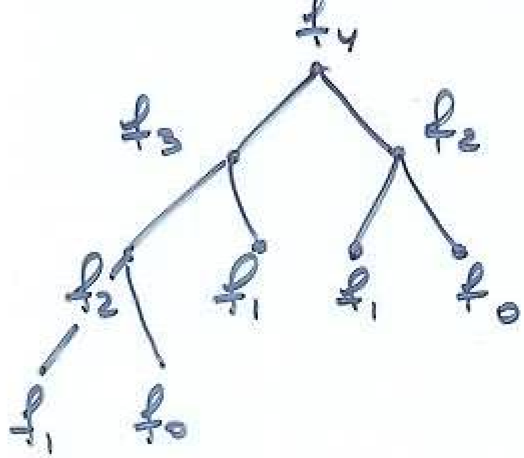
$$\left\{ \begin{array}{l} \text{INPUT } a \\ P_n(a) = P(P_{n-1}(a)) \end{array} \right.$$

USUALLY PERFORMED STRAIGHT

$$a,\ P(a),\ P(P(a)),\ \ldots\ \underbrace{P(P(P\ldots}_{n \text{ TIMES}}(Pa)\ldots)$$

RECURSIVE	ITERATIVE
PROCEDURE *factorial(n)* IF $n=1$ THEN *factorial(n)* := 1 ELSE *factorial(n)* := $n*$ *factorial(n-1)*	PROCEDURE *iterfactorial (n)* $x := 1$ FOR $i := 1$ TO n $x := i * x$ $\{x$ is $n!\}$
PROCEDURE *fibonacci(n)* IF $n=0$ THEN *fibonacci(n)* := 0 ELSE IF $n=1$ THEN *fibonacci(n)* := 1 ELSE *fibonacci(n)* := *fibonacci(n-1)* + *fibonacci(n-2)*	PROCEDURE *itfibonacci (n)* IF $n=0$ THEN $y := 0$ ELSE BEGIN $x := 0$ $y := 1$ FOR $i := 1$ to $n-1$ BEGIN $z := x+y$, $x := y$, $y := z$ END END {y is nth Fibonacci num}

RECURSIVE FIBONACCI COMPUTATION



$f_{n+1}-1$ ADDITIONS TO FIND f_n

(EXERCISE!)

EXPONENTIAL

ITERATIVE FIBONACCI COMPUTATION

$f_0, f_1, f_2 = f_1 + f_0, f_3 = f_2 + f_1, \ldots, f_n = f_{n-1} + f_{n-2}$

$n-1$ ADDITIONS / LINEAR

HW 3.4: 4, 14