CS 280 PRELIM 2
Grading Guide/Comments

1a. 2 pts for C(6, 2). –1 for using P(6, 2). –2 for using 6!
3 pts for $5^2 21^4$. 1 pt for getting some part correct. 2 for minor mistakes.

1b. 2 pts for C(6, 2), -1 for using P(6, 2), -2 for using 6!
3 pts for P(5, 2)P(21,4). –1 each for using C instead of P.

2. Comments:
Most errors were on part b. Almost none were for part a. Some were for part c.
Comments for b:

1) There need to be 15 people to satisfy three being born on the same day of the week. There are 25 people needed for three being born in the same month. You cannot multiply those together to get 375, since then those three may not be the same (i.e. you have three in one case, and three in the other).
2) You need 3 people as the minimum number, not 2. Please read the question carefully.

Misc. Comments:
Having two people born on the same month and the same day of the week does not make them being born on the same date. Take March 1 and March 15 for example.
You cannot take max(15, 25), since this would not guarantee the same three people for both cases.
Finally, the minimal solution to the equation
Ceil(N / 84) = 3 is 169, not 253.

Comments for part c:
You cannot divide the distribution of names into odd and even. The cases all ages are odd, and all ages are even do not cover every possibility.
Taking the ceiling of 98/50 does not go according to the pigeonhole principle. We have 98 slots, and 50 objects, not the other way around.

3. (5 points per question)

(a) 5 points are given for the correct answer, 84.
Unevaluated correct answers, C(9, 3) and C(9, 6) were also given full credit.

If the answer recognized ‘01’ as a unit, but used the incorrect approach (such as using stars-and-bars to solve the problem), 3 marks were *taken off*.

Answers which disregarded the conditions, such as 9! or 15!, were given no credit.

(b) 5 points are given for the correct answer, 5 984.
Unevaluated correct answers, C(34, 3) and C(34, 31) were also given full credit.

Off-by-one errors, such as C(34, 4) and C(34,30) were penalized by 1 mark.

Answers that did not take note of the condition that each variable must be *positive* (not just *non-negative*) were penalized by 2 marks. Answers such as C(38, 3) and C(38, 35) received only 3 out of 5 possible marks.

(Note: Off-by-one answers that did not take note of the "*positive*" condition were penalized by 3 marks altogether, for example, answers such as C(38, 4) received only 2 out of 5 possible marks.)

Answers which were incorrect but acknowledged that $x_i \leq 32$ for all i were given at most 1 out of 5 possible marks.

4. (a) Totally, 5 points. To find the number of elements in $|A_1 \cup A_2 \cup A_3|$, principle of inclusion-exclusion should be used:

$$|A_1 \cup A_2 \cup A_3| = |A_1| + |A_2| + |A_3| - (|A_1 \cap A_2| + |A_1 \cap A_3| + |A_2 \cap A_3|) + |A_1 \cap A_2 \cap A_3|$$
$$= 100 + 100 + 100 - (25 + 25 + 25) + 10$$
$$= 235$$

In the above inclusion-exclusion expression:
$|A_1| + |A_2| + |A_3|$ is worth 1 point,
$- (|A_1 \cap A_2| + |A_1 \cap A_3| + |A_2 \cap A_3|)$ is worth 2 points,
$|A_1 \cap A_2 \cap A_3|$ is worth 2 points.

If there is a sign mistake, 1 point is lost.
If the inclusion-exclusion expression is right, but the result is not 235, then 1 point is lost.

(b) Totally, 5 points. To determine whether A and B are independent events, definition of independence should be used:

$$P(A_1).P(A_2) \stackrel{?}{=} P(A_1 \cap A_2)$$

In this expression:

- showing $P(A_1) = P(A_2) = 100/235 = .181077$ is worth 2 points,
- showing $P(A_1 \cap A_2) = 25/235 = .10638$ is worth 2 points,
- concluding that A_1 and A_2 are not independent by stating $P(A_1).P(A_2)$ is not equal to $P(A_1 \cap A_2)$ is worth 1 point.

- If the conclusion follows from the incorrect reasoning:
  "A_1 and A_2 are not disjoint, so they can't be independent"
  then 4 points are lost.

- If the answer is “A and B are independent” 5 points are lost.
- If the answer in (a) is wrong and $P(A_1)$, $P(A_2)$ or $P(A_1 \cap A_2)$ are wrong due to that, then no points are lost.

4. (a) This part was generally correct, few people made sign mistakes and addition-subtraction errors.

(b) -Most common mistake was using a wrong definition for independence, such as:

- $P(A_1) + P(A_2) = P(A_1 \cup A_2)$
- $P(A_1).P(A_2) = P(A_1 \cup A_2)$
- …

(Only 3 points are lost if $P(A_1)$, $P(A_2)$ are calculated correctly)

-Another common mistake was concluding with incorrect reasoning:
“A_1 and A_2 are not disjoint, so they can’t be independent”. This is not true in general, because there exist independent events that are not disjoint. (See page 271-example 5)
-Also, some people confused the independence of events with “linear independence” which is a totally different thing.

Question 5

The following guide applies to both parts (a) and (b), each worth 5 points.

-2.5 points if the dice are considered indistinguishable, e.g. rolling (3,1,1) is considered the same as (1,3,1) and (1,1,3).

If the approach to the problem was one involving permutations, then -2 points for neglecting to account for duplicates.

-0.5 points for listing all the right cases, but not computing the probability correctly.

-3 points if "rolling a 5" is interpreted as rolling a 5 on any of the dice (i.e. the note about the sum in the question was ignored.)

-2 points if the total number of outcomes was computed incorrectly (e.g. there are 6*6=36 outcomes with 2 dice, not 12.)

Question 6 (7 marks)

4 marks maximum for an incorrect answer. Many people tried summing over all 11 possible outcomes, but did not end up with the correct answer because of the complexity of the arithmetic. This was taken into consideration and marks were awarded for such working. The best way is still to make use of the independence of the dice and calculate the expected value of a single dice. Still if you managed to arrive at the final answer by calculating all possibilities, full credit was awarded too.

CS 280 – Discrete Structures
Grading Guide – Prelim 2

Problem 7a)
Total score was 8 points, broken up as follows: 2 points for the initial conditions, and 6 points for the recurrence relation. Since a recurrence relation was required, a general formula that wasn't a recurrence but almost worked, received 1 or 2 points. In the solution, the recurrence relation is the sum of 3 expressions. Each expression was worth 2 points. If you had part of an expression correct, you probably received 1 point for that expression. Some people got recurrence relations different from the one in the solution. These were tested for a_1 to a_6 (possibly further), and if the explanation also made sense, they probably received full credit.

Problem 7b)
Total score was 5 points. We tried our best to treat this question as separate from (7a). So if you correctly implemented your recurrence relation from (7a) here, you received full credit, even if your recurrence in (7a) was wrong. Ideally, you would have got the correct recurrence relation to (7a) and implemented it here. If you did this and got the right answer (43), you received all 5 points. If you made a mistake in the recurrence implementation here, you probably lost 1 point. Finally, if you treated this question separately from (7a) and didn't use any type of recurrence, but used combinations to get your answer, you received full credit if your answer was correct, and lost about 2 points if there was some error in logic. In general, this question carried 3 points for the right idea and 2 points for accuracy.