CS280 Fall 2001 Prelim 2

Full name: Student ID: If you want your exam returned in private, then write 'PRIVATE RETURN' here:

Statement of integrity: I did not, and will not, break the rules of academic integrity on this exam.

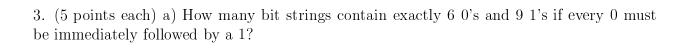
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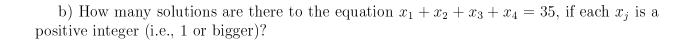
You must **show your work** and/or give clear explanations or reasons for each answer you give. If you use a formula, show all parts of the formula before simplifying. Correct answers without explanation will be worth 0. Also, please indicate clearly your final answer (e.g. by drawing a box around it or similarly highlighting it). Please leave answers such as $C(4,2)7^3$ unevaluated (i.e., you don't need to expand or calculate the final number).

- 1. (5 points each) The English alphabet contains 5 vowels and 21 consonants.
- a) How many strings of 6 lowercase English letters contain exactly 2 vowels if each letter may be used as often as you like?

b) How many strings of 6 lowercase English letters contain exactly 2 vowels if no letter is allowed to be used more than once?

| 2. (5 points each) a) What's the minimum number of people that must be chosen to be sure that at least 2 have the same first initial? | |
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| b) What's the minimum number of people that must be chosen to be sure that at least 3 have the same birth month and and were born on the same day of the week (Sat, Sun, Mon, etc)? | |
| c) Suppose there are 50 people with ages between 1 and 98 (1 and 98 are allowed). Show | |
| that either there are 2 people with the same age or two whose ages are consecutive integers. | |





4. (5 points each) a) Find the number of elements in
$$A_1 \cup A_2 \cup A_3$$
 if there are 100 elements in each set and there are 25 common elements in each pair and 10 elements in the intersection of all 3 sets.

b) Suppose an experiment consists of choosing one of the elements of $A_1 \cup A_2 \cup A_3$ at random with equal probability (where these three sets have the number of elements and intersections as given in part a). Are the events A_1 and A_2 independent?

| 5. (5 points each) The dice in this question are standard six-sided fair dice, and rolling a number with more than one die refers to the sum of the numbers showing on the dice. a) What is the probability of rolling a 5 with 2 dice? |
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| b) What is the probability of rolling a 5 with 3 dice? |
| b) what is the probability of folling a 5 with 5 dice: |
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| 6. (7 points) What is the expected sum that appears on 2 dice, where each of the dice is biased so that a 3 appears with probability .3 and the other 5 numbers all have equal probability? (For this problem, do the arithmetic to determine the expected value as a number). |
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| 7. a) (8 points) Find a recurrence relation for the number of binary strings of length n that contain 2 consecutive 0's. Also, give the initial conditions for this recurrence relation. |
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| b) (5 points) How many strings of length $n=6$ have 2 consecutive 0's? |
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