

CS280 Fall 2001 Prelim 2

Full name:

Student ID:

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Statement of integrity: I did not, and will not, break the rules of academic integrity on this exam.

(Signature)

You must **show your work** and/or give clear explanations or reasons for each answer you give. If you use a formula, show all parts of the formula before simplifying. Correct answers without explanation will be worth 0. Also, please indicate clearly your final answer (e.g. by drawing a box around it or similarly highlighting it). Please leave answers such as $C(4, 2)7^3$ unevaluated (i.e., you don't need to expand or calculate the final number).

1. (5 points each) The English alphabet contains 5 vowels and 21 consonants.

a) How many strings of 6 lowercase English letters contain exactly 2 vowels if each letter may be used as often as you like?

b) How many strings of 6 lowercase English letters contain exactly 2 vowels if no letter is allowed to be used more than once?

2. (5 points each) a) What's the minimum number of people that must be chosen to be sure that at least 2 have the same first initial?

b) What's the minimum number of people that must be chosen to be sure that at least 3 have the same birth month and were born on the same day of the week (Sat, Sun, Mon, etc)?

c) Suppose there are 50 people with ages between 1 and 98 (1 and 98 are allowed). Show that either there are 2 people with the same age or two whose ages are consecutive integers.

3. (5 points each) a) How many bit strings contain exactly 6 0's and 9 1's if every 0 must be immediately followed by a 1?

b) How many solutions are there to the equation $x_1 + x_2 + x_3 + x_4 = 35$, if each x_j is a positive integer (i.e., 1 or bigger)?

4. (5 points each) a) Find the number of elements in $A_1 \cup A_2 \cup A_3$ if there are 100 elements in each set and there are 25 common elements in each pair and 10 elements in the intersection of all 3 sets.

b) Suppose an experiment consists of choosing one of the elements of $A_1 \cup A_2 \cup A_3$ at random with equal probability (where these three sets have the number of elements and intersections as given in part a). Are the events A_1 and A_2 independent?

5. (5 points each) The dice in this question are standard six-sided fair dice, and rolling a number with more than one die refers to the sum of the numbers showing on the dice.

a) What is the probability of rolling a 5 with 2 dice?

b) What is the probability of rolling a 5 with 3 dice?

6. (7 points) What is the expected sum that appears on 2 dice, where each of the dice is biased so that a 3 appears with probability .3 and the other 5 numbers all have equal probability? (For this problem, do the arithmetic to determine the expected value as a number).

7. a) (8 points) Find a recurrence relation for the number of binary strings of length n that contain 2 consecutive 0's. Also, give the initial conditions for this recurrence relation.

b) (5 points) How many strings of length $n = 6$ have 2 consecutive 0's?