CS 280 – Discrete Structures
Prelim 1 – Grading Guide

1
For parts **a** and **c**, you must have a correct proof and draw the correct conclusion to get all the points.
If you used a truth table, each row that is wrong gets 1 pt off. You get another point off if your conclusion does not follow from your truth table.
Other proofs get partial credit based on how correct they are.
In the future, make sure the columns in your truth table are aligned. Many people lost points because they used different values of p and q from column to column.

For part **b**, you must show at least one counter example or have a correct proof to get all the points.
Showing that the LHS is **syntactically** different from the RHS is insufficient. You must show that they are **logically** different. You'll get 1 or 2 points off based on how correct your answer is.
You don't lose any points if your truth table is wrong as long as the counter example is correct.

2a)
Total points: 4
1 point for saying "true"
3 points for giving an example of the proposition, proving that it is indeed true. If you gave some other reasoning that was incorrect, you received 0 or 1 point here.
Most people got this question right.

2b)
Total points: 4
1 point for saying "false"
3 points for your reason. Note that here an example would not work as a reason because the proposition has both the ∃ and ∀ quantifiers. Your reason needs to take into account the ∃ and ∀ quantifiers. If your explanation made sense but did not take these into account, you received just 1 point here. Many people said something like "whatever the value of x and y, there is some real number z between x and y". These people also received just 1 point for this. (If your graded solution has two of the quantifiers circled, this is the mistake you made.) This statement doesn't really take the quantifiers into account. The quantifiers say that the z value needs to satisfy ALL values of x and y. Clearly this is not true, since x or y can "catch up" to z, whatever z's value. But since z has only 1 value, it cannot "catch up" to all values of x or y.

2c)
Total points: 4
1 point for saying "true"

3 points for your reason. Many people considered one side of the OR for negative numbers and the other side for positive numbers, hence reaching the answer “false”. You received just 1 point if you did this. (If your graded solution has the OR circled, this is the mistake you made.) You need to consider *both* sides of the OR for *each* value of x. You will see that for each value of x, either the left hand side of the OR is true, or the right hand side is true, hence making the proposition true.

3a) if any of the subsets were missed, 1 point was subtracted.
Everybody got at least 1 point.

Some mistake:
a)did not notice {{0}}.
b)thought that {0} and 0 were the same and ommited one of the two.

Most people got all of the points on this one

3b
at least one point was given for something that was legeable.
Examples received no credit.
Venn diagrams alone also received no credit.

x-element in the new set.
Most common approach:
1) x in A x not in C (1pt)
2) x in C x not in B (1pt)
3) x in C, x not in C (1pt)
4) no such x (1pt)

Second most common approach:
1)A-C = A intersect ~C --(1pt)
2)C-B = C intersect ~B /
3)their intersection is A intersect ~C intersect C intersect ~B (1pt)
4)A intersect EMPTYSET intersect ~B (1pt)
5)EMPTYSET (1pt)

Arguments stating
C-B is a subset of C and no Subset of C is in A-C as no
elements of C are in A-C
were also good.

Some mistakes:
1) x is not in either A, B, nor C
(even if it were true, it doesn't follow that x is nowhere)

2) saying one of the given sets could not exist.

Most people got all of the points on this one

4:

The most common error was in some way to assume that f was onto as well as being one-to-one, often by assuming that f^{-1} is well-defined. This assumption was deducted 4 points. Other common problems were assuming that the sets were finite, which received –2, making incorrect logical deductions, which were either –2 or –3 depending on the specific case, and giving an incorrect definition of injective, which was –2.

5. (5 points per question)

(a) 5 points are given for the correct answer, 20100.
Un-computed correct answers, such as ½ x 200 x 201 were also given full credit.

If the answer was incorrect...

... but the formula used was correct (formula for computing the sum of an arithmetic progression), 3 points were awarded.

... but the student attempts to sum the series by breaking the series up into different component parts, (e.g. sum from 1 to 10, 11 to 20,..., 91 to 100) and the sum of each individual parts were correct, 3 points were awarded.

(b) 5 points are given for the correct answer, $-\frac{1}{4}(3^{27} - 3)$.
The computed version, –1 906 399 371 246, was also given full credit.
The computed version, given to 4 significant figures and above, i.e. 1.906×10^{12}, was also awarded full credit if the working was correct. However, if the working was wrong, up to 2 marks were deducted (see below for further details). If the answer was 1.906×10^{12}, but the working leading to it was $-\frac{1}{4} \times 3^{27}$, for instance, 2 marks will be taken off.

If the answer was correct...

... simply because all the terms were individually computed and re-listed ($3 - 9 + 27 - 81 + \ldots - 3^{26}$), 2 points were awarded

... but the terms were re-listed and computed in pairs $(3, -9), (27, -81), \ldots (3^{25}, -3^{26})$, and eventually written as the sum of $6 \times (-3)^{2n}$ (but not computed) for n = 0 to 12, 3 points were awarded.

... without enlisting the help of the G.P. formula (i.e. the terms were individually computed and summed using a calculator), full credit was given. Even though this is highly tedious and definitely not the efficient way to do it, we feel that anyone who does so deserves marks for being extremely patient.

If the answer was incorrect...

... but the formula used was correct (formula for computing the sum of an geometric progression), 3 points were awarded.

... but the formula used was correct (formula for computing the sum of an geometric progression) and it was an "off-by-1" error, i.e. $-\frac{1}{4}(3^{26} - 3)$ or $-\frac{1}{4}(3^{28} - 3)$, 4 points were awarded.

... but the formula used was *almost* correct (formula for computing the sum of an geometric progression to infinity, for instance), 2 points were awarded.

... but the terms were re-listed and separated into positive terms (3, 27, 243, ...) and negative terms (9, 81, 729, ...), and summed up individually (but incorrectly), 3 points were awarded.

6a) For each part, one mark was given for identifying the existence or non-existence of a solution. Another mark was awarded for the explanations.
A few had problems with 6aii) which requires one to find a value for x. A quick way to check whether your answer is correct is to plug it into the original modular equation and see if it works.

b) 5 marks for correct answer with explanation, credit was given to relevant workings.
A few people misinterpreted the question. The number given was the product of the two integers and not one of the integers. Otherwise, the question is relatively straightforward.

7.

Total Points: 10
Induction Hypothesis: 2 points. 1 point deducted for minor errors in the statement of the hypothesis.
Base Case: 2 points.
Induction Step: 6 points. 3 points deducted for statements of the form 5 divides 11^k-1 without any proof. Similarly stating that any positive integer power of 11 ends with a 1 and so subtracting 6 must make the result divisible by 5 is not sufficient. 1 or 2 points deducted for computational errors depending on the severity of the error.