**CS280 Fall 2001 Prelim 1**

Full name: Student ID:

Statement of integrity: I did not, and will not, break the rules of academic integrity on this exam.

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(Signature)

You must **show your work** and/or give clear explanations or reasons for each answer you give. If you use a formula, show all parts of the formula before simplifying. Correct answers without explanation will be worth 0. Also, please indicate clearly your final answer (e.g. by drawing a box around it or similarly highlighting it).

1. (4 points each) Determine which of the following propositions are tautologies:
   a) $(p \rightarrow q) \leftrightarrow (\neg q \rightarrow \neg p)$

   b) $((p \rightarrow q) \rightarrow r) \leftrightarrow (p \rightarrow (q \rightarrow r))$ (Pay attention to the parentheses!)

   c) $(s \rightarrow p) \vee (\neg p \wedge s)$

2. (4 points each) Determine whether each of the following statements is true or false. The domain of discourse in each case is the real numbers.

a) $\exists x \exists y((x > y) \rightarrow (x^2 < y^2))$

b) $\forall x \exists z \forall y((x < y) \rightarrow ((z > x) \wedge (z < y)))$

c) $\forall x(((x > 0) \rightarrow (x^3 < 0)) \vee ((x^3 < 0) \rightarrow (x > 0)))$

3. (5 points each)

a) Let $A = \{\emptyset, \{\emptyset\}\}$. Find $P(A)$, the power set of $A$.

b) Show that $(A - C) \cap (C - B) = \emptyset$. You must give a logical argument, not just show Venn diagrams.

4. (10 points) Let $X$ and $Y$ be sets and suppose there is an injection (1-to-1 function) $f : X \to Y$ (but $f$ is not necessarily onto). Prove that there is a surjection (onto function) $g : Y \to X$ (i.e, for each $x \in X$, there exists $y \in Y$ so that $g(y) = x$).

5. (5 points each)
a) Find $\sum_{n=1}^{200} n$

b) Find $\sum_{n=0}^{25} 3(-3)^n$

6. a) (2 points each) Which of the following have solutions? Explain why or why not. If there is a solution, use the Euclidean algorithm to find it.
i) $8x \equiv 1 \bmod 12$

ii) $7x \equiv 1 \bmod 30$

iii) $100x \equiv 1 \bmod 102$

b) (5 points) If the product of two integers is $2^7 3^8 5^2 7^{11}$ and their greatest common divisor is $2^3 3^4 5$, what is their least common multiple? Explain.

7. (10 points) Prove by induction that 5 divides $11^n - 6$ for all positive integers $n$.