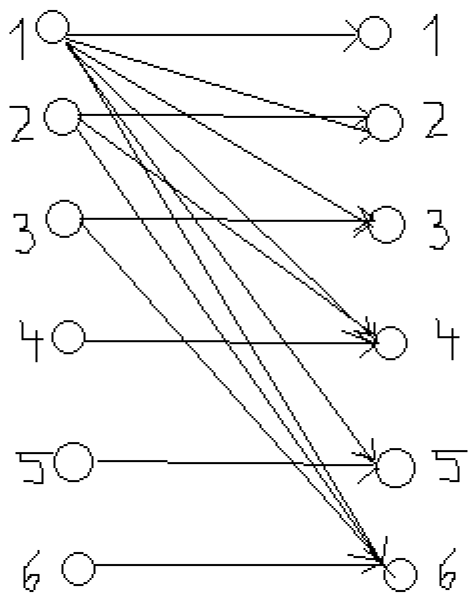
Sec 6.1

2) a) { (1,1), (1,2), (1,3), (1,4), (1,5), (1,6), (2,2), (2,4), (2,6), (3,3), (3,6), (4,4), (5,5), (6,6) }

2) b and c)



R	1	2	3	4	5	6
1	x	x	x	x	x	x
2		x		x		x
3			x			x
4				x		
5					x	
6						x

14) a) R^{-1} = { (b,a) | (a,b) ∈ R}. Then it's clear that R^{-1} = { (a,b) | a > b}.
14) b) The complement of R = { (a,b) | a ≥ b}

20) Here, both R and S are on the set {1,2,3,4}. Thus, if (a,b) ∈ S·R, then there exists a c such that (a,c) ∈ R and (c, b) ∈ S. Thus, S·R = { (1,1), (1,2), (2,1), (2,2) }

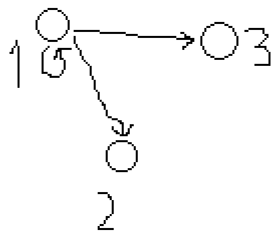
28) a) R ∪ S is reflexive. Proof: ∀a, (a,a) ∈ S, and (a,a) ∈ R. Since R ∪ S contains all ordered pairs that are in R and all ordered pairs that are in S, we see that (a,a) ∈ R ∪ S, hence R ∪ S is reflexive.
28) b) R ∩ S is reflexive. Proof: ∀a, (a,a) ∈ S, and (a,a) ∈ R. Since R ∩ S contains all ordered pairs that are in both R and S, we see that (a,a) ∈ R ∩ S, hence R ∩ S is reflexive.
28) e) S·R is reflexive. Proof: ∀a, (a,a) ∈ S, and (a,a) ∈ R. Since S·R contains all ordered pairs of the form (a,c) where (a,b) ∈ R and (b,c) ∈ S for some element b, we see that (a,a) ∈ S·R, hence S·R is reflexive.

Sec 6.3

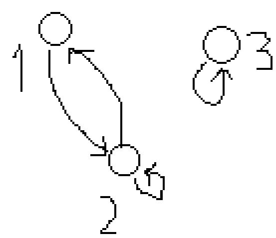
4) a) Symmetric since the matrix is symmetric, reflexive since all entries on the diagonal are present, and transitive because the only elements of the relation are (2,2) plus the full 2 by 2 relation on the elements {1,3}.

4) b) Antisymmetric since each 1 above the diagonal has a symmetric 0 below the diagonal and each 1 below the diagonal has a symmetric 0 above the diagonal, and transitive because the only elements are (1,2), (2,2), and (3,2) and the only way to combine these as in the definition of transitive is by combining (x, 2) with (2, 2), but then (x, 2) is in the relation, so the relation is transitive. Since some but not all of the elements on the diagonal are 1's, this relation is neither reflexive nor irreflexive.
4) c) Symmetric since the matrix is symmetric, not transitive since (2, 1) and (1, 2) are present but (2,2) is not, and neither reflexive nor irreflexive for the same reason as part (b).

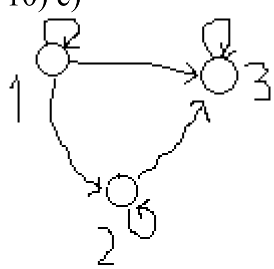
10) a)



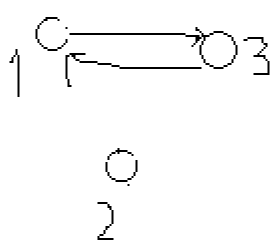
10) b)



10) c)



10) d)



Sec 6.4

10) Let S be the desired relation. Since $(a,b) \in R \Rightarrow (b,a) \in S$, then S must contain $\{(a,b) | a \neq b\}$. To add reflexivity, we add all pairs (a,a). Thus, S = R X R.

26) a) {(a,c), (b,d), (c,a), (d,b), (e,d)} goes to {(a,c), (b,d), (c,a), (d,b), (e,d), (a,a), (b,b), (c,c), (d,d), (e,b)} which goes to {(a,a), (a,c), (b,b), (b,d), (c,a), (c,c), (d,b), (d,d), (e,b), (e,d)}. All later iterations of the for loop lead this result.

26) d) {(a,e), (b,a), (b,d), (c,d), (d,a), (d,c), (e,a), (e,b), (e,c), (e,e)} goes to {(a,a), (a,b), (a,c), (a,e), (b,a), (b,c), (b,d), (b,e), (c,a), (c,c), (c,d), (d,a), (d,c), (d,d), (d,e), (e,a), (e,b), (e,c), (e,d), (e,e)} which goes to {(a,a), (a,b), (a,c), (a,d), (a,e), (b,a), (b,b), (b,c), (b,d), (b,e), (c,a), (c,b), (c,c), (c,d), (c,e), (d,a), (d,b), (d,c), (d,d), (d,e), (e,a), (e,b), (e,c), (e,d), (e,e)} = {a,b,c,d,e} X {a,b,c,d,e}.

Sec 6.5

10) We need to show that it is transitive, symmetric, and reflexive.
Transitive: Need $((a,b),(c,d)) \in R$ and $((c,d), (e,f)) \in R \Rightarrow ((a,b), (e,f)) \in R$. So assume $((a,b),(c,d)) \in R$ and $((c,d), (e,f)) \in R$. Then ad = bc and cf = de. Need af = be. Then:

$c = (ad) / b$
$adf / b = de$
$adf / d = be$
$af = be$.

Symmetric: Need $((a,b),(c,d)) \in R \Rightarrow ((c,d),(a,b)) \in R$. Assume $((a,b),(c,d)) \in R$. Then ad = bc, and we need bc = ad. Of course, we are done.
Reflexive: Need $((a,b),(a,b)) \in R$. That is, we need to show that ab = ab, which is clearly true.

16) a) {0}, {1}, {2}, {3}
16) b) Not an equivalence relation.
16) c) {0}, {1,2}, {3}
16) d) Not an equivalence relation.
16) e) Not an equivalence relation.

24) a) $(c,d) \in [(1,2)] \Leftrightarrow ((1,2),(c,d)) \in R \Leftrightarrow d = 2c$. Then [(1,2)] = { (c,d) | d = 2c }.
24) b) The equivalence class of (a,b) is the set of (x, y) so that x/y = a/b. I.e., this is a way of equating fractions.

32) a) First, let's figure out how the letters correspond to operations.
If R B W is an initial coloring, after a rotation it will be W R B or B W R
If R B W is an initial coloring, after a reflection it will be R W B, W B R, or B R W.
We need to show that R is reflexive, symmetric, and transitive.
Reflexive: Need to show that $(B_1, B_1) \in R$. Rotate B_1 three times to obtain B_1.
Symmetric: Need to show that $(B_1, B_2) \in R \Rightarrow (B_2, B_1) \in R$. Assume $(B_1, B_2) \in R$. Then B_2 was obtained from B_1 by rotations and reflections. Let rot1, rot2, rot3, …, ref1, ref2,

… be the sequence of operations needed to get B_2. Take B_2. For each rot_i, rotate it in the opposite direction. For each ref_i, reflect it in the same way. We get B_1.
Transitive: Need to show that $(B_1, B_2) \in R$ and $(B_2, B_3) \in R \Rightarrow (B_1, B_3) \in R$. Then there was a sequence of rotations and reflections to get B_2 from B_1, and B_3 from B_2. Repeat the rotations from B_1 to B_2. Now take into account the reflections. They will either have the beads in the same order as before (that is, if we started with ABC, the same sequence will be ABC, CAB, and BCA), or inverse (that is, if we started with ABC, the same sequence will be CBA, ACB, BAC). If it is the same, repeat the rotations from B_2 to B_3. Otherwise, do them in the opposite direction. Finally, repeat the reflections, first from B_1 to B_2, then from B_2 to B_3. We get B_3 from B_1.
32) b) We observe that given a coloring and a configuration with the same colors, we can obtain any other configuration of those colors. Proof:
Let ABC be the initial colors. We know there are 3! = 6 possible permutations of these colors. We have: ABC rotates to CAB which rotates to BCA which rotates to ABC. This covers 3 permutations.
Now if we reflect each of those by switching the last two colors, we have the other 3 permutations.
Note: this proof would imply part a.
The equivalence classes thus are sets with each element having the same colors.
Let's count them: we can have all three beads the same color, two beads the same color, or no two beads the same color. Thus, the number of equivalence classes is
$C(3,1) + C(3,1) * C(2,1) + C(3,3) = 3 + 6 + 1 = 10$. The classes are as described above.