CS 280 – Discrete Structures
Solutions to Assignment 1

1.1, 12)
No. Rephrasing the given information, "The barber shaves Person X if and only if Person X does not shave himself." In other words, both of the following are true:

If the barber shaves X, X does not shave himself.
If X does not shave himself, the barber shaves X.

But what if barber = X? Then the first of these statements would be:

If the barber shaves the barber, the barber does not shave himself.

This is clearly false, so there cannot be a barber that satisfies the given information.

1.1, 14a)
For cannibals that always tell the truth:
Question: Are you a liar?
Answer: No (he is telling the truth by saying that he always tells the truth).

For cannibals that always lie:
Question: Are you a liar?
Answer: No (he is lying by saying that he is not a liar).

Since all cannibals would have the same answer to this question, the explorer cannot differentiate between the truthful and lying cannibals, so this question does not work.

1.1, 14b)
Any question for which the explorer already knows the answer will work. For example, the question, "Are you a cannibal?" will cause the lying cannibals to say "no" and the truthful cannibals to say "yes", thus enabling the explorer to differentiate between them.

An alternative question is "If I asked you if you are a liar, would you say yes?" If the cannibal is a truth-teller, then his answer to the question in part (a) would be "no," and since he tells the truth, he would respond to this new question with "no" (note that we didn't actually have to ask the question in part (a), we're just reasoning about what would happen if we did ask that question). If the cannibal is a liar, then his answer in part (a) would be "no," but since he will lie about that also, he will respond to the new question with "yes." A variation is "If I asked you if you are a liar, what would you say?"

1.1, 28a) 2

1.1, 28b) 1

1.1, 28c) 2

1.1, 28d) 1 (both conditions are true, and *true XOR true* = false, so the *then* condition does not get executed)

1.1, 28e) 2

1.1, 42a)
If Alice is telling the truth, then Carlos did it, but then Diana is also telling the truth.
So Alice must be lying, and so Carlos is innocent.

If John is telling the truth, then John is innocent, but Carlos's and Diana's statements contradict each other, so one of them is true and the other is false. So in this case, John is not the only one telling the truth.
Therefore John must be lying. This implies that John is guilty.

Answer: John did it.

1.2, 18)

You may use a truth table to show equivalence, but it's good to also be able to do this without the use of truth tables, as follows:

(p XOR q) is false when:
both variables are true or
both variables are false

So ~(p XOR q) is true when:
both variables are true or
both variables are false
This is the definition of (p←→q).

Therefore, both expressions are logically equivalent.

1.2, 8a)

p	q	p OR q	~p	~p AND (p OR q)	(~p AND (p OR q)) → q
0	0	0	1	0	1
0	1	1	1	1	1
1	0	1	0	0	1
1	1	1	0	0	1

Since the final expression evaluates to 1 for all values of p and q, the expression is a tautology.

1.2, 8b)

a, b, and c are new variables introduced below for simplicity.

p	q	r	p → q (a)	q → r (b)	p → r (c)	a AND b	(a AND b) → c
0	0	0	1	1	1	1	1
0	0	1	1	1	1	1	1
0	1	0	1	0	1	0	1
0	1	1	1	1	1	1	1
1	0	0	0	1	0	0	1
1	0	1	0	1	1	0	1
1	1	0	1	0	0	0	1
1	1	1	1	1	1	1	1

Since the final expression evaluates to 1 for all values of p, q, and r, the expression is a tautology.

1.2, 24)

p AND q AND ~r

1.2, 34a)

(p NOR p) = ~(p OR p) = ~(p)

1.2, 34b)

(p NOR q) NOR (p NOR q) = ~(p NOR q) {from 1.2, 34a, above}
= ~(~(p OR q)) = (p OR q)

1.2, 34c)

From (a), we see that the ~ operator can be represented by NOR operators alone.
From (b), we see that the OR operator can be represented by NOR operators alone.
Exercise 29 shows that the ~ and the OR operators together form a functionally complete collection of logical operators.

Representing ~ and OR operators with NOR operators alone (in accordance with parts a and b), Exercise 29 shows that the NOR operator forms a functionally complete collection of logical operators.

1.3, 8a) Randy Goldberg is enrolled in CS 252.

1.3, 8b) At least one student is enrolled in Math 695.

1.3, 8c) Carol Sitea is enrolled in at least one class.

1.3, 8d) At least one student is simultaneously enrolled in Math 222 and CS 252.

1.3, 8e) There exist two students such that one of them is enrolled in all the classes that the other is enrolled in.

1.3, 8f) There exist two students that are enrolled in exactly the same classes.

1.3, 14a) ~I(Jerry)

1.3, 14b) ~C(Rachel, Chelsea)

1.3, 14c) $\forall$x (~C(Jan, x) AND ~C(Sharon, x))
Alternative possible interpretation: ~C(Jan, Sharon)

1.3, 14d) $\forall$x ~C(Bob, x)

1.3, 14e) $\forall$x ((x $\neq$ Joseph) $\rightarrow$ C(Sanjay, x))
Another possible interpretation: $\forall$x ((x $\neq$ Joseph) $\leftrightarrow$ C(Sanjay, x))

1.3, 14f) $\exists$x ~I(x)

1.3, 14g) $\exists$x ~I(x)
Alternative: ~$\forall$x I(x)

1.3, 14h) $\exists$x$\forall$y (I(x) AND ((y $\neq$ x) $\rightarrow$ ~I(y)))

1.3, 14i) $\exists$x$\forall$y (~I(x) AND ((y $\neq$ x) $\rightarrow$ I(y))

1.3, 14j) $\forall$x$\exists$y (I(x) $\rightarrow$ ((y $\neq$ x) AND C(x, y)))

1.3, 22a) True (x = square root of 2 = real number)

1.3, 22b) False (x = square root of –1 = complex number)

1.3, 22c) True (squaring any real number gives a real number)

1.3, 22d) False (x = square of y; y = square root of x; if x is negative, y is complex)

1.3, 22e) True (for all values of y, if x = 0 then xy = 0)

1.3, 22f) False (breaking basic rule of arithmetic with real numbers)

1.3, 22g) True (whatever the value of x, if y = 1/x then xy = 1, as long as x is not zero which would cause division by zero in calculation of y)

1.3, 22h) False (in this case x is specified first, in which case there is at most one value of y for which xy = 1)

1.3, 34a) $\forall x\ (P(x) \rightarrow \sim S(x))$

1.3, 34b) $\forall x\ (R(x) \rightarrow S(x))$
Alternative: $\sim\exists x\ (R(x) \text{ AND } \sim S(x))$

1.3, 34c) $\forall x\ (Q(x) \rightarrow P(x))$

1.3, 34d) $\forall x\ (Q(x) \rightarrow \sim R(x))$

1.3, 34e) Yes, (d) can be concluded from (a), (b), and (c) as follows:
All my poultry are ducks (c).
Ducks do not waltz (a).
So all my poultry do not waltz.
All officers are willing to waltz (b).
Since my poultry do not waltz and officers waltz, my poultry cannot be officers.