CS 222: Introduction to Scientific Computing
Spring 2001
Practice Prelim 1

Handed out: Fri., Feb. 23.

This was a timed 80-minutes exam. This exam was closed-book, but a cribsheet (one page) was allowed.

1. [**5 points**] Write down the formula for a Newton-form degree-n polynomial.

2. [**5 points**] Name an end-condition for cubic splines. (Any of the end conditions covered in lecture or the book is acceptable.) Write down (as a formula or equation) what this end-condition means.

3. [**5 points**] Explain how to vectorize the following loops. Here, `v` is a column vector of length `m` and and `w` is a column vector of length `n`.

```
a = zeros(m,n);
for i = 1 : m
  for j = 1 : n
    a(i,j) = v(i)*w(j);
  end
end
```

4. [**10 points**] Write down a single vectorized Matlab statement that implements the m-point Newton-Cotes quadrature rule to integrate $f(x) = \exp(x)$ over the interval $[0, 1]$.

   Your statement may presume that there is already a scalar variable for the number of points named `m` defined. Also, there is already a column vector `w` with m entries defined that holds the correct Newton-Cotes weights $w_0, \ldots, w_{m-1}$ for the interval $[0, 1]$.

5. [**15 points**] Consider solving the linear system

$$\begin{aligned} 10^{-6}x_1 + x_2 &= 3, \\ x_1 - x_2 &= 1. \end{aligned}$$

   using plain Gaussian elimination (no pivoting). By working through the steps of forward and backward substitution, pinpoint the particular operation in the solution procedure in which a serious cancellation error occurs. For your information, the two factors of the matrix are

$$L = \begin{pmatrix} 1 & 0 \\ 10^6 & 1 \end{pmatrix} \text{ and } U = \begin{pmatrix} 10^{-6} & 1 \\ 0 & -1 - 10^6 \end{pmatrix}.$$

   As further possibly helpful information, the solution of $L\mathbf{w} = [3; 1]$ is

$$\mathbf{w} = [3; -3 \cdot 10^6 + 1].$$

6. **[10 points]** Consider a sequence of data points $(x_1, y_1), \ldots, (x_n, y_n)$ such that $x_1 < x_2 < \ldots < x_n$ and $y_1 < y_2 < \ldots < y_n$. These data points appear to be describing an increasing function. Nonetheless, show by means of an explicit example that the polynomial interpolant for this data (a polynomial of degree $n-1$ or less) is not necessarily an increasing function over the whole interval $[x_1, x_n]$. [Hint: n can be fairly small.]

7. **[15 points]** Consider the following interpolation problem. Given two data points (x_1, y_1) and (x_2, y_2) such that $x_1 < x_2$ and a scalar a, find a quadratic function $p(x)$ that interpolates the two data points (i.e., satisfies $p(x_1) = y_1$ and $p(x_2) = y_2$) and satisfies the following additional condition

$$\int_{x_1}^{x_2} p(x)\, dx = a.$$

Write down the linear equations that must hold for the coefficients of p to satisfy these conditions.

8. **[15 points]** Figure out a one-point quadrature rule of the form $Q[f]_0^1 = w_1 f(x_1)$ (i.e, figure out w_1 and x_1) that is exact for all functions of the form $f(x) = p + q\sqrt{x}$ for all choices of p and q integrated over the interval $[0, 1]$. In other words, for all functions of this form, the following identity should hold:

$$\int_0^1 f(x)\, dx = w_1 f(x_1).$$