

Representing Java Values

Recitation 2





Decimal Integers

$$d_{n-1}d_{n-2}\dots d_0$$

$$\sum_{i=0}^{n-1} d_i(10^i)$$

$$131 = (100 * 1) + (10 * 3) + 1$$

$$= (10^2 * 1) + (10^1 * 3) + (10^0 * 1)$$



Binary Integers

$$b_{n-1}b_{n-2}\dots b_0$$

$$\sum_{i=0}^{n-1} b_i(2^i)$$

$$131 = (128 * 1) + (2 * 1) + 1$$

$$= (2^7 * 1) + (2^1 * 1) + (2^0 * 1)$$

$$= 10000011_2$$



Exercise

Convert 7 and 11 to binary



Two's Complement

Definition: It's a signed n-bit representation where the most significant bit stands for -2^{n-1} instead of 2^{n-1}

$$b_{n-1}b_{n-2}\dots b_0$$
$$-b_{n-1}(2^{n-1}) + \sum_{i=0}^{n-2} b_i(2^i)$$



Two's Complement Example

$$11000001_2 = -128 + 64 + 1$$
$$= -63$$

$$11111111_2 = -128 + 64 + 32 + 16 + 8 + 4 + 2 + 1$$
$$= -1$$



Exercise

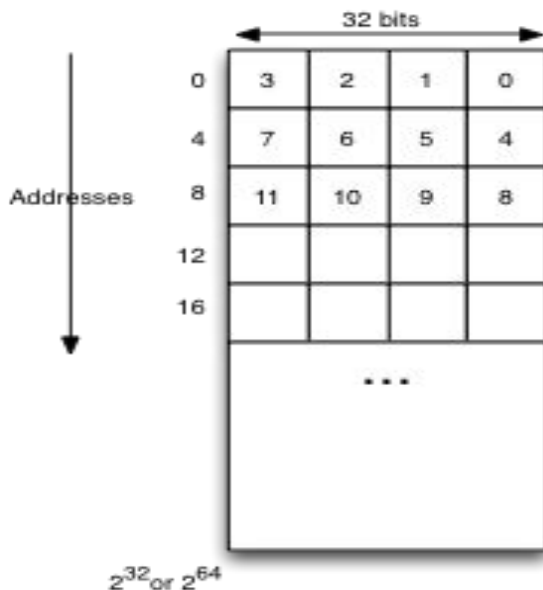
Convert 3 and -3 to Two's complement

(Assume that the numbers are represented using 4 bits)



Memory

- Computer memory is a grid with a bit stored in every cell
- Each address names a group of 8 bits (a **byte**)
- Computer memory can read the four bytes beginning at an address. These four bytes are called a **word**





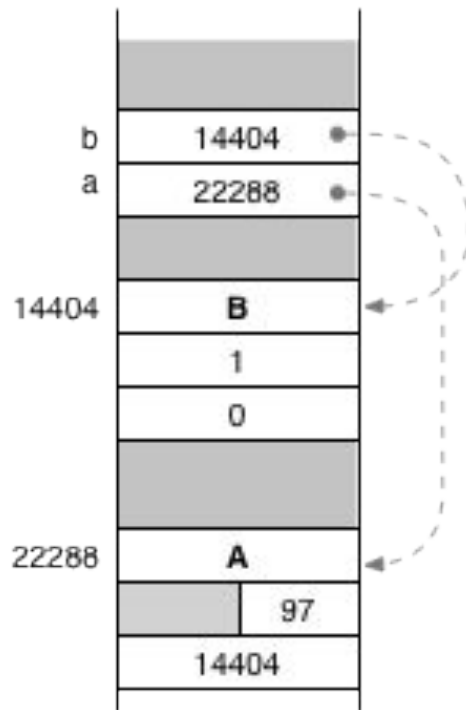
Variables

- Variables are assigned an integer number of words (even if it needs less space)
- `char c = 'a'; long x = 1;`

c	10012			0	97
x	10016	0	0	0	1
	10020	0	0	0	0

Objects

```
class A {  
    char c;  
    B y;  
}  
  
class B {  
    long z;  
}  
  
...  
B b = new B();  
b.z = 1;  
A a = new A();  
a.c = 'a';  
a.y = b;
```





Decimal Scientific Notation

- Consists of an integer n between 0 and 9
- A rational number between 0 and 1
- A factor of 10 raised to an integer power

$$7.234 * 10^{(-5)}$$



Floating Point Representation

Consists of the following components:

- sign (s): 0 or 1
- exponent (exp): integer
- mantissa (m): sequence of binary digits such that $1 \leq 1.m < 2$

$$(-1)^s \times 2^{\text{exp}} \times (1.m)$$



Floating Point Pitfalls!!

- Due to precision related errors check for closeness instead of equality
 - $f1 == f2$
 - $|f1 - f2| < \varepsilon$
- Floating point addition is not commutative
 - $1 + 10^{-40} - 1 \neq 1 - 1 + 10^{-40}$