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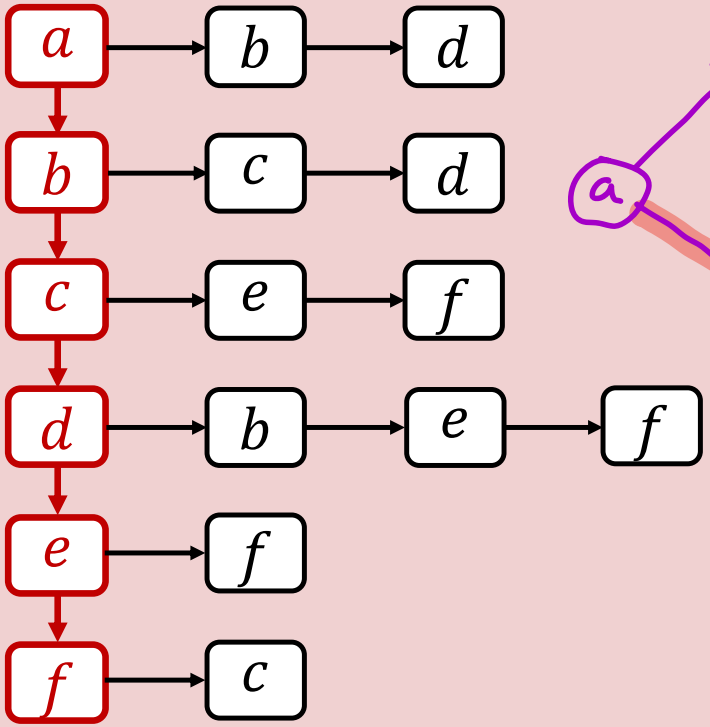
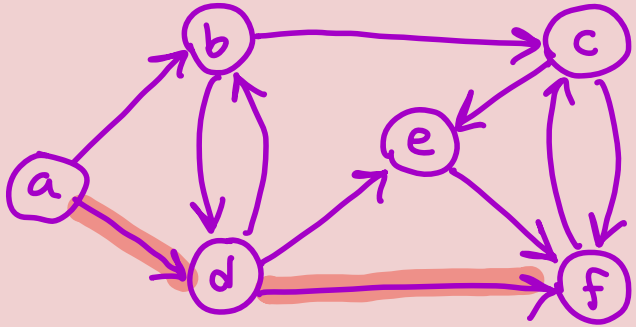
What is the length of the shortest $a \rightsquigarrow f$ path in the graph whose adjacency list representation is below?





1 (A)

2 (B)

3 (C)

4 (D)

Lecture 22: Graph Traversals

CS 2110

November 11, 2025

Today's Learning Outcomes

90. Describe adjacency list and matrix representations of a graph and compare the space/time complexities of operations on each of these representations.

91. Perform BFS and DFS traversals of a given graph by hand and in code.

92. Visualize the state of a graph traversal at a given point of its execution, describing each node as either undiscovered, discovered, visited, and/or settled.

Recall: (Improved) Adjacency List Representation

```
class AdjListGraph implements Graph<...> {
    record AdjListEdge(...) implements Edge<...> { }

    static class AdjListVertex implements Vertex<...> {
        String label;
        HashMap<String, AdjListEdge> outEdges;
    }

    private HashMap<String, AdjListVertex> vertices;

    // methods
}
```

Space Complexity: $O(|V| + |E|)$

$|V|$: vertices map; $|E|$: all outEdges maps

Runtime Complexities:

Check for a vertex: $O(1)$ `vertices.containsKey()`

Check for an edge/neighbor: $O(1)$

check for tail, get tail vertex, check for head

Iterate over all vertices: $O(|V|)$ `vertices.values()`

Iterate over all edges: $O(|V| + |E|) = O(|E|)$

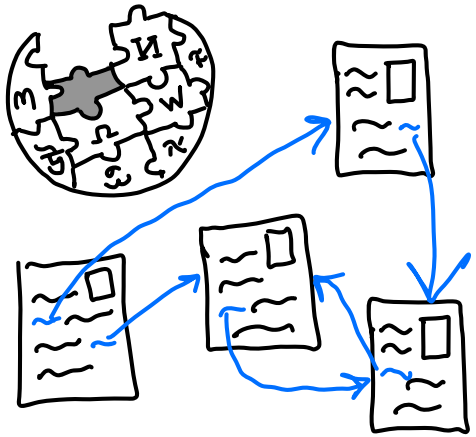
Iterate over neighborhood: $O(|V|)$

Graph Traversals

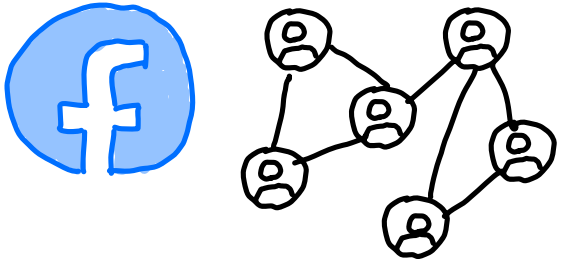
Procedure to systematically visit vertices/edges in a graph in an order that respects "local structure".

- Follow outgoing edge from one vertex to move to its neighbor.

Why? Local structure holds rich information



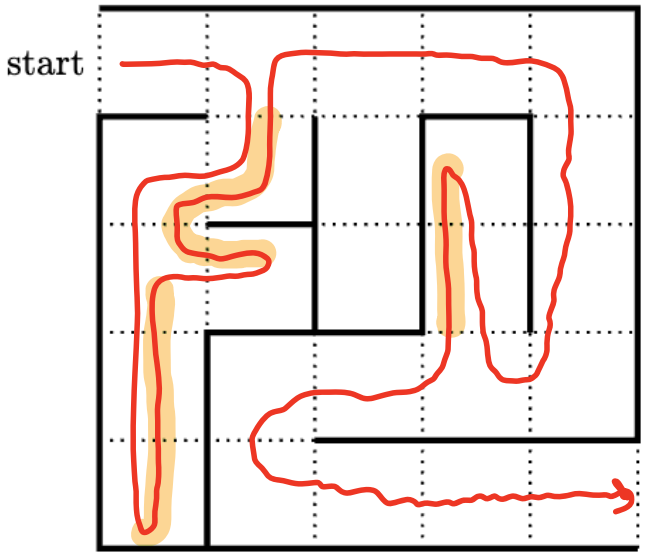
Wikipedia Links



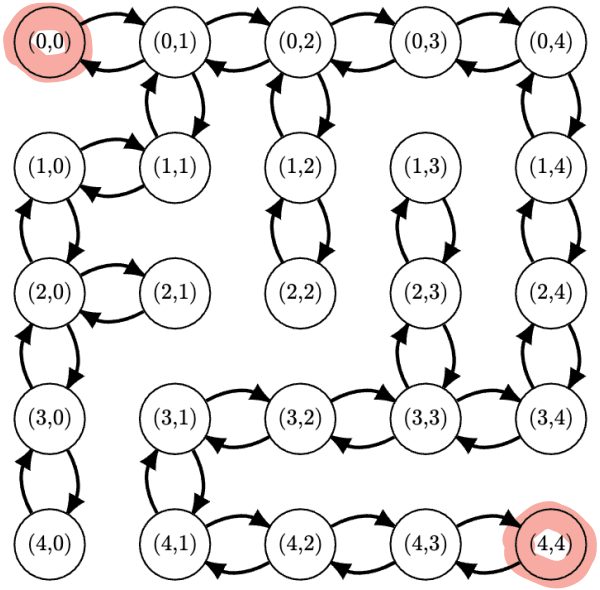
Social Networks

Route Planning

Navigating a Maze



= is there a path from source → dest in this graph



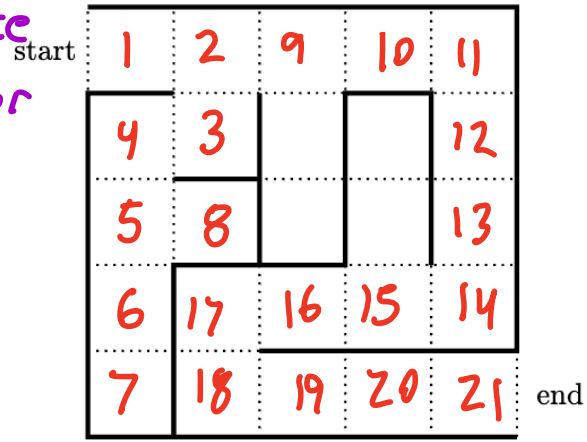
"Are these vertices connected"

Rule: Always in one square, can only see whether it's possible to move to adjacent squares

Idea: "Blindly" follow one path, backtrack if we hit dead end.

Depth-First Search (DFS)

A search procedure that prioritizes following one path as far away from source as possible before backtracking to consider other paths.



- begin at source (current vertex)
- follow an edge to a neighbor
- set off on a DFS from that neighbor
  * Recursion *
- stop if we reach destination * base case *
- unwind (backtrack) if vertex has no neighbors, continue when we find another alternate neighbor to visit

A First Attempt

```
/** Returns whether there is a path from `source`
 *  to `dest` in this graph. */
public static <...> boolean dfs(V source, V dest) {
  if (source == dest) {  return true;  }
  for (E edge : source.outgoingEdges()) {
    if (dfs(edge.head(), dest)) {  return true;  }
  }
  return false;
}
```

base case

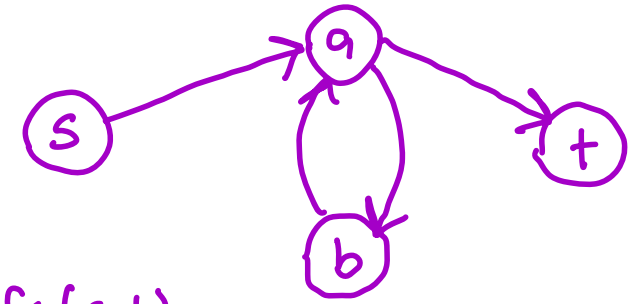
iterate over neighbors

step to neighbor and initiate new search

Need a way to avoid infinite loops

Idea: Keep track of which vertices we've already seen.

What's wrong?



dfs(s,t)

↘

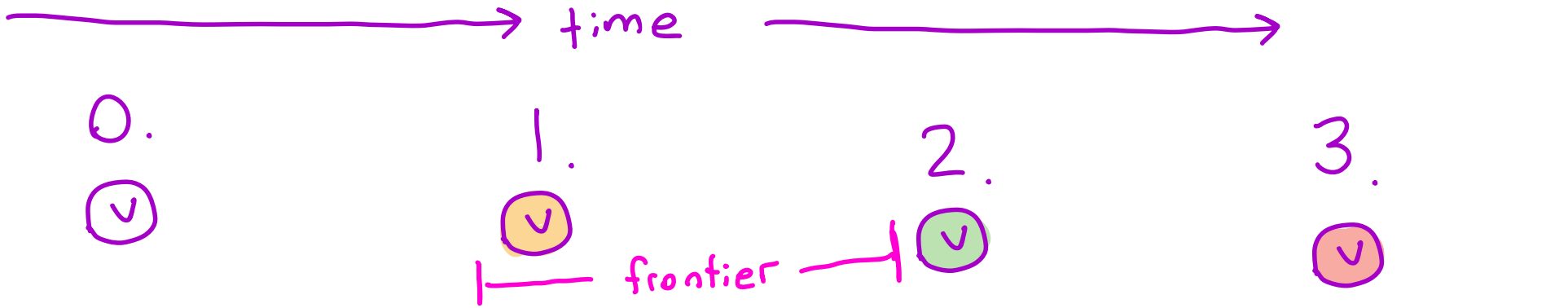
dfs(a,t)

↘ dfs(b,t)

infinite recursion!

↖ ↘ dfs(a,t)

Vertex Status During Search



undiscovered

haven't yet reached vertex in traversal

visiting

follow first incoming edge to vertex

discovered

actively traversing over v's neighbors

settled

finished traversing v's neighbors

In DFS, we want to track which vertices have been discovered

Coding Demo: Fixing our DFS Code

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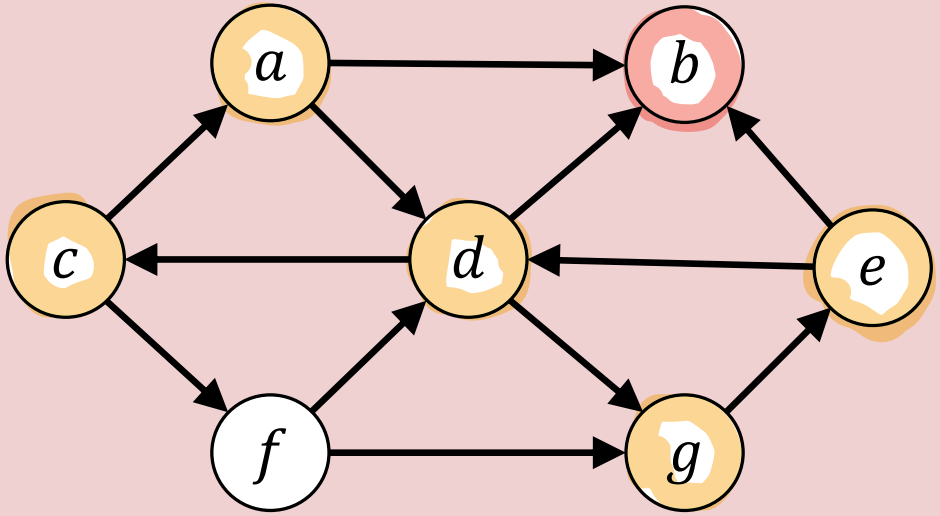
In which order will the vertices be *discovered* during a DFS from c to e of the following graph? Assume neighbors are discovered in alphabetical order.



skipped b

c, a, d, g, e **(A)**

don't rediscover a

c, a, b, a, d, g, e **(B)**

only backtrack to b

c, a, b, f, d, g, e **(C)**

c, a, b, d, g, e **D**

DFS Complexity

```
boolean dfsRec(V current, V dest, Set<String> discovered) {
  if (current == dest) {  return true; }
  for (E edge : current.outgoingEdges()) {
     V neighbor = edge.head();
     if (!discovered.contains(neighbor.label())) {
        discovered.add(neighbor.label());
        if (dfsRec(neighbor, dest, discovered)) {  return true; }
     }
  }
  return false;
}
```

O(1)

O(|V| iterations)

O(1)·O(|E|) = O(|E|)

O(1)·O(|V|) = O(|V|)

O(|V| invocations)
(one per discovered vertex)

per invocation

* Only O(|E|) iterations total across all calls (better bound than O(|V|²)).

O(|E|)

Overall: O(|V| + |E|)
= O(|E|)

great! linear in graph size.

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What is the space complexity of our DFS implementation?

```
boolean dfsRec(V current, V dest, Set<String> discovered) {
  if (current == dest) {  return true; }
  for (E edge : current.outgoingEdges()) {
     V neighbor = edge.head();
     if (!discovered.contains(neighbor.label())) {
        discovered.add(neighbor.label());
        if (dfsRec(neighbor, dest, discovered)) {  return true; }
     }
  }
  return false;
}
```

grows to O(|V|) size

O(1) extra memory per call

O(|V|) # calls / recursive depth

$O(1)$	**(A)**
$O(\|V\|)$	**(B)**
$O(\|E\|)$	**(C)**
$O(\|V\| + \|E\|)$	**(D)**

Search vs. Traversal

In a graph search, we try to locate a path from the source to one particular vertex (and can usually stop when we discover / settle it)

In a graph traversal, we systematically visit every (reachable) vertex in the graph and do something when we get there.

Two natural orders for DFS

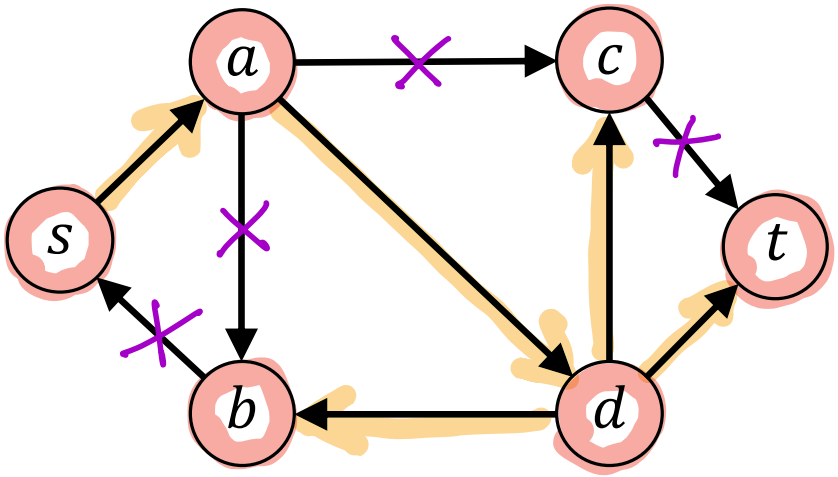
(Discovery)

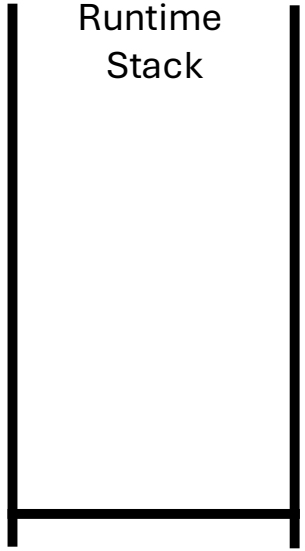
1) Visitation Order ≈ pre-order

2) Settlement Order ≈ post-order

Coding Demo: DFS Traversal Actions

Visitation vs Settlement Orders





Visitation: s a d b t c

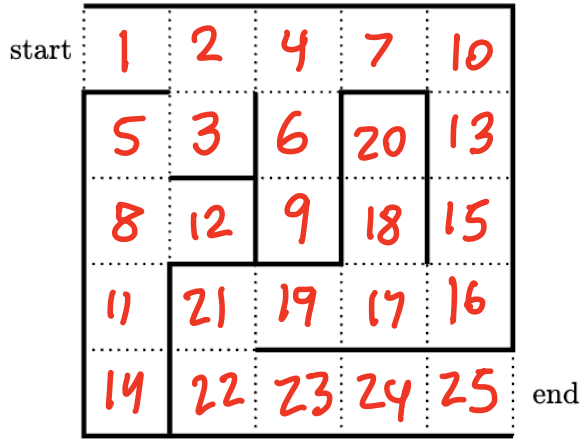
Settlement: b t c d a s

Breadth-First Search

DFS focuses in on one particular path

Alternate approach: Fan out and walk along all paths simultaneously

Breadth-First Search (BFS) strategy



If there's a short path, we're guaranteed to find it fast, but fanning out expensive if we can "guess" correct path and focus on it.

Need a way to organize incoming messages from scouts. Can queue up their opportunities to report back.

Coding Demo: BFS

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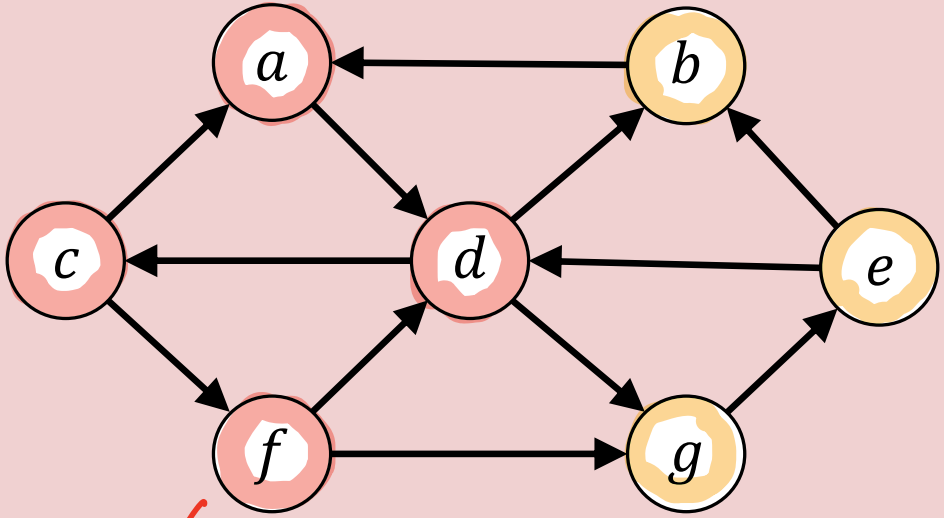
In which order will the vertices be visited in a BFS traversal of the following graph starting from c? Assume neighbors are discovered in alphabetical order.



c d d f g b e

c, a, f, d, g, b, e **(A)**

c, a, d, b, f, g, e **(B)**

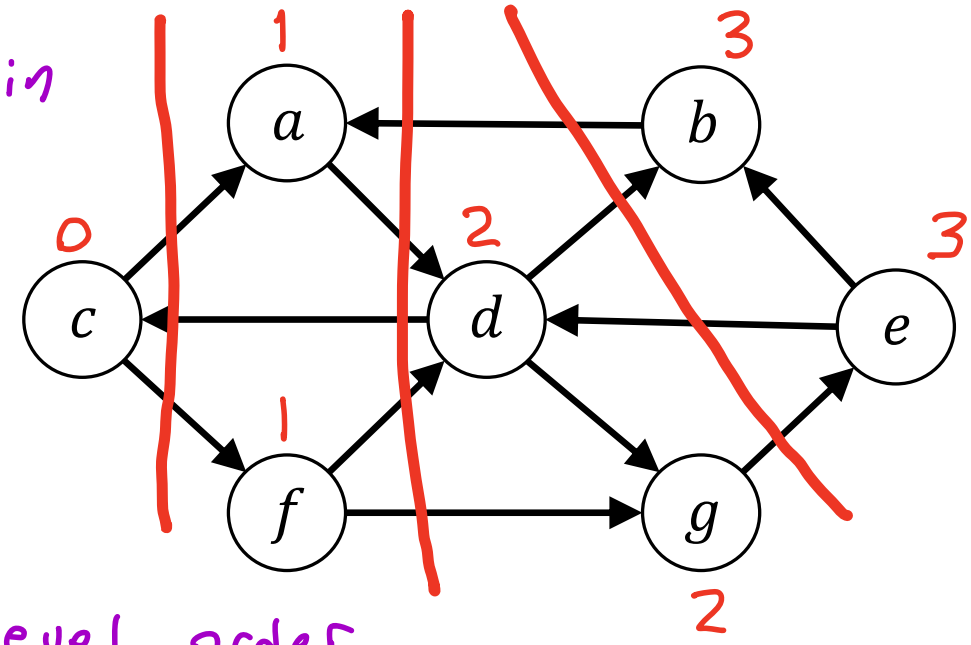
c, a, d, f, g, b, e **(C)**

c, a, f, d, g, e, b **(D)**

Traversal Levels

BFS sweeps across graph in layers (or levels), moving outward from source

level of vertex v = length of shortest s ⇝ v path



Vertices are traversed in level order

We'll take inspiration from this next lecture when we discuss shortest path algorithms.

BFS Complexity

```
while(!frontier.isEmpty()) {
    V v = frontier.remove();
    if (v == dest) { return true; }

    for (E edge : v.outgoingEdges()) {
        V neighbor = edge.head();
        if (!discovered.contains(neighbor.label())) {
            discovered.add(neighbor.label());
            frontier.add(neighbor);
        }
    }
}
return false;
```

O(|V|) iterations
- each vertex added when discovered
one vertex removed per iteration

O(1) · O(|V|)

O(1) · O(|E|) iterations per outer loop iteration
but O(|E|) total iterations

O(1) · O(|E|)

O(1) · O(|V|)

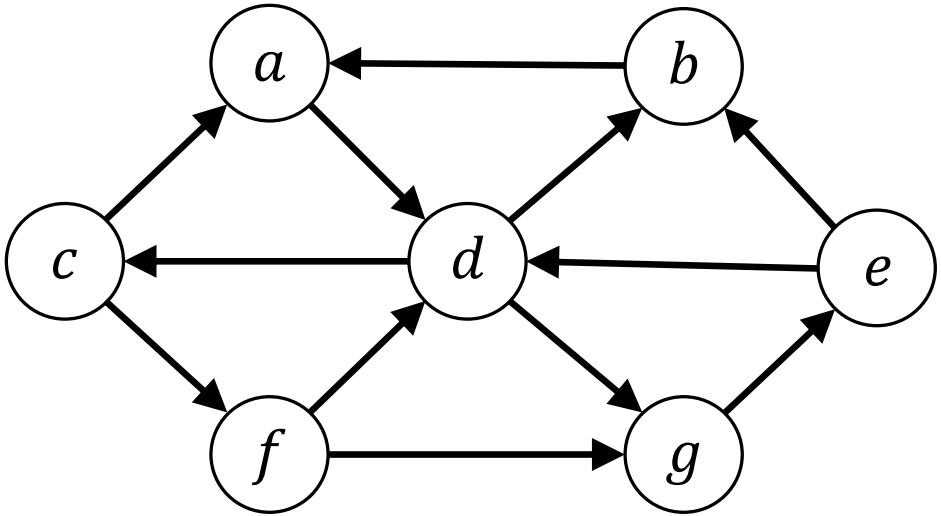
O(1) · O(|V|)

O(|E|)
=
Another O(|V| + |E|) algorithm

Space complexity O(|V|)
discovered + frontier

Setting up Next Lecture

Highlight the edges that discovered new vertices during BFS



What structure do these edges form?

How do these edges help us find paths from the source vertex?