Name: NetID:

Prelim 2 Solution

CS 2110, November 19, 2015, 5:30 PM

	1	2	3	4	5	6	Total
Question	True False	Short Answer	Complexity	Searching Sorting Invariants	Trees	Graphs	
Max	20	15	13	14	17	21	100
Score							
Grader							

The exam is closed book and closed notes. Do not begin until instructed.

You have **90 minutes**. Good luck!

Write your name and Cornell **NetID** at the top of EACH page! There are 6 questions on 8 numbered pages, front and back. Check that you have all the pages. When you hand in your exam, make sure your booklet is still stapled together. If not, please use our stapler to reattach all your pages!

We have scrap paper available. If you do a lot of crossing out and rewriting, you might want to write code on scrap paper first and then copy it to the exam, so that we can make sense of what you handed in.

Write your answers in the space provided. Ambiguous answers will be considered incorrect. You should be able to fit your answers easily into the space provided.

In some places, we have abbreviated or condensed code to reduce the number of pages that must be printed for the exam. In others, code has been obfuscated to make the problem more difficult. This does not mean that its good style.

1. True / False (20 points)

a)	T	F	Calculating the depth of a tree always requires examining all of its nodes.
b)	T	F	Let t be a node of a tree. Determining whether t is a leaf takes constant time.
c)	T	F	Let t be a BST with n values, not necessarily balanced. The worst-case time for inserting a value into t is $O(\log n)$. [Since it is not balanced, all left subtrees could be empty, making it look like a linked list.]
d)	T	F	To obtain the values of a BST in ascending order, use a preorder traversal. [inorder]
e)	T	F	Given only the preorder and inorder traversals of a binary tree, one can construct the tree (uniquely).
f)	T	F	The language of a grammar that contains no recursively defined nonterminals is finite.
g)	T	F	A connected graph with n nodes must have at least n edges. [It must have at least n-1 edges, e.g. a 2-node connected graph needs 1 edge.]
h)	T	F	Topological sort works only if the graph it is processing is acyclic.
i)	T	F	A bipartite graph must have an even number of nodes. [The bipartite graph shown in lectures notes has an odd number of nodes.]
j)	T	F	When using the adjacency matrix representation of a directed graph, to determine whether there exists an edge from node u to node v requires only constant time.
k)	T	F	Breadth-first search can be written recursively or iteratively, but depth-first search can be written only iteratively. [BFS uses a queue, making it unnatural to implement recursively.]
l)	T	F	Dijkstra's shortest-path on a graph with all edge weights being 2 is a depth-first search, and not a breadth-first search. [It's a breadth-first search.]
m)	T	F	In the **best case**, it is not possible to use a comparison based sorting algorithm to sort an array in $O(n)$ time. [Look at slides 31..24 of lecture 10.]
n)	T	F	All `JButtons` in a GUI must be "listened to" with the same `actionPerformed` procedure; it's not possible to have different `actionPerformed` procedures for different buttons. [We showed different listeners in lecture.]
o)	T	F	Our code style guidelines say to declare all local variables used in a method at the beginning of the method, in order to save time allocating and deallocating space for them. [Place them as close to first use as possible.]
p)	T	F	`Integer[]` is not a subclass of `Object[]`, just as `HashSet<Integer>` is not a subclass of `HashSet<Object>`. [Slides 11..13 of lecture 16.]
q)	T	F	Methods in a class `C` with an inner class `IC` can call methods in `IC` even though they are `private`.
r)	T	F	If the value of function `equals(Object)` in a class depends only on fields `b` and `c`, function `hashCode()` in that class has to depend also on only those two fields.
s)	T	F	One can't use a "for-each" statement to process the elements of a `HashSet` because the elements of a `HashSet` are not ordered in any way.
t)	T	F	The purpose of an object of class `Iterator` is to enumerate the values in some collection.

2. Short Answer (15 points)

2.a Hashing (9 points)

Consider implementing a set using hashing with linear probing. Assume an array of 6 elements of class `Integer`, as shown below. We define the hash function $f(i) = (2 * i) \mod 6$, where 6 is the table size. For example, hashing 4 gives 8 mod 6, which is 2.

0	1	2	3	4	5
0 ~~null~~	6 ~~null~~	4 ~~null~~	1 ~~null~~	null	null

(i) 5 points Insert the values 4, 1, 0, 1, and 6 into the set, in that order —write the values in the appropriate element in the table above, crossing off the value currently in that element. Do not re-size the array, even though this is the standard way to implement a hash table.

(ii) 2 points Now remove the value 1 from the set. Do you simply set the array element to `null`? Explain why or why not.

No. We set the value of `isInSet` to `false`. If we simply set the array element to `null`, then the linear probing algorithm might stop searching for an element before it should.

(iii) 2 points Consider the insertion of values in point (i) above. Which insertion (if any) caused the load factor to surpass 0.25?

The second insertion (the first "1" in the list of values above) into the set changed the load factor to 2/6, or .333.

2.b Exception Handling 6 points

(i) 2 points Consider the statement below, appearing in a method `m`, where `b` is an int array. Does its execution result in a `RuntimeException` being thrown out to the call of `m`? Write your answer and an explanation for it to the right of the statement.

```
try {
    int x=  b[-5];
}
catch (Exception e) {
    throw new RuntimeException();
}
```

Yes. The line `throw new RuntimeException();` in the catch block above throws a `RuntimeException` that is not caught in `m`.

(ii) 4 points To the right below, write down what is printed by the `println` statements during execution of the call `mm(1)`, where method `mm()` is defined as follows:

```
public static void mm(int x) {
    try {
       System.out.println("11");
       int b= 5/(x-1);
       System.out.println("12");
       return;
    } catch (RuntimeException e) {
       System.out.println("13");
       int c= 5/(x-1);
       System.out.println("14");
    }
    System.out.println("15");
    int d= 5/x;
    System.out.println("16");
    return;
}
```

11
13 The division in the catch block throws an exception, which is not caught.

3. Complexity (13 points)

(a) 4 points For each of the functions f below, state the function $g(n)$ such that $f(n)$ is $O(g(n))$. $g(n)$ should be as small as possible. (e.g. one could say that $f(n) = 2n^2$ is $O(n^3)$, but the best answer, the one we want, is $O(n^2)$.)

(i) $f(n) = \log(n) + n + n^3$. $g(n) = n^3$

(ii) $f(n) = 2n + \frac{3000}{n} + 42$. $g(n) = n$

(iii) $f(n) = 2^{n+12} + 400n^4$. $g(n) = 2^n$

(iv) $f(n) = 55$ $g(n) = 1$

(b) 3 points State the tightest (smallest) asymptotic time complexity (in terms of n) of the following statement sequence:

```
int s= 0;
for (int k= 0; k < n; k= k+1) {
   for (int j= k+5; j > k; j= j-1) {
       s= s + j*k;
    }
}
```

$O(n)$. The inner loop iterates no more than 5 times, so it is O(1).

(c) 6 points Give a formal proof that $f(n) = 3000 + 2n$ is $O(n)$.

```
        f(n)
  =        <definition of f>
        3000 + 2n
  <=      <for n >= 1>
        3000n + 2n
   =        <arithmetic>
        3002n
So, select c = 3002, N = 1
```

4. Searching, Sorting, and Invariants (14 points)

(a) 6 points Assume that this procedure has already been written:

```
/** Merge sorted segments b[h..j] and b[j+1..k] so that b[h..k] is sorted. */
public static void merge(int[] b, int h, int j, int k) {...}
```

Below, write the body of procedure `MS`, completely in Java.

Solution

```
    /** Sort b[h..k], using the mergesort algorithm. */
    public static void MS(int[] b, int h, int k) {
        if (h >= k) return; // Segments of size <= 1 are in sorted order
        int m = (h + k) / 2;
        MS(b, h, m);
        MS(b, m + 1, k);
        merge(b, h, m, k);
    }
```

(b) 8 points Below is the precondition and postcondition of an algorithm to swap the negative values in $b[h..k]$ into the beginning of $b[h..k]$. Note that h is not necessarily 0 and k is not necessarily `b.length-1`. Do not change h and k. You may assume the following procedure has already been written:

```
/** Swap b[i] and b[j]. */
public static void swap(int[] b, int i, int j) {...}
```

Complete procedure `p` below using a loop that follows the given loop invariant to accomplish this task.

		h k
Precondition:	b	unknown

		h	k
Postcondition:	b	< 0	≥ 0

		h	e	f k
Invariant:	b	unknown	< 0	≥ 0

Solution

```
/** Modify b[h..k] as defined above. */
public static void p(int[] b, int h, int k) {
    int e= k + 1;    OR  int f= k;
    int f= k;        OR  int e= f+1;
    while (e != h) {            // 2 pts for init making inv true
        if (b[e - 1] < 0) {     // 2 pts for inv && !cond implies result
            e= e - 1;           // 2 pts for body making progress
        } else {                // 2 pts for maintaining inv
            swap(b, e - 1, f);  // pts could be deducted for doing
            e= e - 1;           // what should not be done
            f= f - 1;           // e.g. changing h and k, starting a 0
        }
    }
}
```

5. Trees (17 points)

5.a Binary Search Trees (9 points)

Assume that each node of a binary search tree (BST), of class `Node`, has these fields:

- `Node left`: the left subtree (`null` if empty)
- `Node right`: the right subtree (`null` if empty)
- `int data`: the data of the node

Write the body of the following method:
Solution

```
/** Return true if v is in tree t and false otherwise.
  * Takes O(d) time, where d is the maximum depth of t
  * Precondition t is a BST --t being null means an empty tree */
public static boolean contains(Node t, int v) {
    if (t == null) return false;
    if (v == t.data) return true;
    if (v < t.data) return t.left.contains(v);
    return t.right.contains(v);
}
```

Should be contains(t.left, v)

Should be contains(t.right, v)

5.b Heaps (8 points)

Consider writing heapsort to sort an int array b. Complete the implementation of step (2) in the method below. You do not have to concern yourself with the implementation of step (1). In implementing step (2):

- Remember that $b[0]$ is the root of the heap and is the largest value (it is a max-heap).
- Assume that function `int poll(int[] b, int k)` has already been written and can be used. It assumes that $b[0..k-1]$ is a heap, removes the root, does what is necessary to make $b[0..k-2]$ back into a heap, and returns the removed value.

Solution

```
/** Sort array b using heapsort */
public static void heapsort(int[] b) {
   // (1) Make b[0..b.length-1] into a max-heap (largest value ends up in b[0])
   heapify(b);

   // (2) Poll values from heap and put them into their sorted position in b
   for (int i = b.length - 1; i >= 0; i--) {
        b[i]= poll(b, i + 1);
   }
}
```

6. Graphs (21 points)

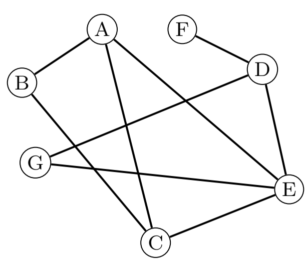
(a) 3 points There are two basic ways to implement a graph: (1) an adjacency matrix and (2) an adjacency list. Let a graph have n nodes and e edges. Below, for each point, state to the right which representation of a graph has that property:

(i) Uses space $O(n+e)$: Adjacency list

(ii) Takes time $O(n)$ to iterate over the edges that leave given node n: Adjacency matrix

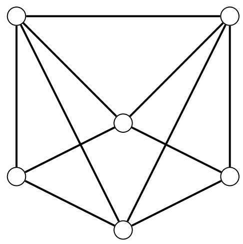
(iii) Takes time $O(1)$ to determine whether there is an edge from node n_1 to node n_2: Adjacency matrix

(b) 5 points For the graph below, give a list of the nodes that are visited by the breadth-first search algorithm starting at node A, in the order visited. Whenever there is a choice of processing nodes in any way, process them in alphabetical order by their names. Example: to do something with nodes G, A, D, first do A, then D, and then G.



A, B, C, E, D, G, F

(c) 3 points Is the following graph a planar graph? Write yes or no to its right.



Yes. Bend each of the two long edges that cross another edge out around the node so it does not cross the other edge.

(d) 3 points State the difference between Kruskal's algorithm and Prim's algorithm for constructing a spanning tree of an undirected graph.
Kruskal's algorithm selects the smallest edge that does not introduce a cycle and adds it to the tree at each iteration. Prim's algorithm starts with a specific node and selects the smallest edge leaving the connected component being built at each iteration.

(e) 7 points Complete recursive algorithm `dfs`, given below. Do not be concerned about how marking occurs. You may simply say "mark n" and "if n is unmarked".
Solution

```
/** Mark all nodes that are reachable along unmarked paths from n.
  * Precondition: n is unmarked. */
public static void dfs(Node n) {
    mark n;
    for (Node v : n.neighbors) {// You could other ways to describe neighbors.
        if (v is unmarked) {    // We were not particular.
            dfs(v);
        }
    }
}
```