TREES II

Lecture 12
CS2110 – Spring 2018

Announcements

2

- Prelim 1 is Tonight, bring your student ID
  - **5:30PM EXAM**
  - OLH155: netids starting aa to dh
  - OLH255: netids starting di to ji
  - PHL101: netids starting jj to ks (Plus students who switched from the 7:30 exam)
  - **7:30PM EXAM (314 Students)**
  - OLH155: netids starting kt to rz
  - OLH255: netids starting sa to wl
  - PHL101: netids starting wm to zz (Plus students who switched from the 5:30 exam)

Comparing Data Structures

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Data Structure	add(val x)	lookup(int i)	search(val x)
Array [2, 1, 3, 0]	$O(n)$	$O(1)$	$O(n)$
Linked List 2→1→3→0	$O(1)$	$O(n)$	$O(n)$
Binary Tree (1; 2, 3)	$O(1)$	$O(n)$	$O(n)$
BST (2; 1, 3)	$O(height)$	$O(height)$	$O(height)$

Binary Search Trees

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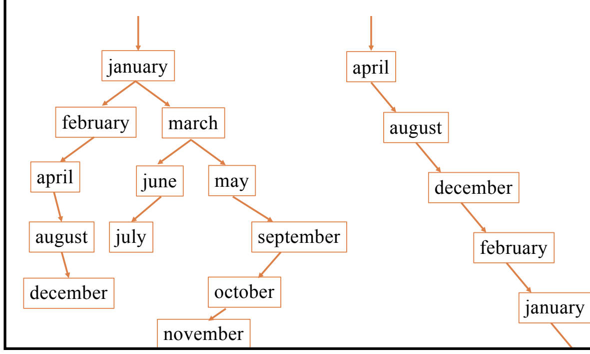

Red-Black Trees

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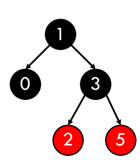
- Self-balancing BST
- Each node has one extra bit of information "color"
- Constraints on how nodes can be colored enforces approximate balance



Red-Black Trees

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1) A red-black tree is a binary search tree.
2) Every node is either red or black.
3) The root is black.
4) If a node is red, then its (non-null) children are black.
5) For each node, every path to a decendant null node contains the same number of black nodes.

RB Tree Quiz

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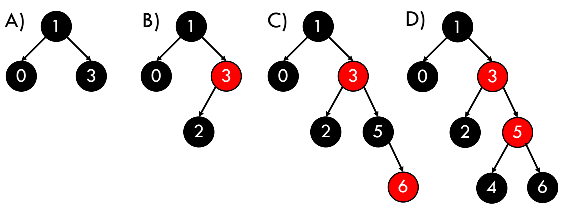
- Which of the following are red-black trees?



YES NO YES NO

Class for a RBNode

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```
class RBNode<T> {
   private T value;
   private Color color;
   private RBNode<T> parent;
   private RBNode<T> left, right;

   /** Constructor: one-node tree with value x */
   public RBNode (T v, Color c) { value= d; color= c; }

   ...
}
```

Null if the node is the root of the tree.

Either might be null if the subtree is empty.

Insert

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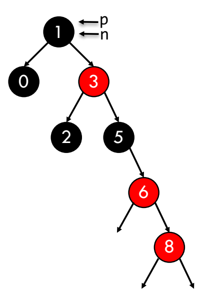
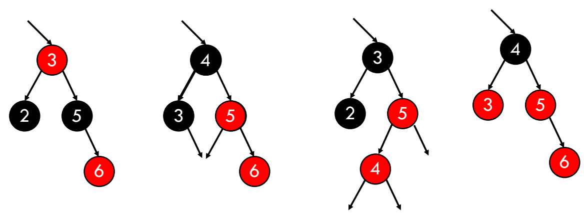
```
Insert(RBTree t, int v){
  Node p;
  Node n= t.root;
  while(n != null){
    p= n;
    if(v < n.value){n= n.left}
    else{n= n.right}
  }
  Node vnode= new Node(v, RED)
  if(p == NULL){
    t.root= vnode;
  } else if(v < p.value){
    p.left= vnode; vnode.parent= p;
  } else{
    p.right= vnode; vnode.parent= p;
  }
  fixTree(t, vnode);
}
```



fixTree

10



Case 1:
parent is black

Case 2:
parent is red
uncle is black
node on outside

Case 3:
parent is red
uncle is black
node on inside

Case 4:
parent is red
uncle is red

Rotations

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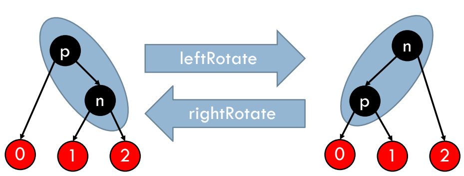

fixTree

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```
fixTree(RBTree t, RBNode n){
  while(n.parent.color == RED){ // not Case 1
    if(n.parent.parent.right == n.parent){
      Node uncle = n.parent.parent.left;
      if(uncle.color == BLACK) { // Case 2 or 3
        if(n.parent.left == n) { rightRotate(n);} //3
        n.parent.color== BLACK;
        n.parent.parent.color= RED;
        leftRotate(n.parent.parent);
      } else { //uncle.color == RED // Case 4
        n.parent.color= BLACK;
        uncle.color= BLACK;
        n.parent.parent.color= RED;
        n= n.parent.parent;
      }
    } else {...} // n.parent.parent.left == n.parent
  }
  t.root.color == BLACK;// fix root
}
```

Search

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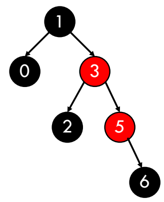
- Red-black trees are a special case of binary search trees
- Search works exactly the same as in any BST
- Time: $O(height)$



What is the max height?

14

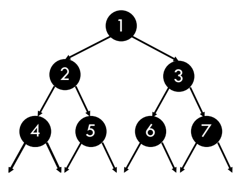
- Observation 1: Every binary tree must have a null node with depth $\leq \log(n+1)$

What is the max height?

15

- Observation 1: Every binary tree must have a null node with depth $\leq \log(n+1)$

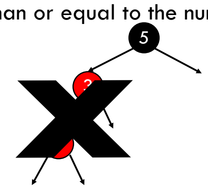
n	log(n+1)
1	1
2	1.584
3	2
4	2.321
5	2.584
6	2.807
7	3
8	3.169
9	3.321
10	3.249



What is the max height?

16

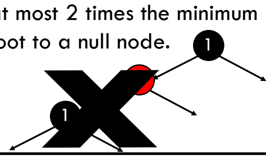
- Observation 1: Every binary tree must have a null node with depth $\leq \log(n+1)$
- Observation 2: In a red-black tree, the number of red nodes in a path from the root to a null node is less than or equal to the number of black nodes.



What is the max height?

17

- Observation 1: Every binary tree must have a null node with depth $\leq \log(n+1)$
- Observation 2: In a red-black tree, the number of red nodes in a path from the root to a null node is less than or equal to the number of black nodes.
- Observation 3: The maximum path length from the root to a null node is at most 2 times the minimum path length from the root to a null node.



What is the max height?

18

- Observation 1: Every binary tree must have a null node with depth $\leq \log(n+1)$
- Observation 2: In a red-black tree, the number of red nodes in a path from the root to a null node is less than or equal to the number of black nodes.
- Observation 3: The maximum path length from the root to a null node is at most 2 times the minimum path length from the root to a null node.

$$h = \max_{root \to null} path\ len \leq 2 \cdot \min_{root \to null} path\ len \leq 2\log(n+1)$$

$$h \text{ is } O(\log n)$$

Comparing Data Structures

19

Data Structure	add(val x)	lookup(int i)	search(val x)
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Linked List 2→1→3→0	$O(1)$	$O(n)$	$O(n)$
Binary Tree 1 2 3	$O(1)$	$O(n)$	$O(n)$
BST 2 1 3	$O(height)$	$O(height)$	$O(height)$
RB Tree 2 1 3	$O(\log n)$	$O(\log n)$	$O(\log n)$

Application of Trees: Syntax Trees

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- Most languages (natural and computer) have a recursive, hierarchical structure
- This structure is *implicit* in ordinary textual representation
- Recursive structure can be made *explicit* by representing sentences in the language as trees: Abstract Syntax Trees (ASTs)
- ASTs are easier to optimize, generate code from, etc. than textual representation
- A parser converts textual representations to AST

Applications of Trees: Syntax Trees

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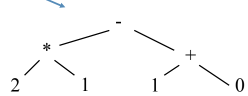
"parsing"

```
2 * 1 – (1 + 0)
```

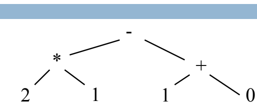
A Java expression as a string.



An expression as a tree.

Pre-order, Post-order, and In-order

22

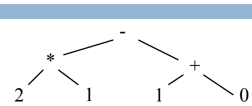


Pre-order traversal:
1. Visit the root
2. Visit the left subtree (in pre-order)
3. Visit the right subtree

- * 2 1 + 1 0

Pre-order, Post-order, and In-order

23



Pre-order traversal **- * 2 1 + 1 0**

Post-order traversal **2 1 * 1 0 + -**
1. Visit the left subtree (in post-order)
2. Visit the right subtree
3. Visit the root

Pre-order, Post-order, and In-order

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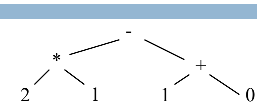


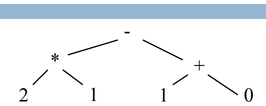
Pre-order traversal **- * 2 1 + 1 0**

Post-order traversal **2 1 * 1 0 + -**

In-order traversal **2 * 1 - 1 + 0**
1. Visit the left subtree (in-order)
2. Visit the root
3. Visit the right subtree

Pre-order, Post-order, and In-order

25



Pre-order traversal	**- * 2 1 + 1 0**
Post-order traversal	**2 1 * 1 0 + -**
In-order traversal	**(2 * 1) - (1 + 0)**

To avoid ambiguity, add parentheses around subtrees that contain operators.

Printing contents of BST (In-Order Traversal)

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Because of ordering rules for a BST, it's easy to print the items in alphabetical order

- Recursively print left subtree
- Print the node
- Recursively print right subtree

```
/** Print BST t in alpha order */
private static void print(TreeNode<T> t) {
   if (t== null) return;
   print(t.left);
   System.out.print(t.value);
   print(t.right);
}
```

In Defense of Postfix Notation

27

- Execute expressions in postfix notation by reading from left to right.
- Numbers: push onto the stack.
- Operators: pop the operands off the stack, do the operation, and push the result onto the stack.

2 1 * 1 0 + -

In Defense of Postfix Notation

28

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1 * 1 0 + -

2

In Defense of Postfix Notation

29

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* 1 0 + -

1
2

In Defense of Postfix Notation

30

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1 0 + -

2

In Defense of Postfix Notation

31

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\+ -

0
1
2

In Defense of Postfix Notation

32

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\-

1
2

In Defense of Postfix Notation

33

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1

In Defense of Postfix Notation

34

- Execute expressions in postfix notation by reading from left to right.
- Numbers: push onto the stack.
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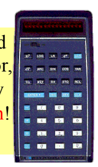
In about 1974, Gries paid $300 for an HP calculator, which had some memory and used postfix notation! Still works.



a.k.a. "reverse Polish notation"

In Defense of Prefix Notation

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- Function calls in most programming languages use prefix notation: like add(37, 5).
- Some languages (Lisp, Scheme, Racket) use prefix notation for *everything* to make the syntax simpler.

```
(define (fib n)
  (if (<= n 2)
      1
      (+ (fib (- n 1) (fib (- n 2)))))
```

Iterator/Iterable

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- There's a pair of Java interfaces designed to make data structures easy to traverse
- You could modify a tree to implement iterable, implement an (in-order, post-order, etc.) iterator and then use a for each loop to traverse the tree!
- In recitation this week, you will modify your linked list from A3 to implement iterable