# TREES

Lecture 12
CS2110 – Spring 2018

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## Important Announcements

2

- A4 is out now and due two weeks from today. Have fun, and start early!

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## Data Structures

3

- There are different ways of storing data, called data structures
- Each data structure has operations that it is good at and operations that it is bad at
- For any application, you want to choose a data structure that is good at the things you do often

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## Example Data Structures

4

| Data Structure | add(val x) | lookup(int i) |
|---|---|---|
| Array [2\|1\|3\|0] | $O(n)$ | $O(1)$ |
| Linked List (2)→(1)→(3)→(0) | $O(1)$ | $O(n)$ |

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## The Problem of Search

5

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## Example Data Structures

6

| Data Structure | add(val x) | lookup(int i) | search(val x) |
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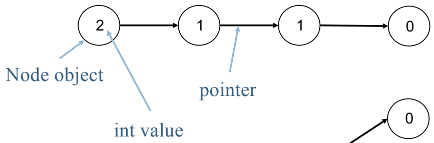
| Array [2\|1\|3\|0] | $O(n)$ | $O(1)$ | $O(n)$ |
| Linked List (2)→(1)→(3)→(0) | $O(1)$ | $O(n)$ | $O(n)$ |

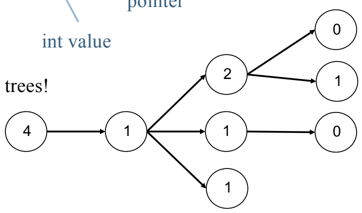
## Tree



Singly linked list:




Today: trees!

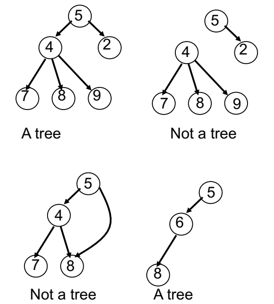



## Tree Overview



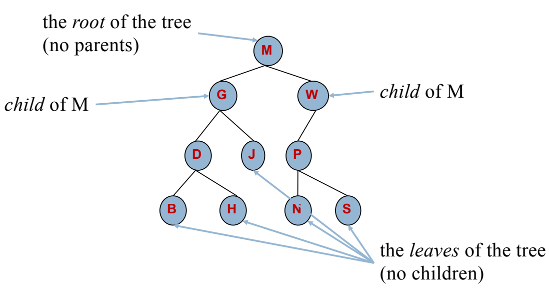
*Tree*: data structure with nodes, similar to linked list

- Each node may have zero or more *successors* (children)
- Each node has exactly one *predecessor* (parent) except the *root*, which has none
- All nodes are reachable from *root*




## Tree Terminology



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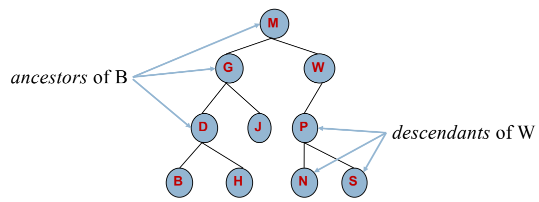
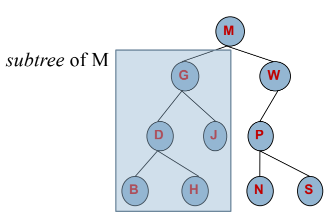



## Tree Terminology

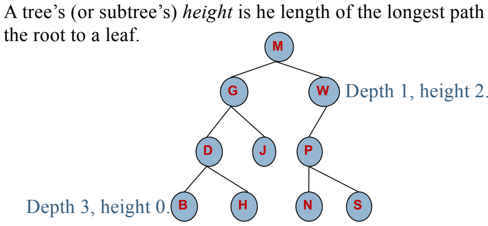





## Tree Terminology



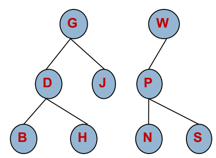
A node's *depth* is the length of the path to the root.
A tree's (or subtree's) *height* is he length of the longest path from the root to a leaf.



## Tree Terminology

13

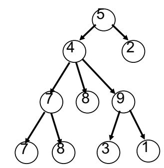
Multiple trees: a *forest*.




## Class for general tree nodes

```
class GTreeNode<T> {
    private T value;
    private List<GTreeNode<T>> children;
    //appropriate constructors, getters,
    //setters, etc.
}
```

Parent contains a list of its children




General tree

## Class for general tree nodes

```
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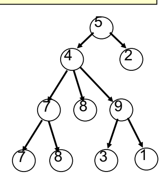
    private T value;
    private List<GTreeNode<T>> children;
    //appropriate constructors, getters,
    //setters, etc.
}
```

Java.util.List is an interface!
It defines the methods that all implementation must implement.
Whoever writes this class gets to decide what implementation to use — ArrayList? LinkedList? Etc.?



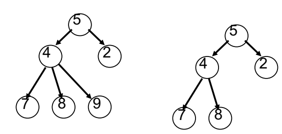

General tree

## Binary Trees

16

A *binary tree* is a particularly important kind of tree where every node as at most two children.

In a binary tree, the two children are called the *left* and *right* children.




Not a binary tree (a *general* tree)

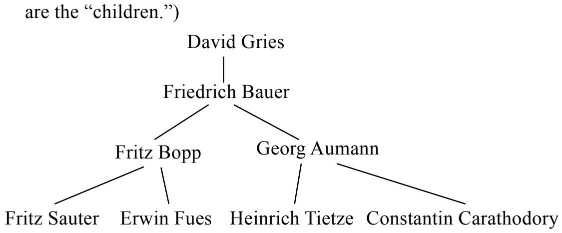
Binary tree

## Binary trees were in A1!

17

You have seen a binary tree in A1.

A PhD object has one or two advisors. (Confusingly, the advisors are the "children.")




## Class for binary tree node

18

```
class TreeNode<T> {
    private T value;
    private TreeNode<T> left, right;

    /** Constructor: one-node tree with datum x */
    public TreeNode (T d) { datum= d; left= null; right= null;}

    /** Constr: Tree with root value x, left tree l, right tree r */
    public TreeNode (T d, TreeNode<T> l, TreeNode<T> r) {
        datum= d; left= l; right= r;
    }
}
```

Either might be null if the subtree is empty.

more methods: getValue, setValue, getLeft, setLeft, etc.

## Binary versus general tree

19

In a binary tree, each node has up to two pointers: to the left subtree and to the right subtree:

- One or both could be **null**, meaning the subtree is empty (remember, a tree is a set of nodes)

In a general tree, a node can have any number of child nodes (and they need not be ordered)

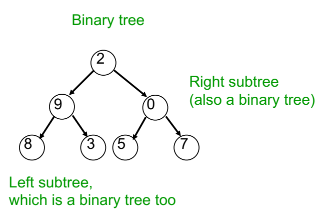
- Very useful in some situations ...
- ... one of which may be in an assignment!

## A Tree is a Recursive Thing

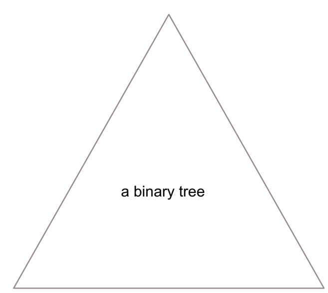
20

A binary tree is either `null` or an object consisting of a value, a left binary tree, and a right binary tree.

## Looking at trees recursively




## Looking at trees recursively

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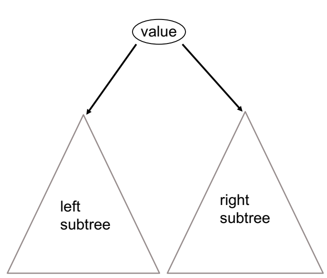
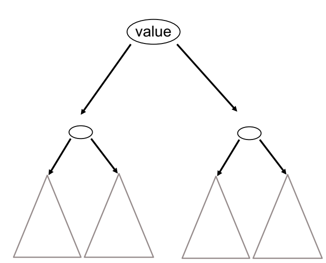



## Looking at trees recursively



## A Recipe for Recursive Functions

25

Base case:
If the input is "easy," just solve the problem directly.

Recursive case:
Get a smaller part of the input (or several parts).
Call the function on the smaller value(s).
Use the recursive result to build a solution for the full input.

## Recursive Functions on Binary Trees

26

Base case:
empty tree (null)
or, possibly, a leaf

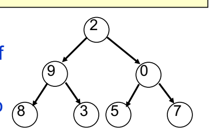
Recursive case:
Call the function on each subtree.
Use the recursive result to build a solution for the full input.

## Searching in a Binary Tree

27

```
/** Return true iff x is the datum in a node of tree  t*/
public static boolean treeSearch(T x, TreeNode<T> t) {
   if (t == null) return false;
   if (x.equals(t.datum)) return true;
   return treeSearch(x, t.left) || treeSearch(x, t.right);
}
```

- Analog of linear search in lists: given tree and an object, find out if object is stored in tree
- Easy to write recursively, harder to write iteratively




## Searching in a Binary Tree

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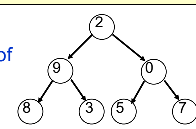
}
```

VERY IMPORTANT!
We sometimes talk of t as the root of the tree.
But we also use t to denote the whole tree.




## Comparing Data Structures

29

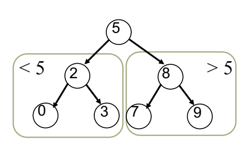
| Data Structure | add(val x) | lookup(int i) | search(val x) |
|---|---|---|---|
| Array [2\|1\|3\|0] | $O(n)$ | $O(1)$ | $O(n)$ |
| Linked List 2→1→3→0 | $O(1)$ | $O(n)$ | $O(n)$ |
| Binary Tree 2 (1, 3) | $O(1)$ | $O(n)$ | $O(n)$ |

## Binary Search Tree (BST)

30

A *binary search tree* is a binary tree that is **ordered** and **has no duplicate values.** In other words, for *every* node:

- All nodes in the left subtree have values that are less than the value in that node, and
- All values in the right subtree are greater.




A BST is the key to making search way faster.

## Building a BST

31

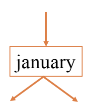
- To insert a new item:
  - Pretend to look for the item
  - Put the new node in the place where you fall off the tree

## Building a BST

32


## Building a BST

33




## Building a BST

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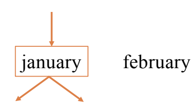


## Building a BST

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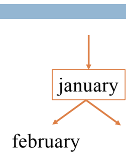
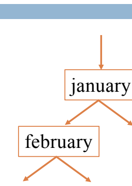


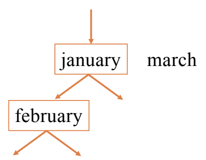

## Building a BST

36



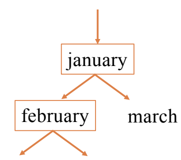
## Building a BST

37




## Building a BST

38




## Building a BST

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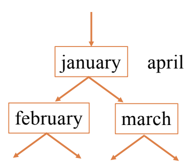


## Building a BST

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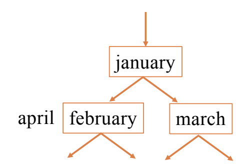
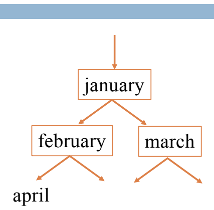


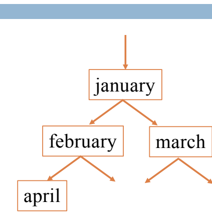

## Building a BST

41



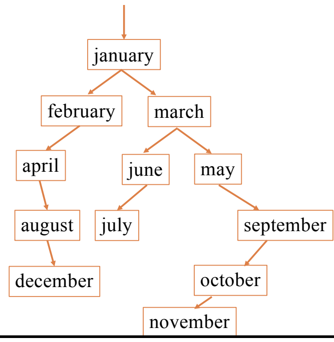

## Building a BST

42



## Building a BST

43




## Printing contents of BST

44

Because of ordering rules for a BST, it's easy to print the items in alphabetical order

- Recursively print left subtree
- Print the node
- Recursively print right subtree

```
/** Print BST t in alpha order */
private static void print(TreeNode<T> t) {
    if (t== null) return;
    print(t.left);
    System.out.print(t.value);
    print(t.right);
}
```

## Tree traversals

45

"Walking" over the whole tree is a tree traversal

- Done often enough that there are standard names

Previous example:
in-order traversal

- Process left subtree
- Process root
- Process right subtree

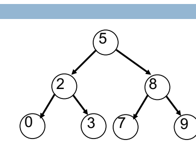
Note: Can do other processing besides printing

Other standard kinds of traversals

- preorder traversal
  - Process root
  - Process left subtree
  - Process right subtree
- postorder traversal
  - Process left subtree
  - Process right subtree
  - Process root
- level-order traversal
  - Not recursive: uses a queue (we'll cover this later)

## Binary Search Tree (BST)

46




Compare binary tree to binary search tree:

```
boolean searchBT(n, v):
  if n==null, return false
  if n.v == v, return true
  return searchBST(n.left, v)
      || searchBST(n.right, v)
```

2 recursive calls

```
boolean searchBST(n, v):
  if n==null, return false
  if n.v == v, return true
  if v < n.v
    return searchBST(n.left, v)
  else
    return searchBST(n.right, v)
```

1 recursive call

## Comparing Data Structures

47

| Data Structure | add(val x) | lookup(int i) | search(val x) |
|---|---|---|---|
| Array [2\|1\|3\|0] | $O(n)$ | $O(1)$ | $O(n)$ |
| Linked List 2→1→3→0 | $O(1)$ | $O(n)$ | $O(n)$ |
| Binary Tree (1; 2, 3) | $O(1)$ | $O(n)$ | $O(n)$ |
| BST (2; 1, 3) | $O(depth)$ | $O(depth)$ | $O(depth)$ |

## Inserting in Alphabetical Order

48

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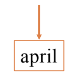
## Inserting in Alphabetical Order

49

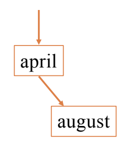



## Inserting in Alphabetical Order

50

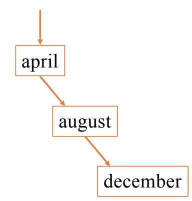

## Inserting in Alphabetical Order

51



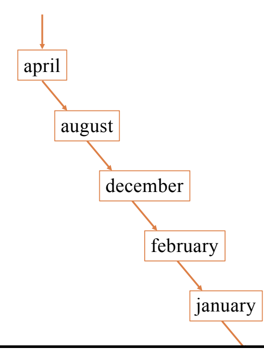

## Inserting in Alphabetical Order

52




## Inserting in Alphabetical Order

53




## Insertion Order Matters

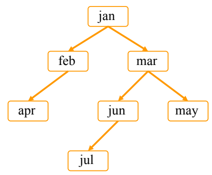
54

- A *balanced* binary tree is one where the two subtrees of any node are about the same size.
- Searching a binary search tree takes O(h) time, where h is the height of the tree.
- In a balanced binary search tree, this is O(log n).
- But if you insert data in sorted order, the tree becomes imbalanced, so searching is O(n).

## Things to think about

55

What if we want to *delete* data from a BST?




A BST works great as long as it's *balanced*.

There are kinds of trees that can *automatically* keep themselves balanced as you insert things!

## Useful facts about binary trees

56

Max # of nodes at depth d: $2^d$

If height of tree is h
- min # of nodes: h + 1
- max #of nodes in tree:
- $2^0 + \ldots + 2^h = 2^{h+1} - 1$

Complete binary tree
- All levels of tree down to a certain depth are completely filled