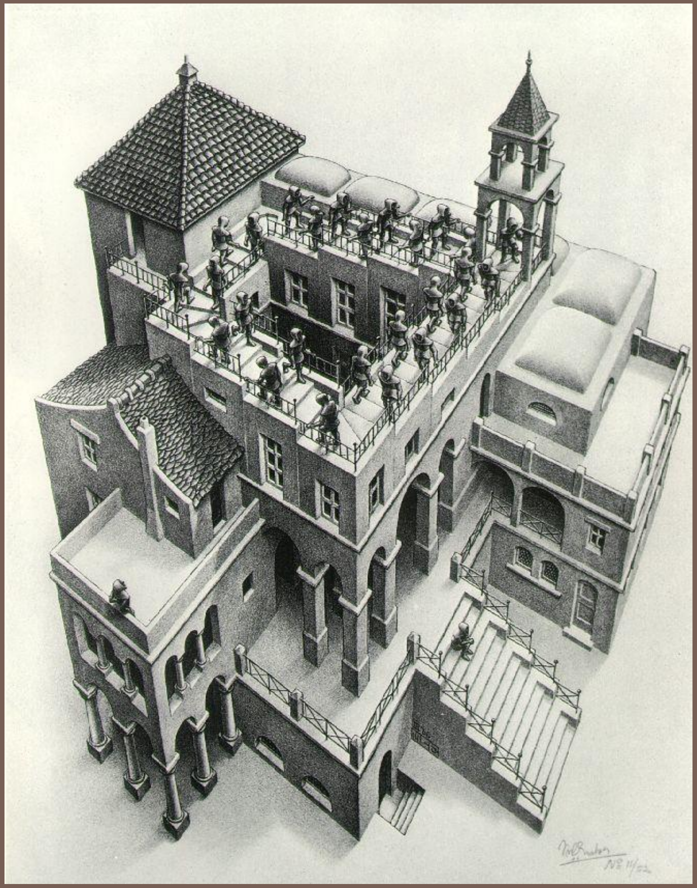


PRIORITY QUEUES AND HEAPS

Lecture 17

CS2110 Fall 2016

Readings and Homework

Read Chapter 26 "A Heap Implementation" to learn about heaps

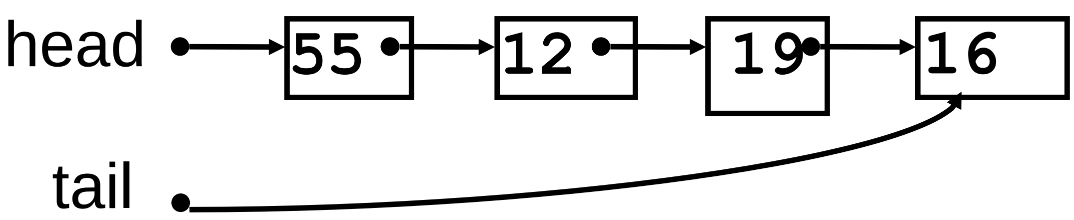
Exercise: Salespeople often make matrices that show all the great features of their product that the competitor's product lacks. Try this for a heap versus a BST. First, try and sell someone on a BST: List some desirable properties of a BST that a heap lacks. Now be the heap salesperson: List some good things about heaps that a BST lacks. Can you think of situations where you would favor one over the other?

With ZipUltra heaps, you've got it made in the shade my friend!

Stacks and queues are restricted lists

- Stack (LIFO) implemented as list
  - **`add()`**, **`remove()`** from front of list (push and pop)
- Queue (FIFO) implemented as list
  - **`add()`** on back of list, **`remove()`** from front of list
- These operations are O(1)

Both efficiently implementable using a singly linked list with head and tail



Interface Bag (not In Java Collections)


```
interface Bag<E>
       implements Iterable {
   void add(E obj);
   boolean contains(E obj);
   boolean remove(E obj);
   int size();
   boolean isEmpty();
   Iterator<E> iterator()
}
```

Also called multiset

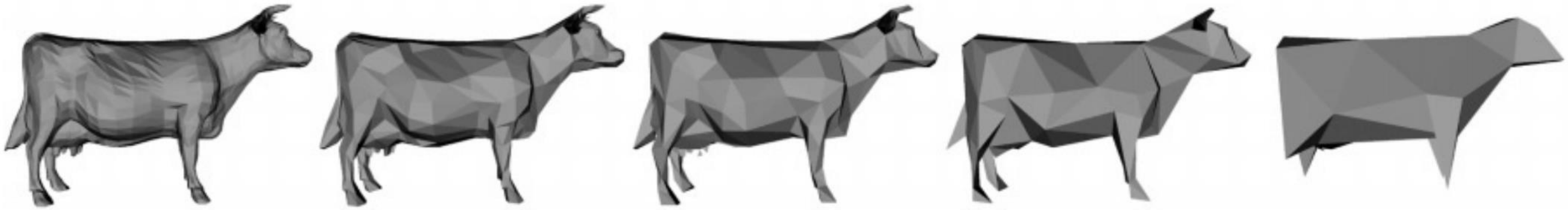
Like a set except that a value can be in it more than once. Example: a bag of coins

Refinements of Bag: Stack, Queue, PriorityQueue

Priority queue

- **`Bag`** in which data items are **`Comparable`**
- Smaller elements (determined by **`compareTo()`**) have higher priority
- **`remove()`** return the element with the highest priority = least element in the **`compareTo()`** ordering
- break ties arbitrarily

Many uses of priority queues (& heaps)



Surface simplification [Garland and Heckbert 1997]

- Event-driven simulation: customers in a line
- Collision detection: "next time of contact" for colliding bodies
- Graph searching: Dijkstra's algorithm, Prim's algorithm
- AI Path Planning: A* search
- Statistics: maintain largest M values in a sequence
- Operating systems: load balancing, interrupt handling
- Discrete optimization: bin packing, scheduling

java.util.PriorityQueue<E>


```
interface PriorityQueue<E> {                           TIME
 boolean add(E e) {...} //insert e.                    log
 void clear() {...} //remove all elems.
 E peek() {...} //return min elem.                     constant
 E poll() {...} //remove/return min elem.              log
 boolean contains(E e)                                 linear
 boolean remove(E e)                                   linear
 int size() {...}                                      constant
 Iterator<E> iterator()
}
```

Priority queues as lists

- Maintain as unordered list
  - `add()` put new element at front – O(1)
  - **poll()** must search the list – O(n)
  - **peek()** must search the list – O(n)

- Maintain as ordered list
  - `add()` must search the list – O(n)
  - **poll()** wamted element at top – O(1)
  - **peek()** O(1)

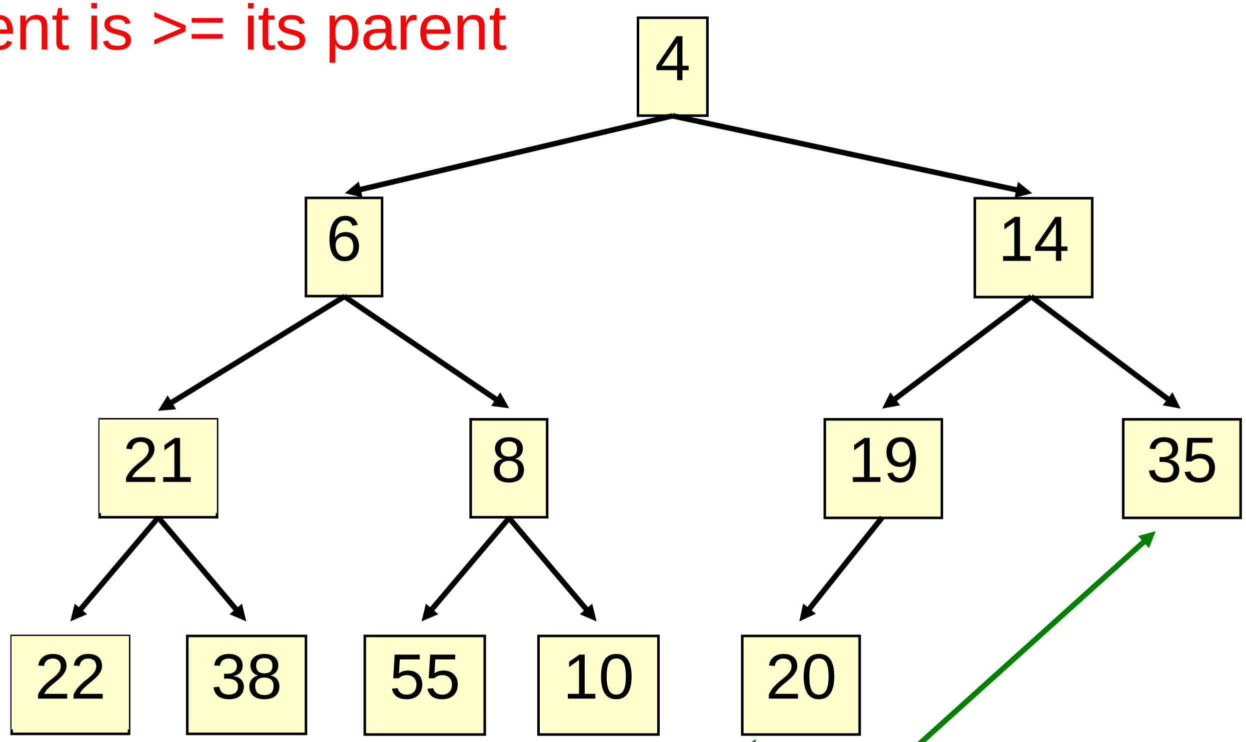
Can we do better?

Heap: binary tree with certain properties

- A *heap* is a concrete data structure that can be used to implement priority queues
- Gives better complexity than either ordered or unordered list implementation:
  - **`add()`**: O(log n) (n is the size of the heap)
  - **`poll()`**: O(log n)
- O(n log n) to process n elements
- Do not confuse with *heap memory*, where the Java virtual machine allocates space for objects – different usage of the word *heap*

Heap: first property

Every element is >= its parent

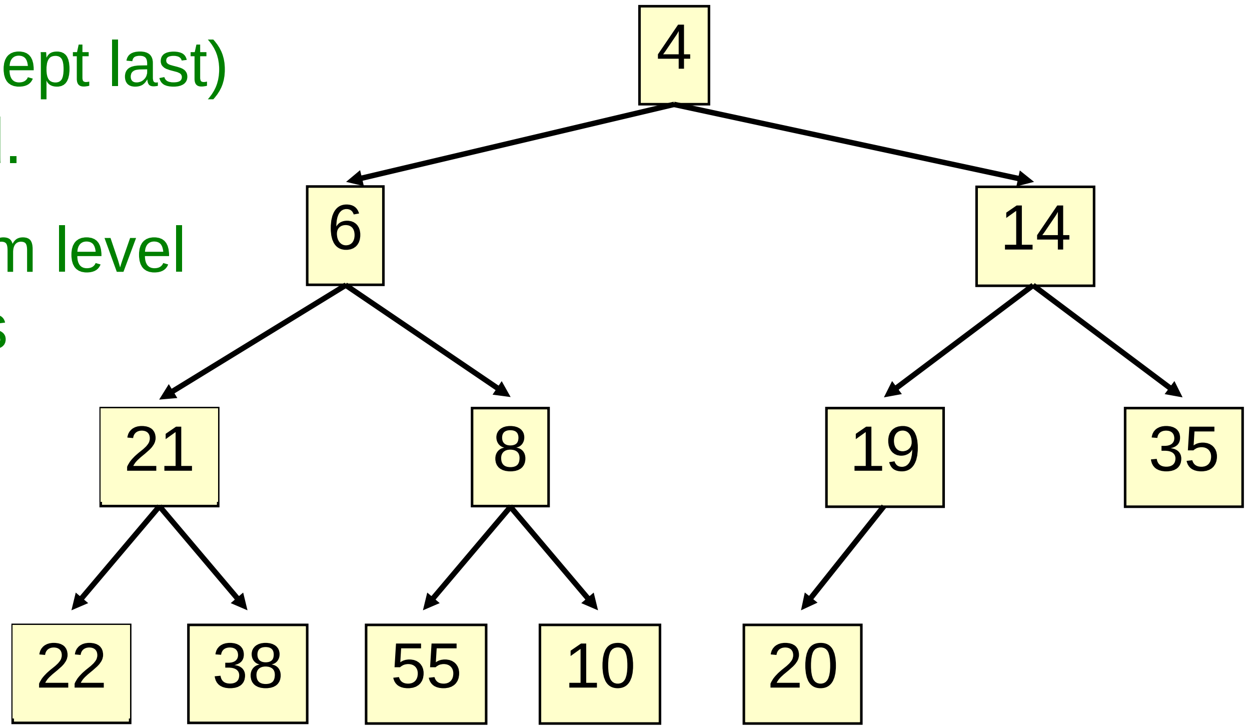


Note: 19, 20 < 35: Smaller elements can be deeper in the tree!

Heap: second property: is complete, has no holes

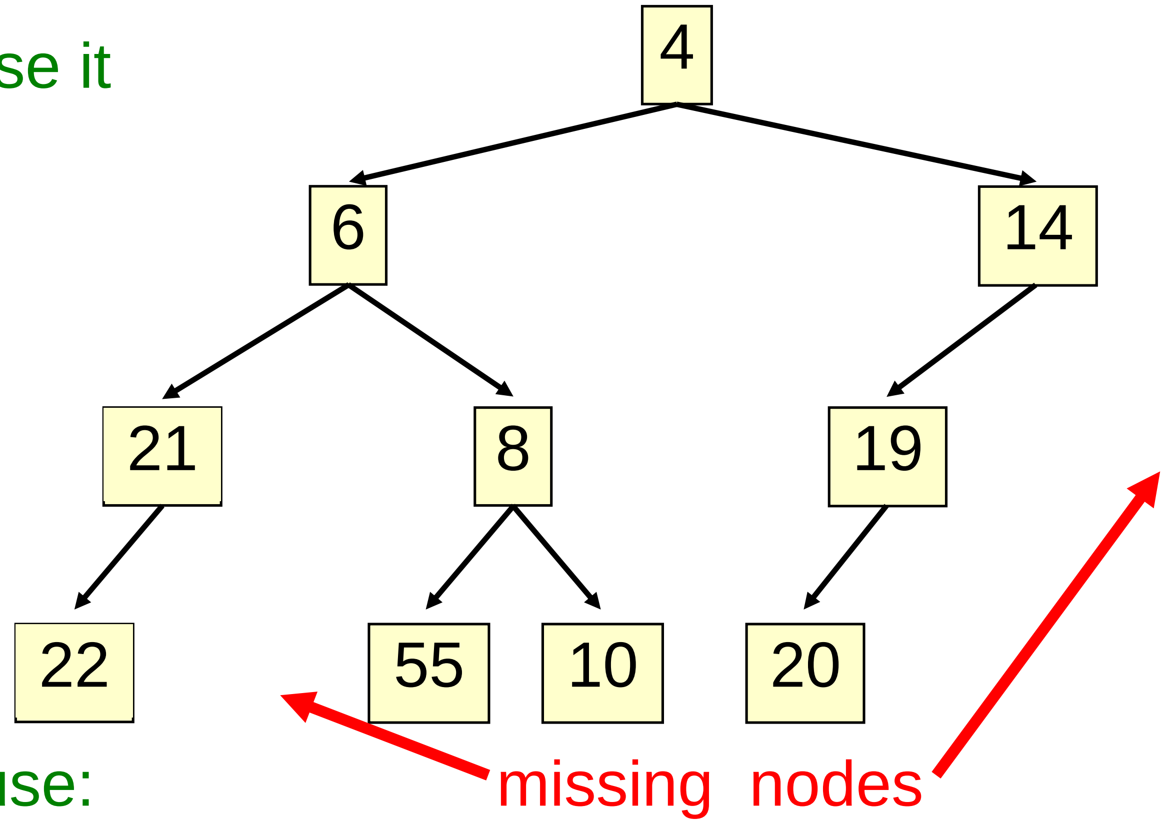
Every level (except last) completely filled.

Nodes on bottom level are as far left as possible.



Heap: Second property: has no “holes”

Not a heap because it has two holes



Not a heap because:

- missing a node on level 2
- bottom level nodes are not as far left as possible

Heap

- Binary tree with data at each node
- Satisfies the *Heap Order Invariant*:

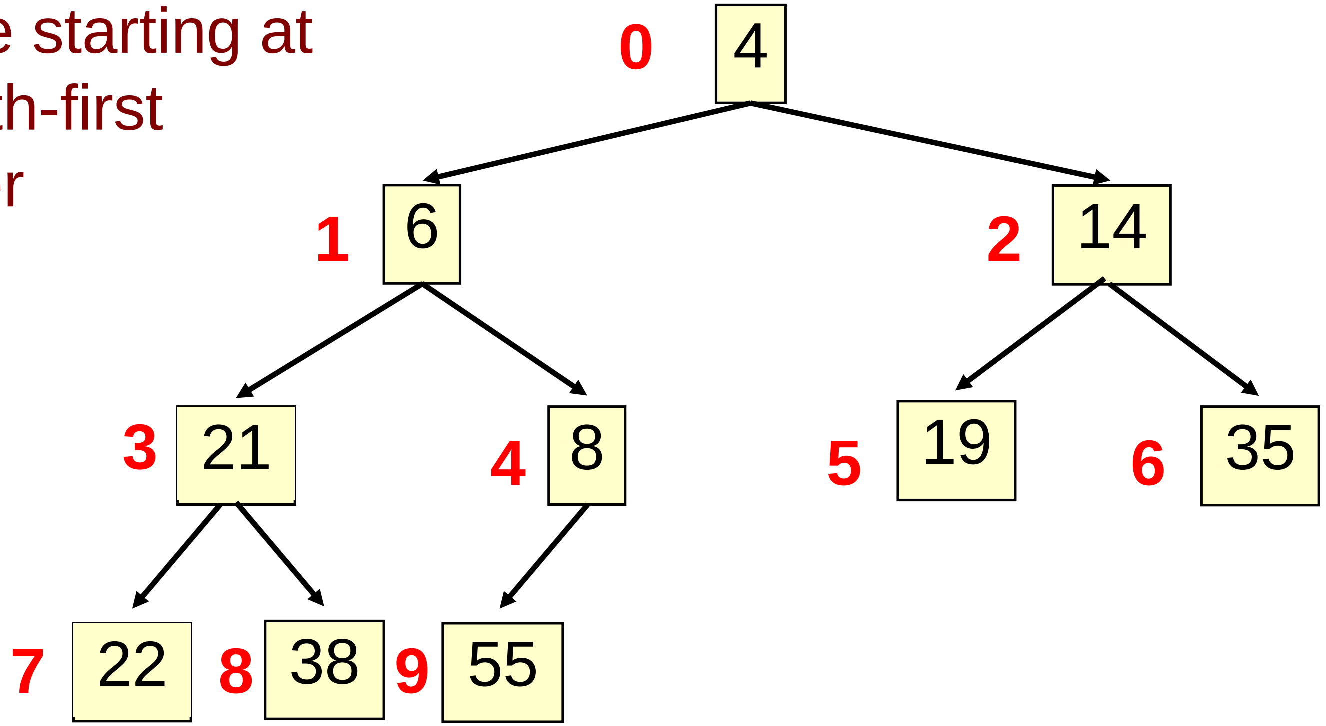
1. Every element is ≥ its parent.

- Binary tree is **complete** (no holes)

2. Every level (except last) completely filled.
Nodes on bottom level are as far left as possible.

Numbering the nodes in a heap

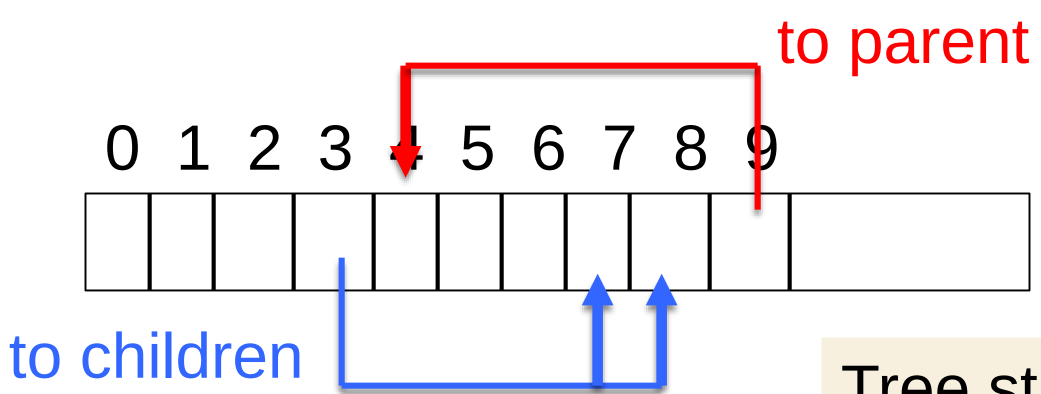
Number node starting at root in breadth-first left-right order



Children of node k are nodes 2k+1 and 2k+2
Parent of node k is node (k-1)/2

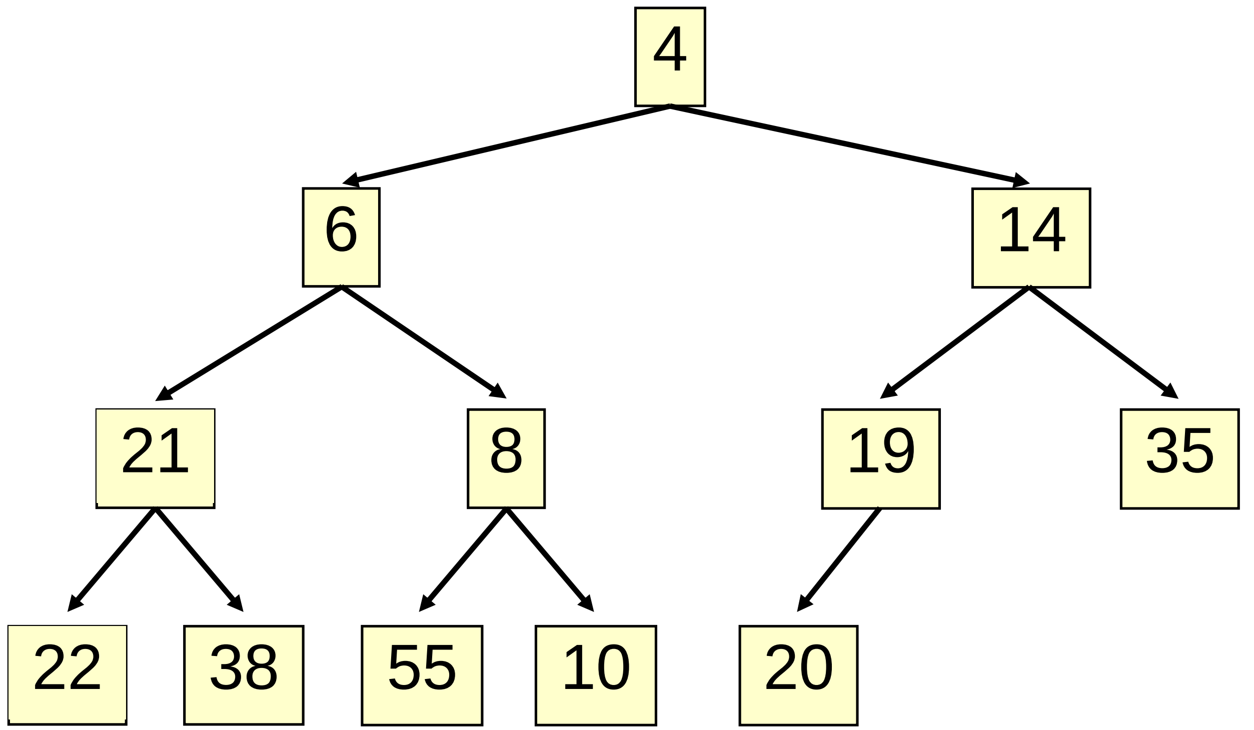
Can store a heap in an array b (could also be ArrayList or Vector)

- Heap nodes in b in order, going across each level from left to right, top to bottom
- Children of b[k] are b[2k + 1] and b[2k + 2]
- Parent of b[k] is b[(k – 1)/2]

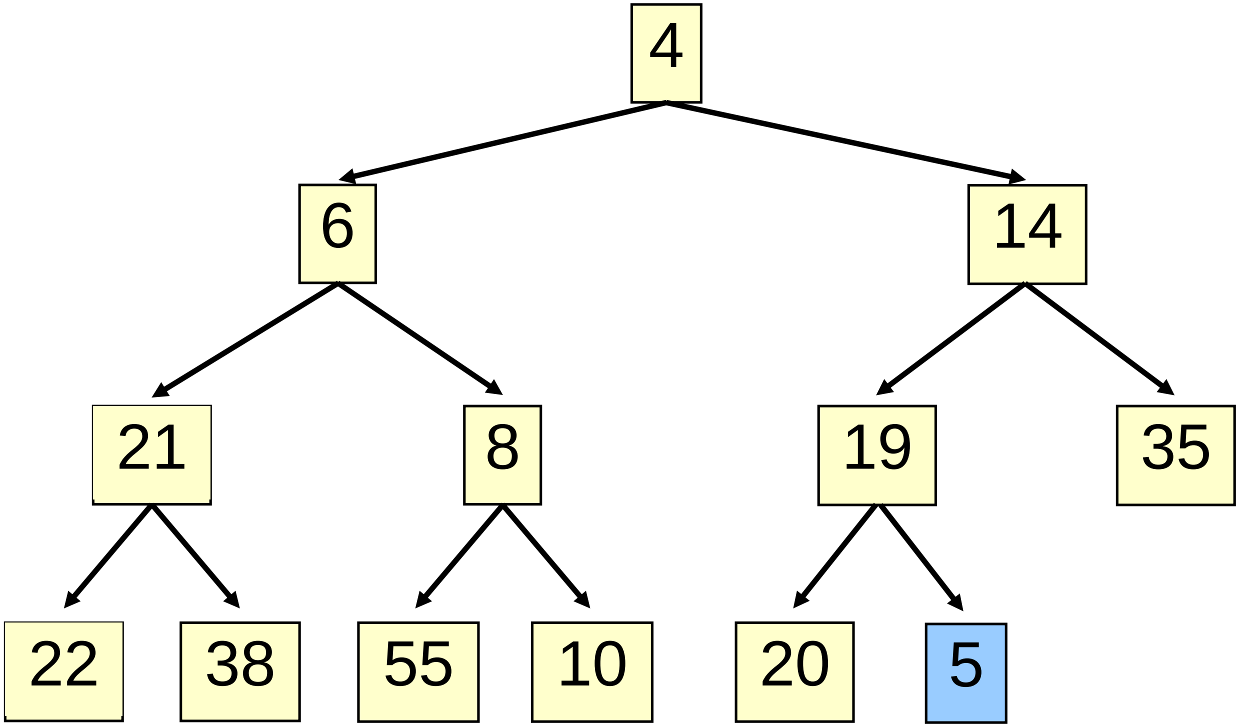


Tree structure is implicit.
No need for explicit links!

add(e)

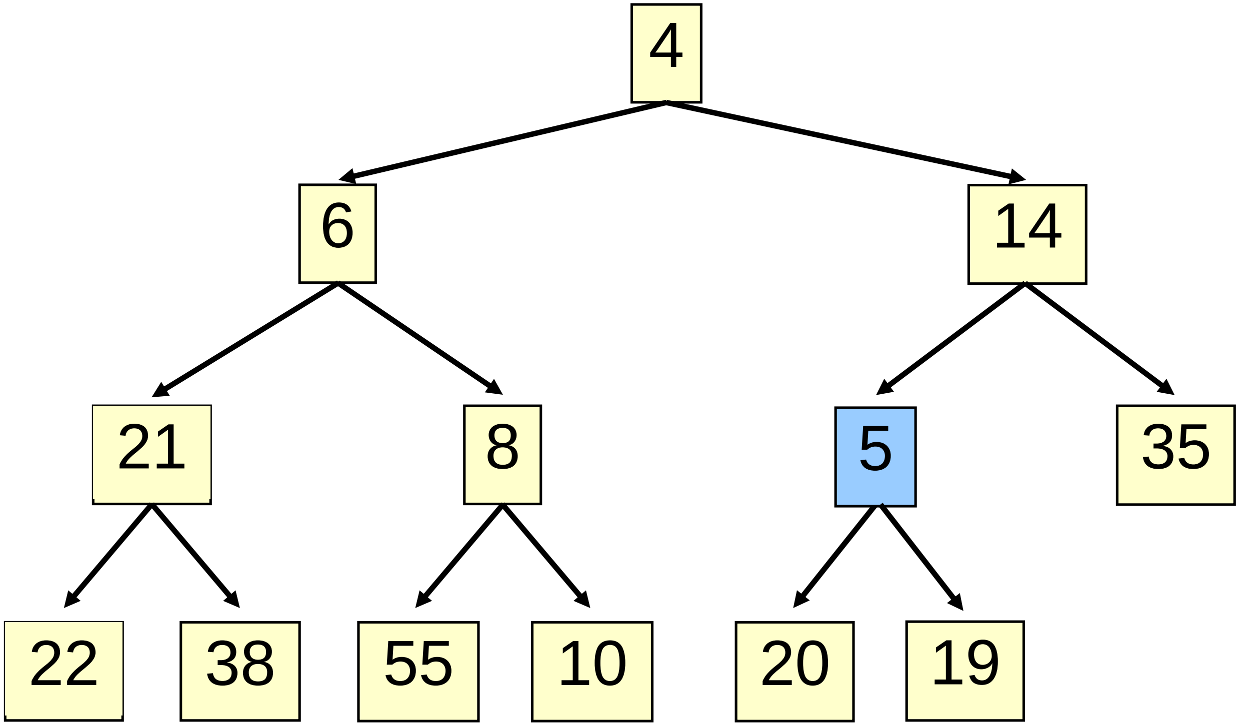


add(e)



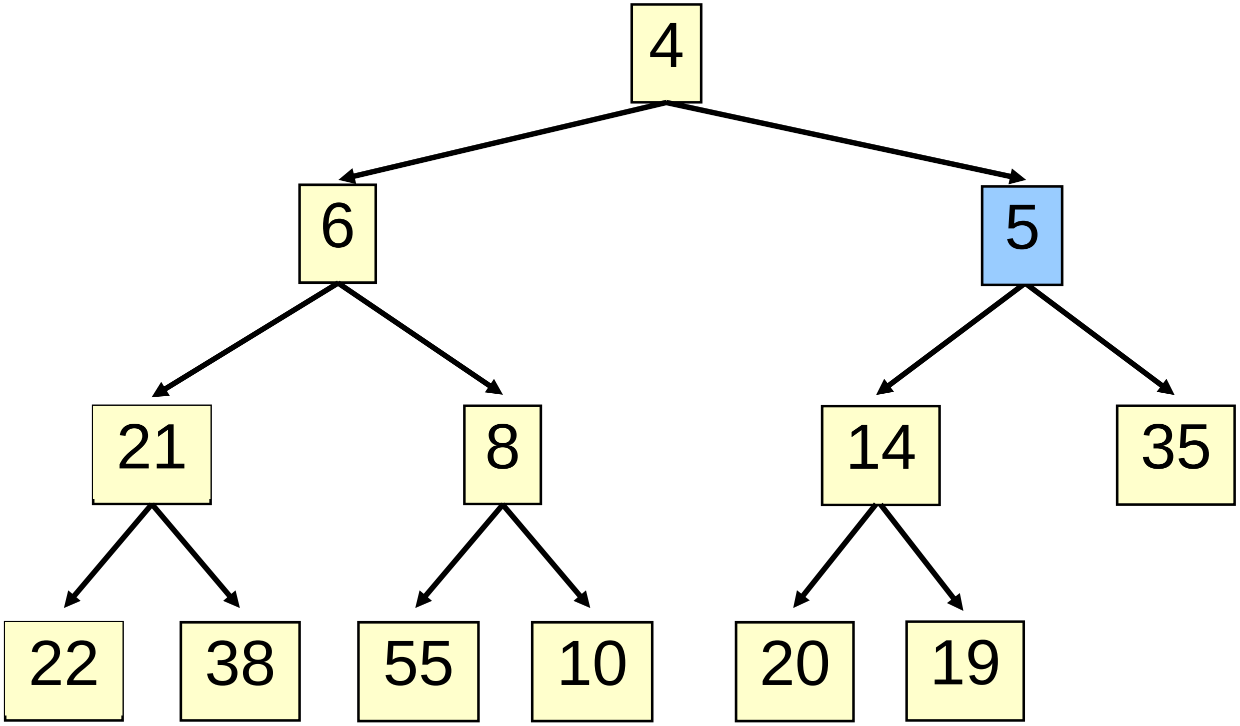
1. Put in the new element in a new node

add()



2. Bubble new element up if less than parent

add()



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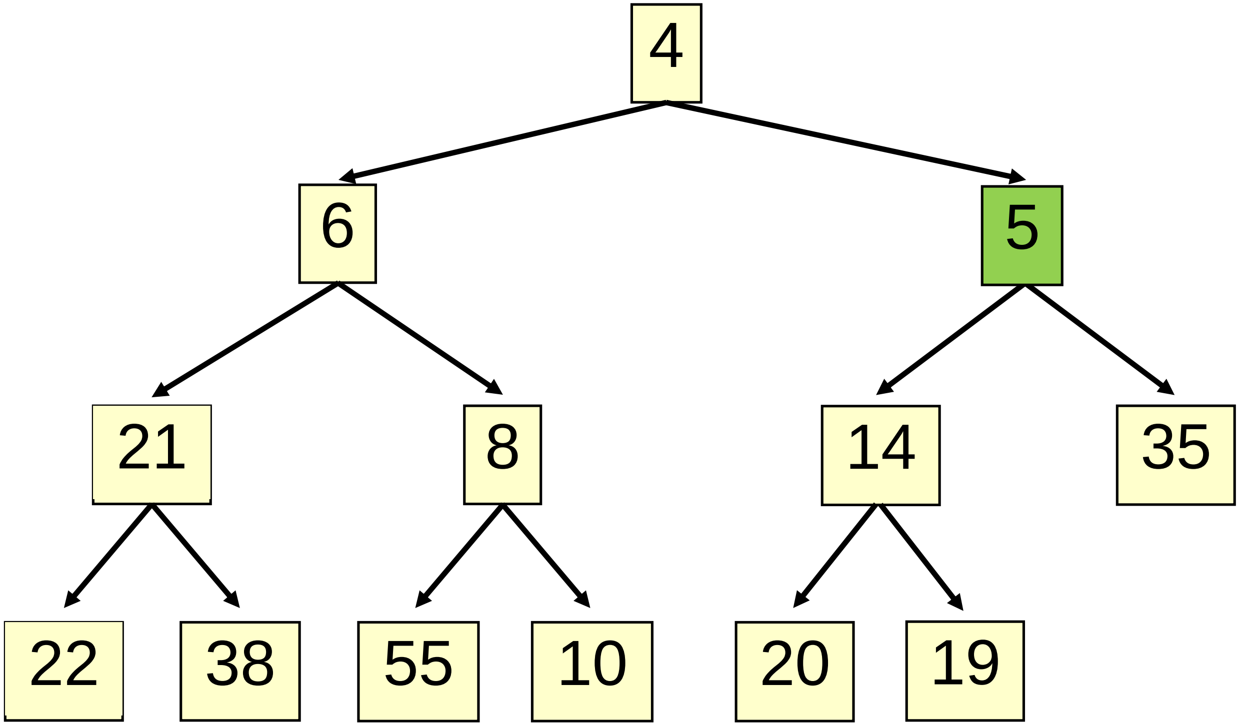

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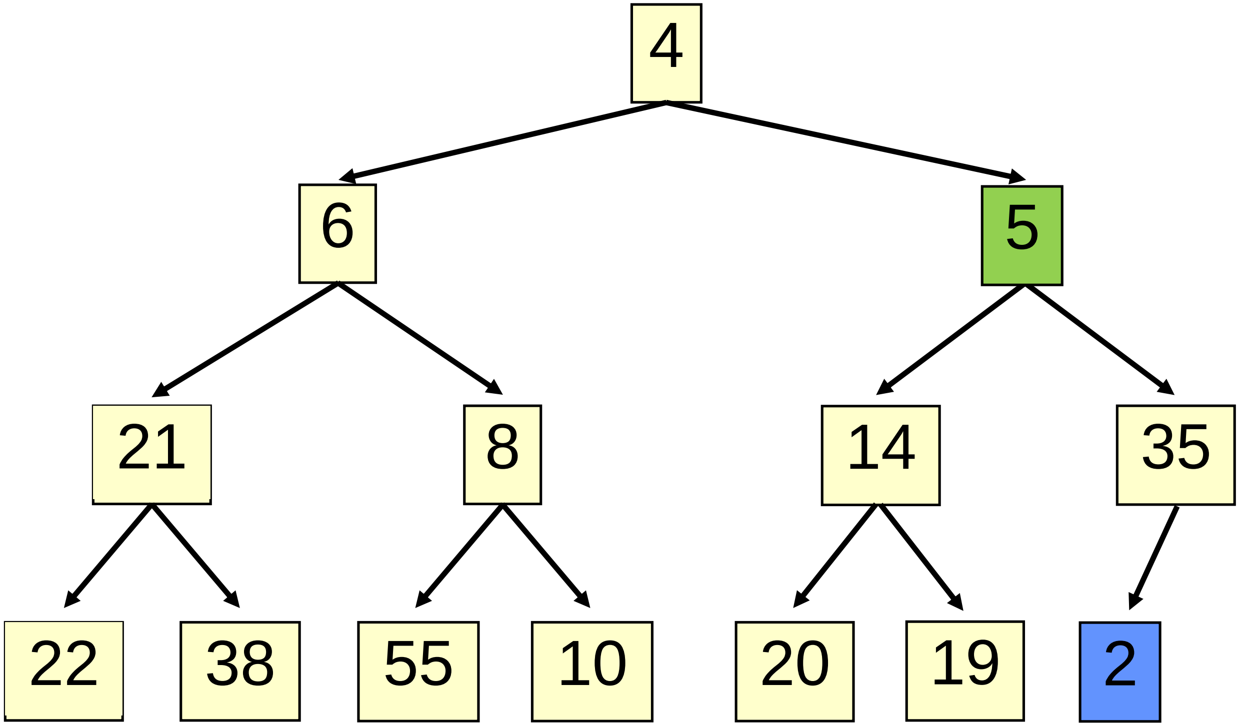

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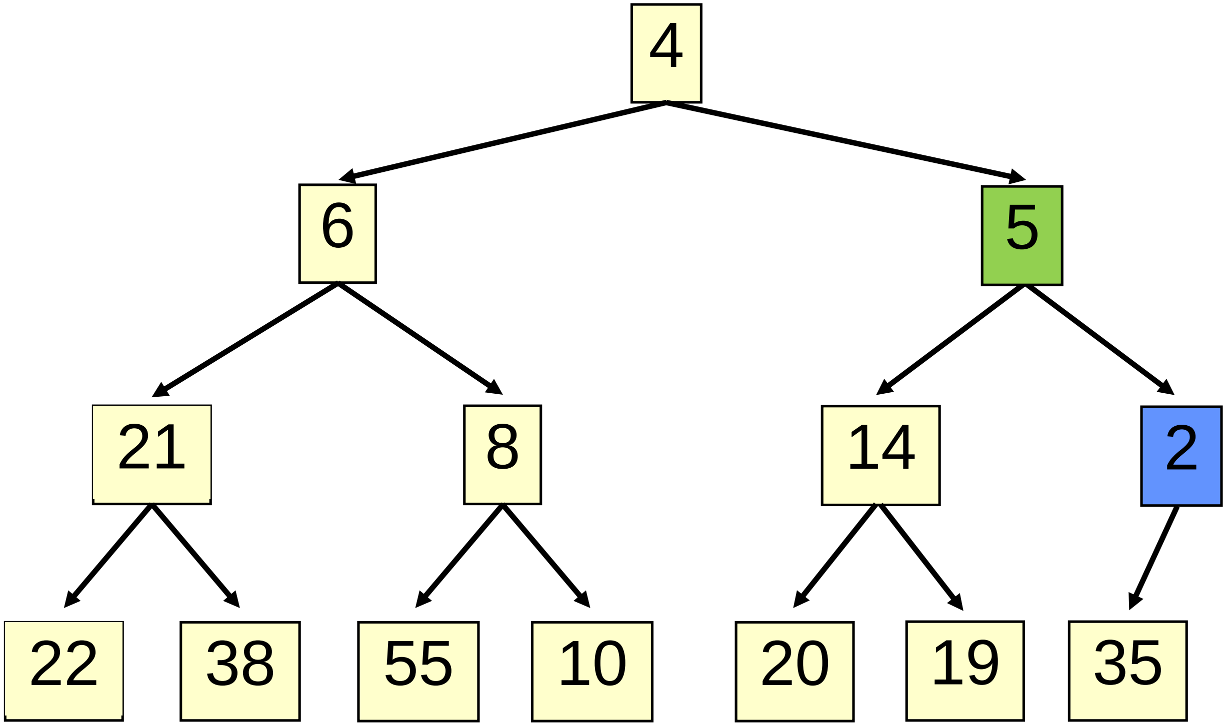

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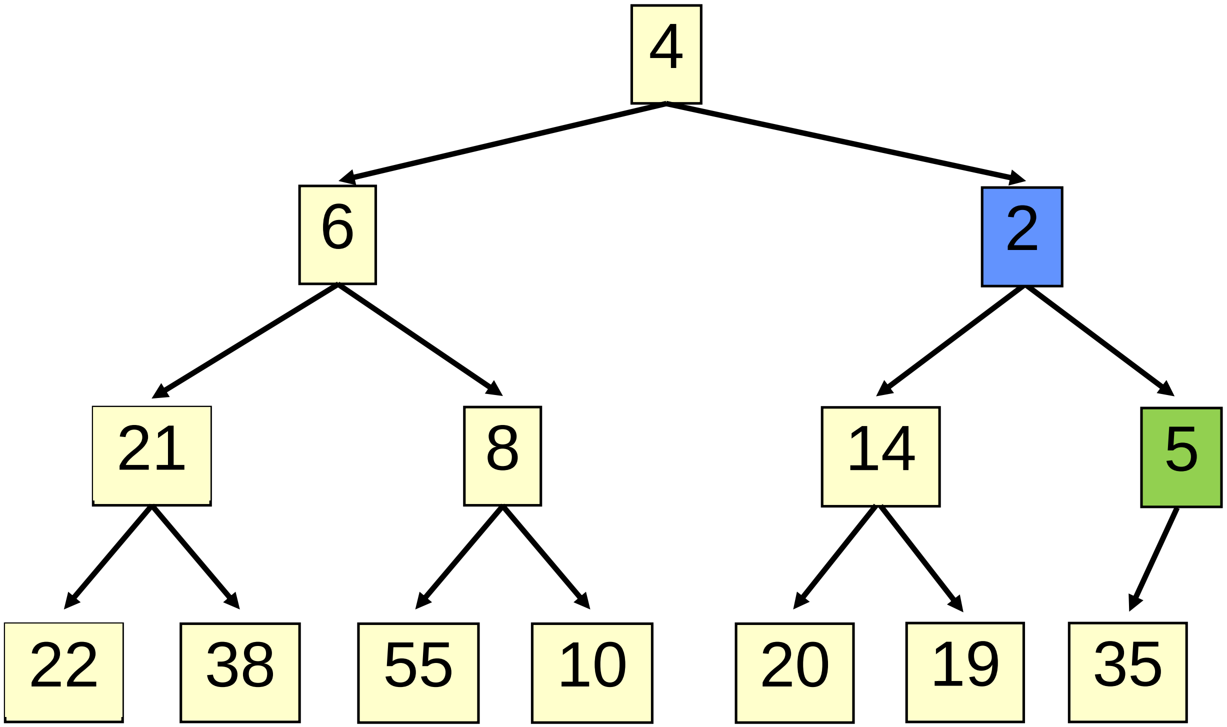

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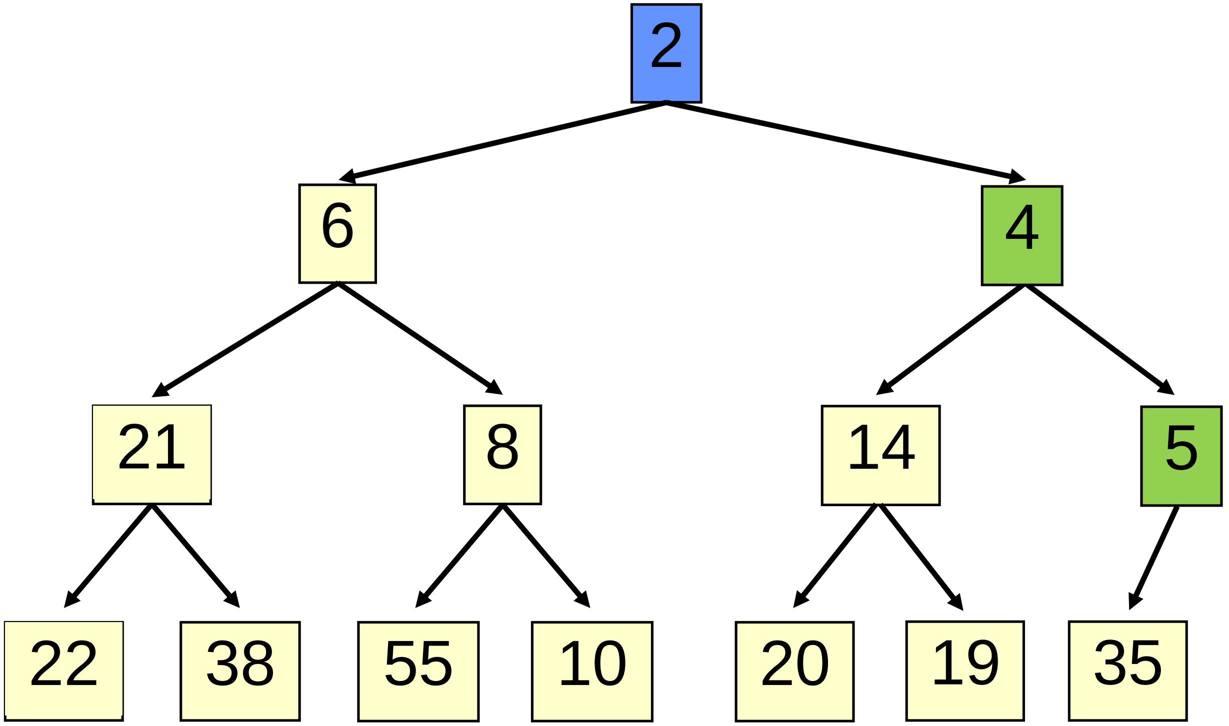

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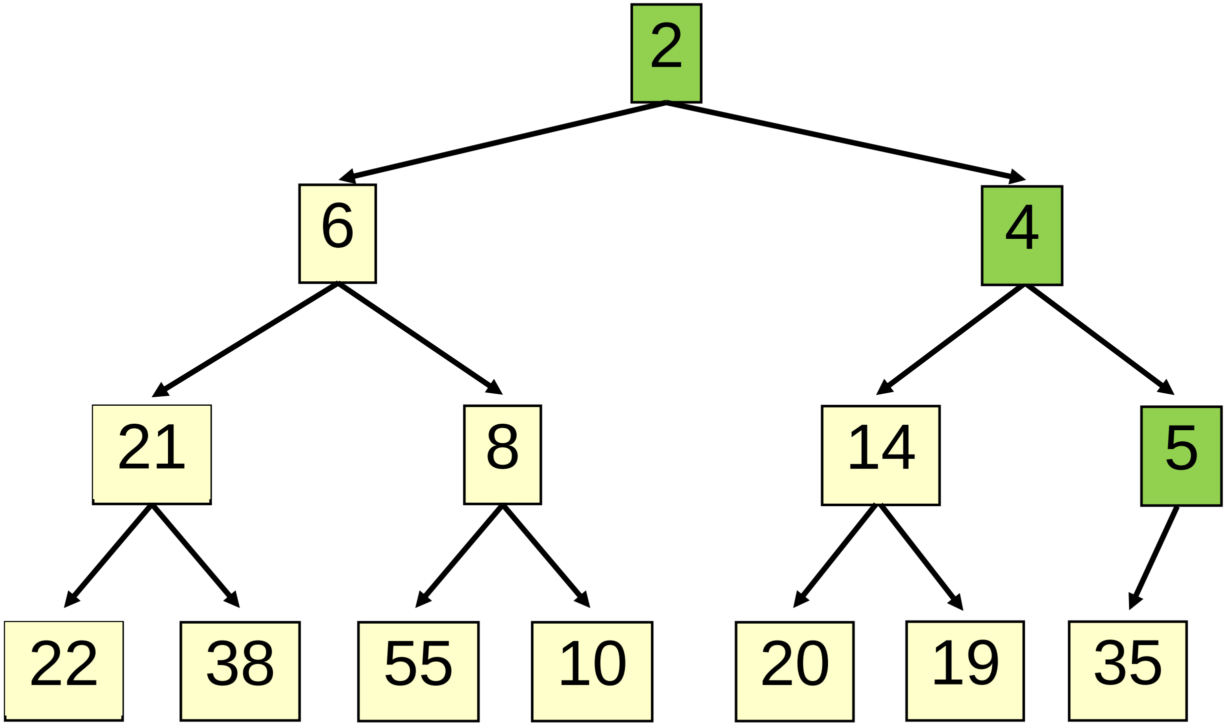


`add(e)`

- Add e at the end of the array
- Bubble e up until it no longer violateds heap order
- The heap invariant is maintained!

`add()` to a tree of size n

- Time is O(log n), since the tree is balanced
  - size of tree is exponential as a function of depth
  - depth of tree is logarithmic as a function of size

add() --assuming there is space

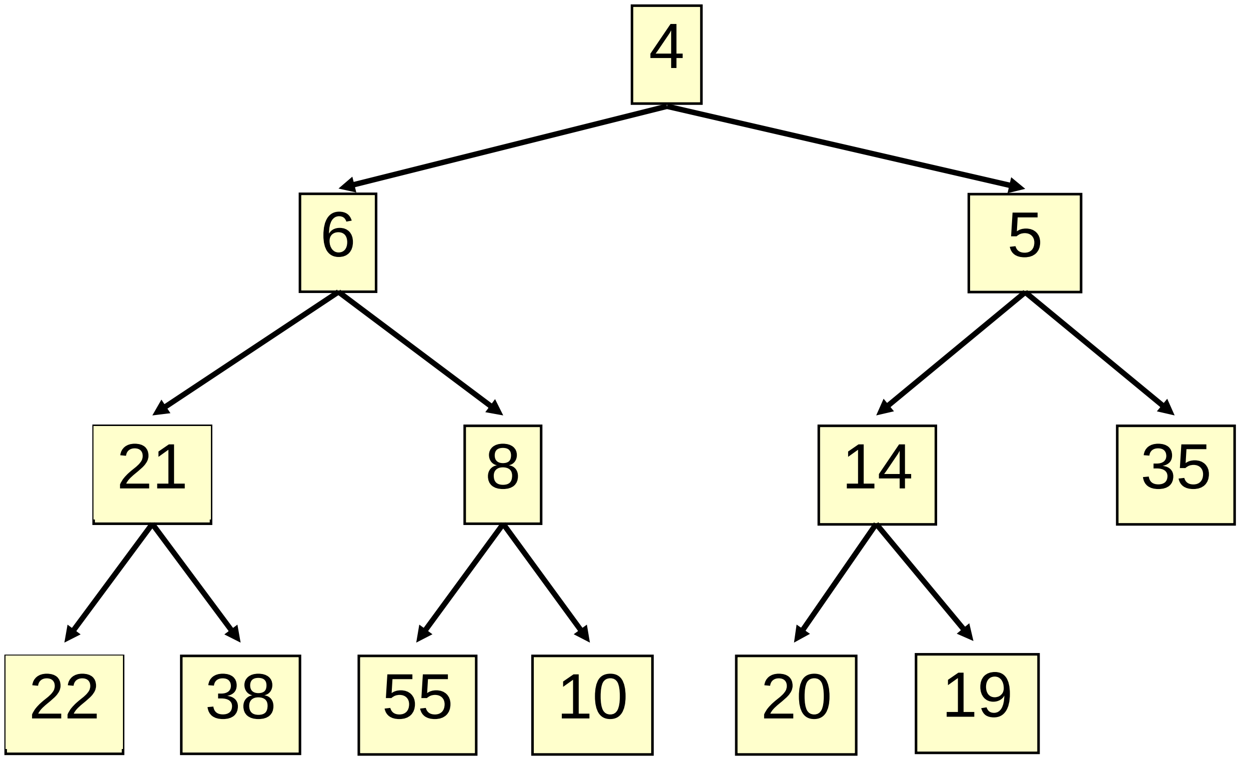

```
/** An instance of a heap */
class Heap<E> {
  E[] b= new E[50];  // heap is b[0..n-1]
  int n= 0;          // heap invariant is true

  /** Add e to the heap */
  public void add(E e) {
    b[n]= e;
    n= n + 1;
    bubbleUp(n - 1); // given on next slide
  }
}
```

`add()`. Remember, heap is in b[0..n-1]


```
class Heap<E> {
  /** Bubble element #k up to its position.
    * Pre: heap inv holds except maybe for k */
  private void bubbleUp(int k) {
    int p= (k-1)/2;
    // inv: p is parent of k and every elmnt
    // except perhaps k is >= its parent
    while (k > 0  &&  b[k].compareTo(b[p]) < 0) {
        swap(b[k], b[p]);
        k= p;
        p= (k-1)/2;
    }
}
```

`poll()`



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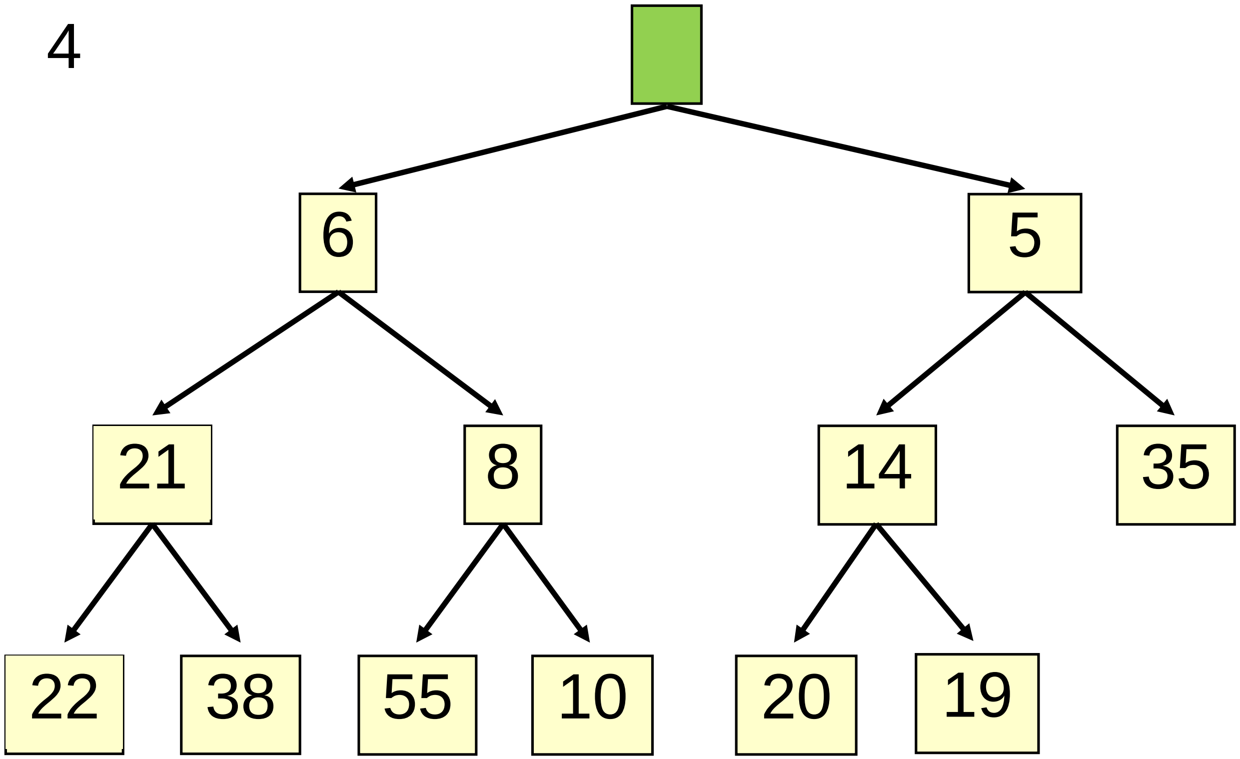
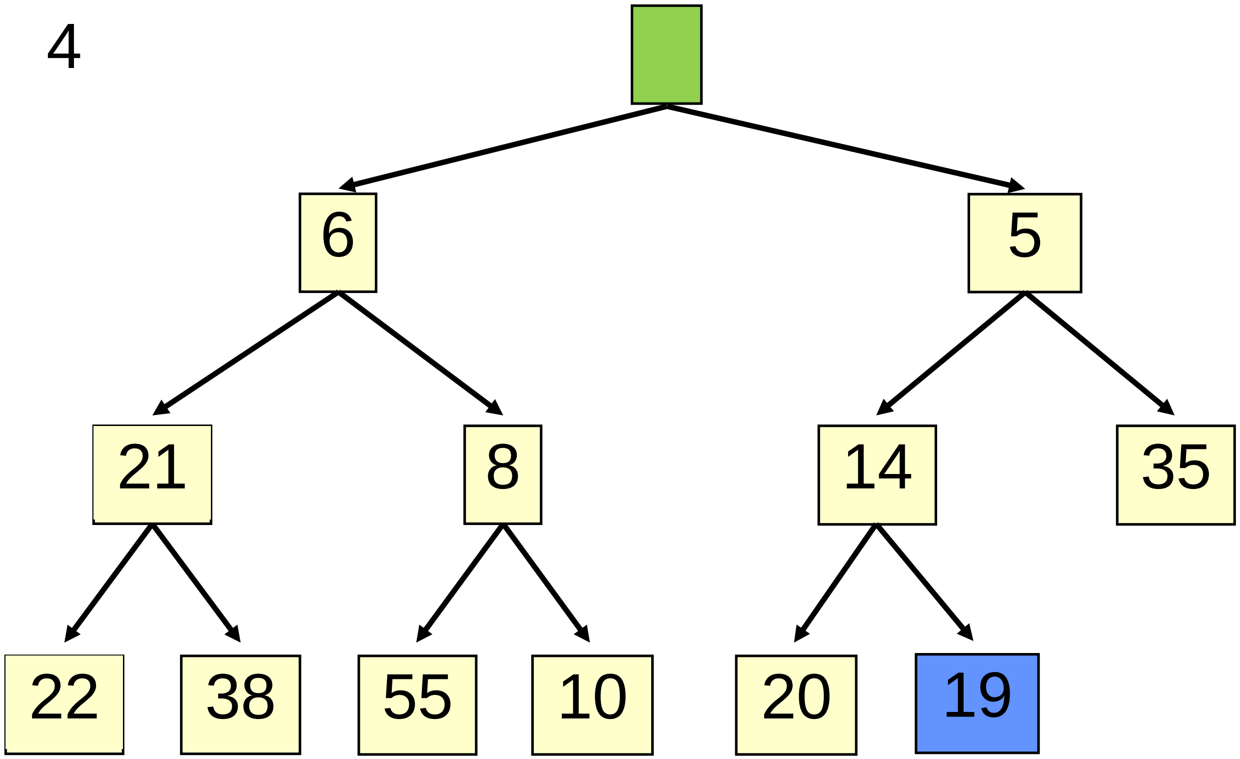


1. Save top element in a local variable

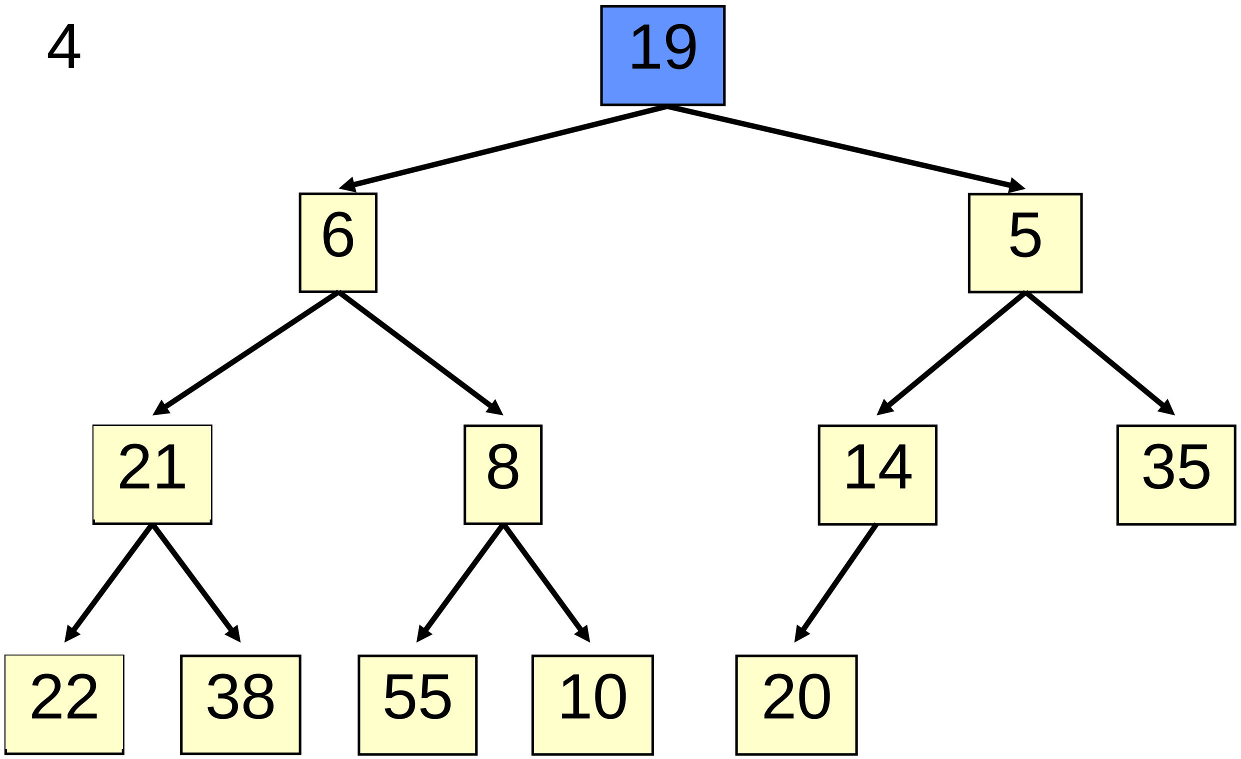
`poll()`



2. Assign last value to the root, delete last value from heap

`poll()`

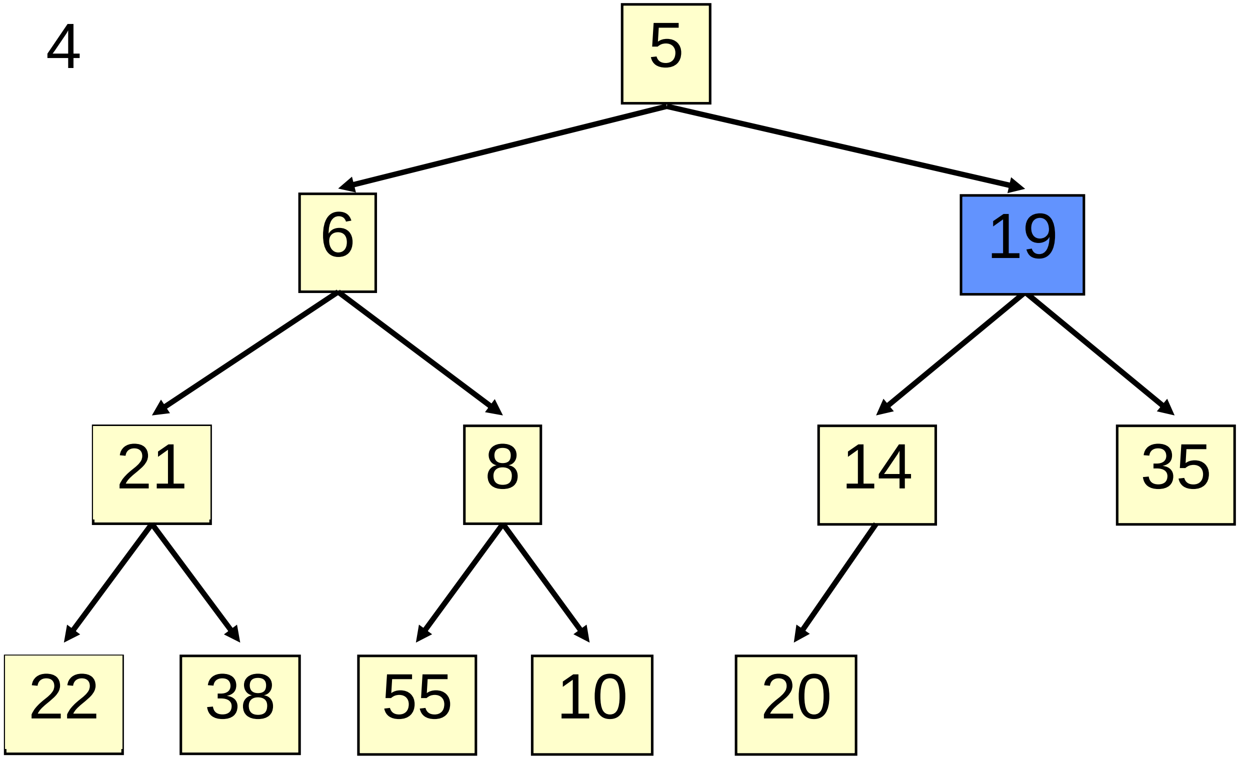
4



3. Bubble root value down

poll()

4



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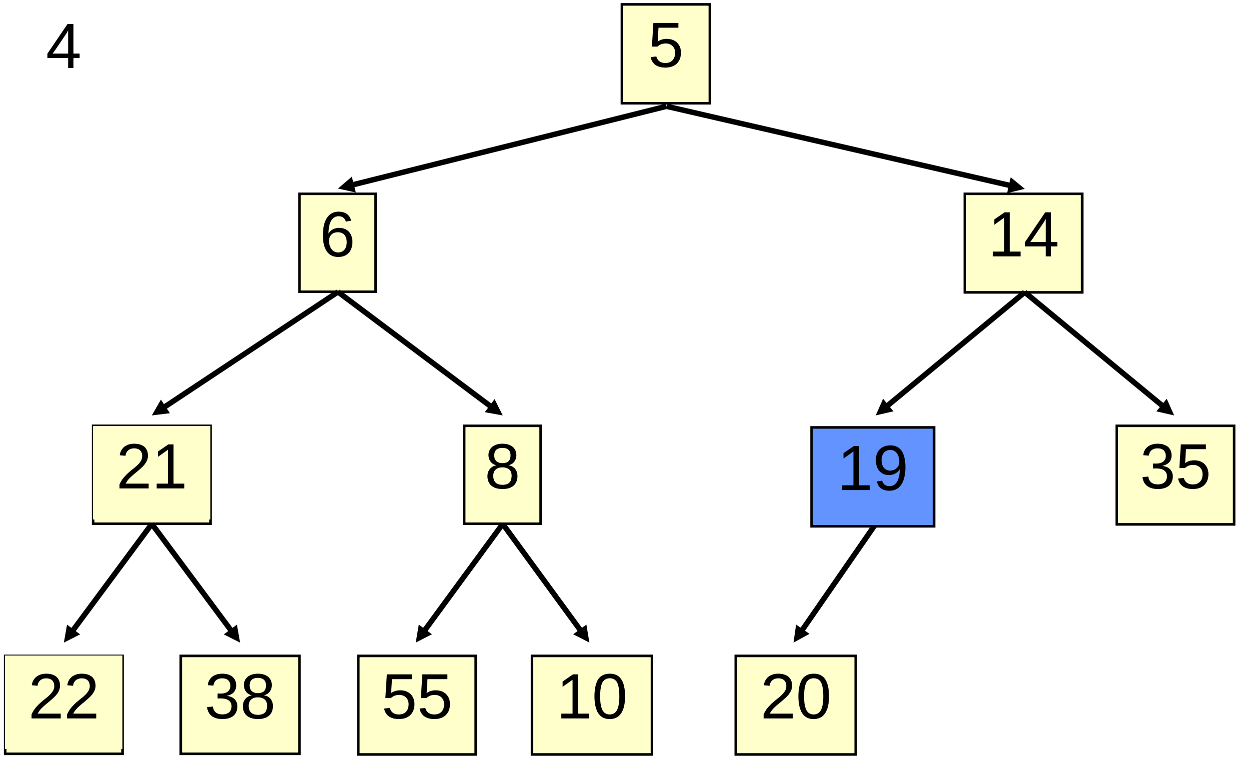

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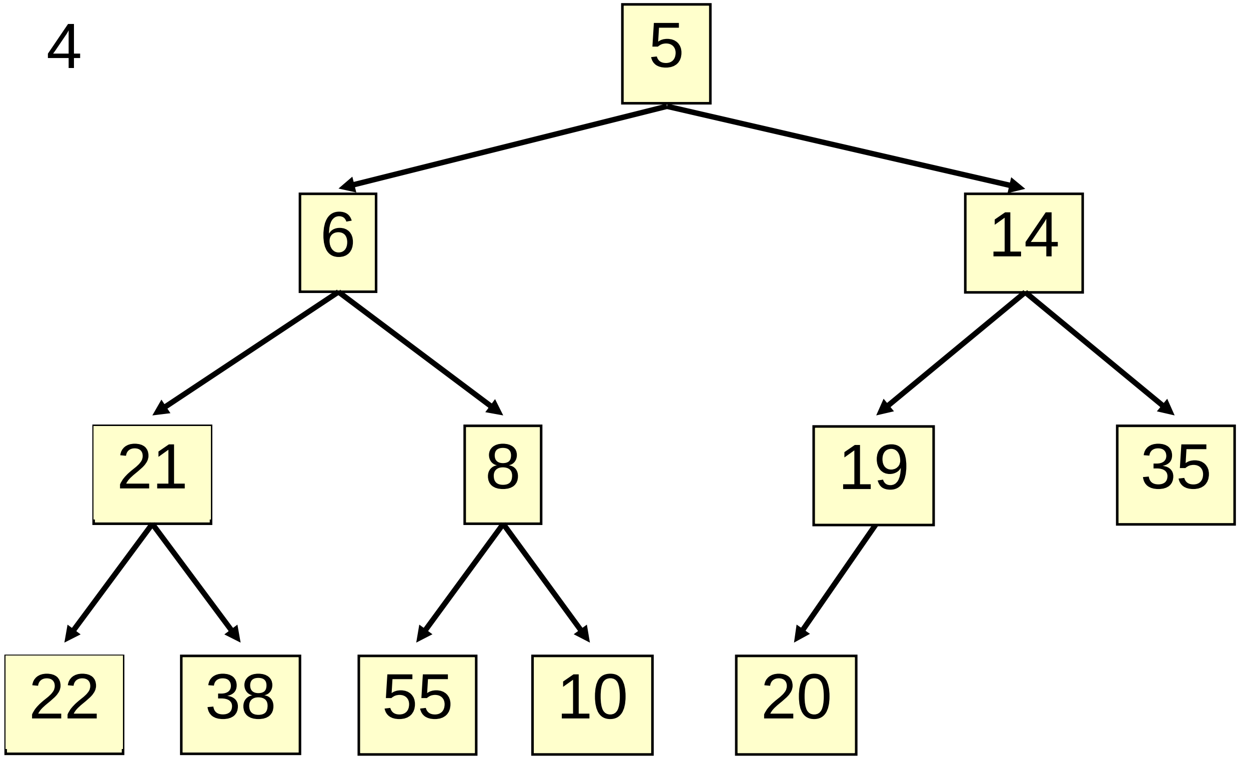

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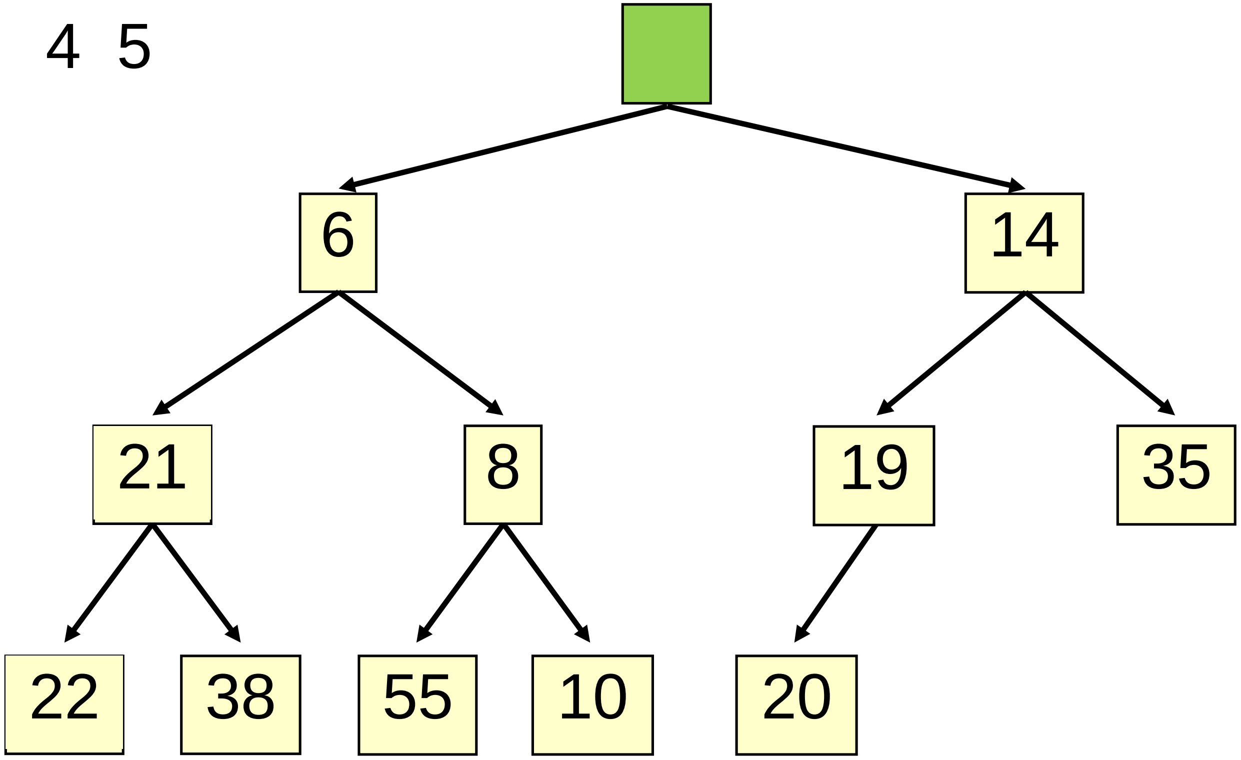
4 5



2. Assign last value to the root, delete last value from heap

`poll()`

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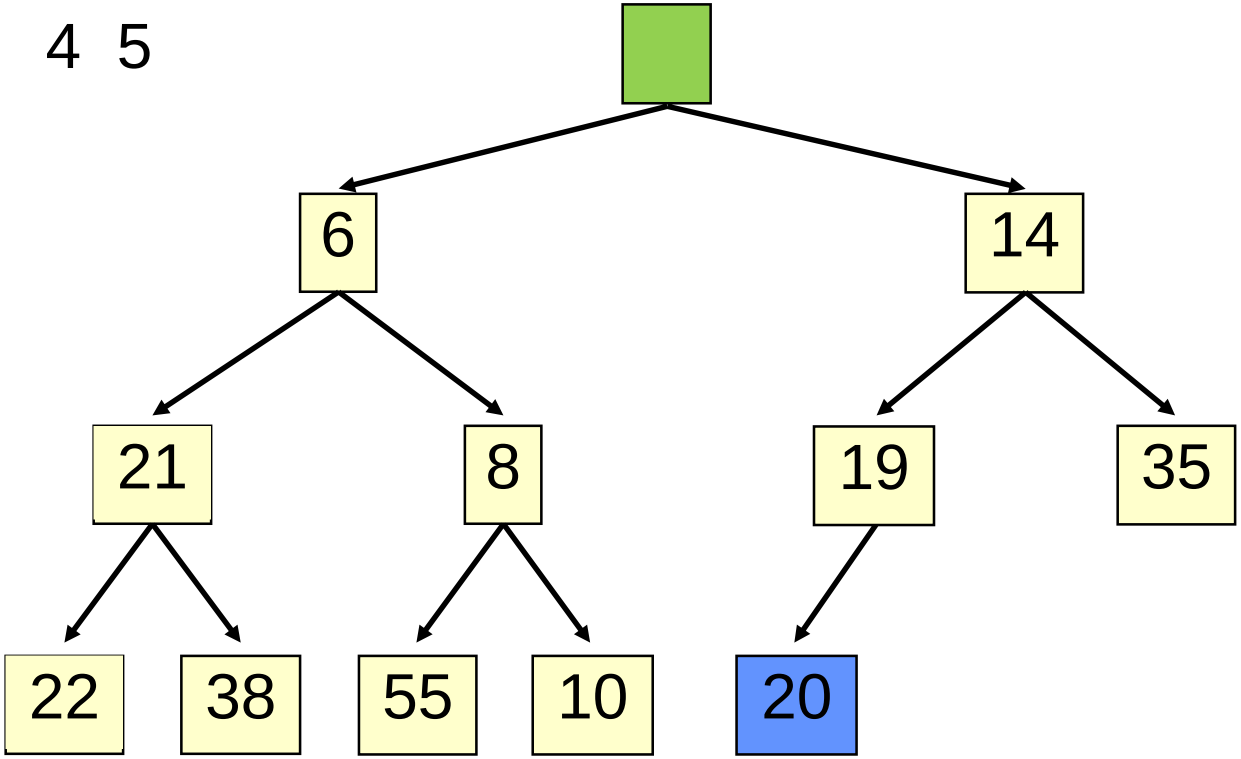

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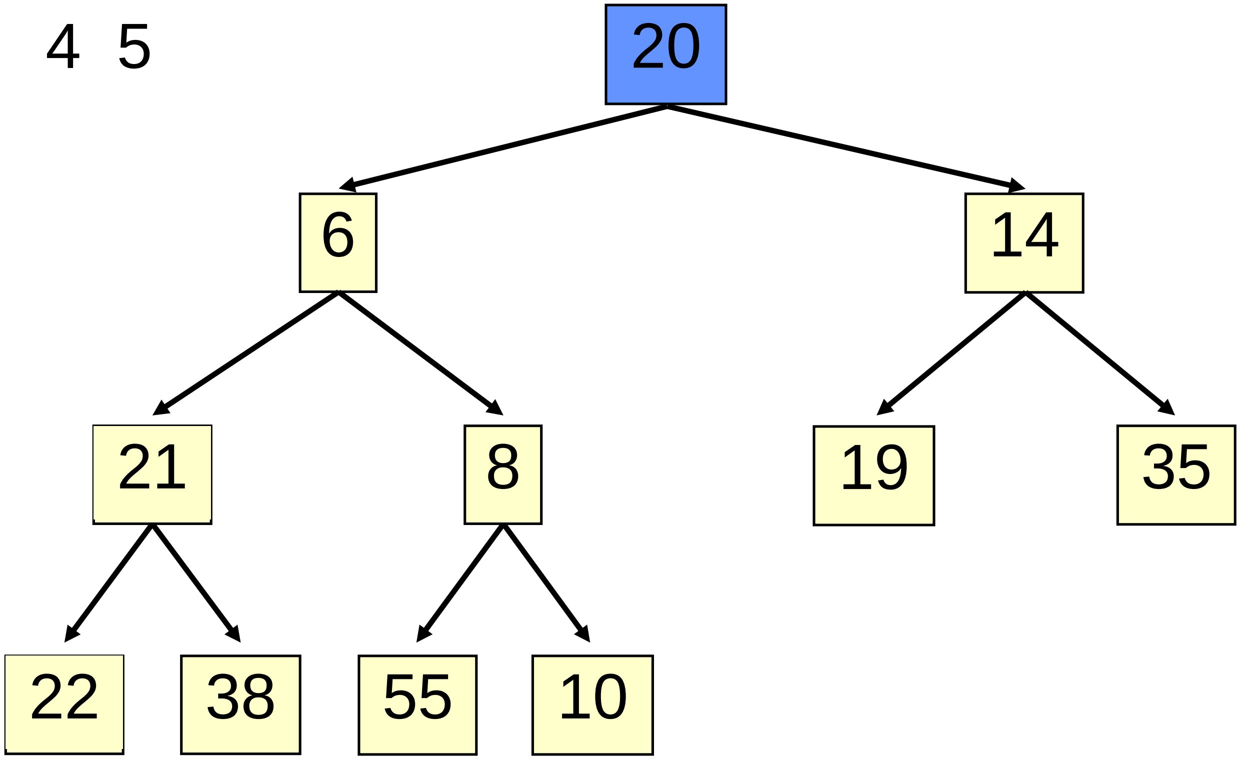

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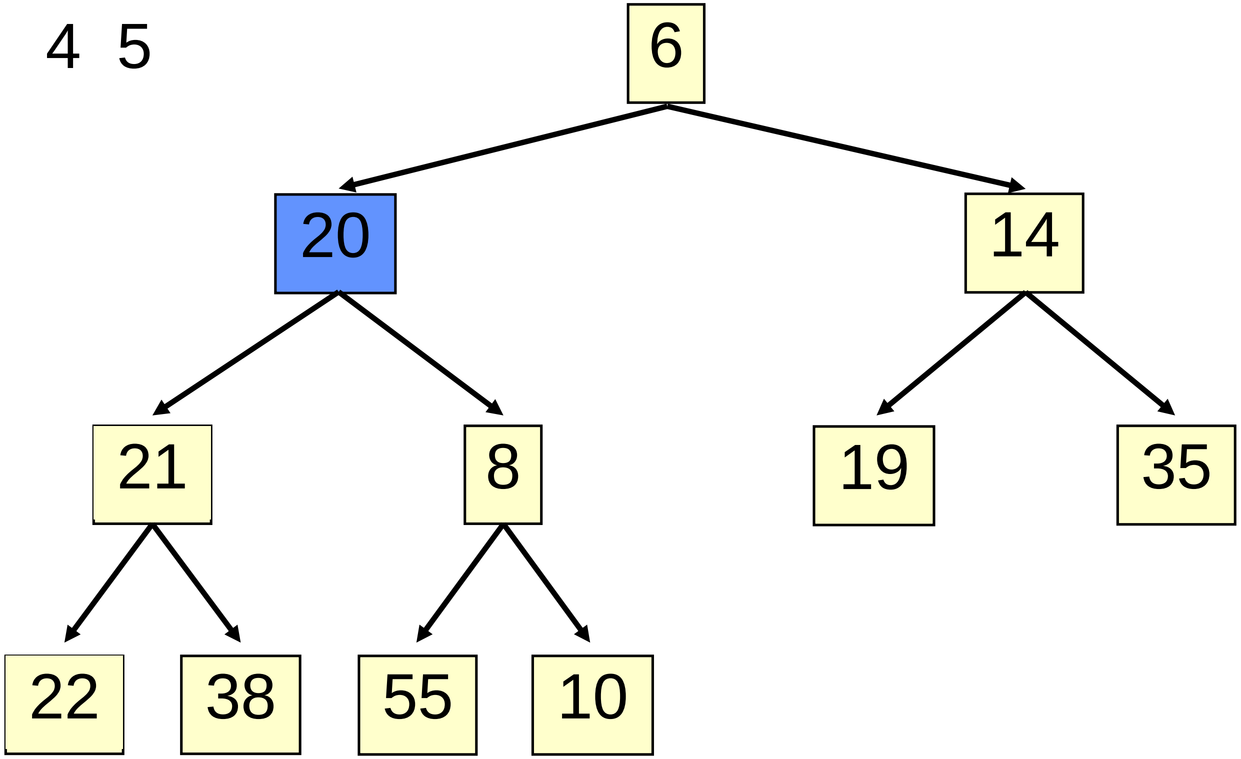

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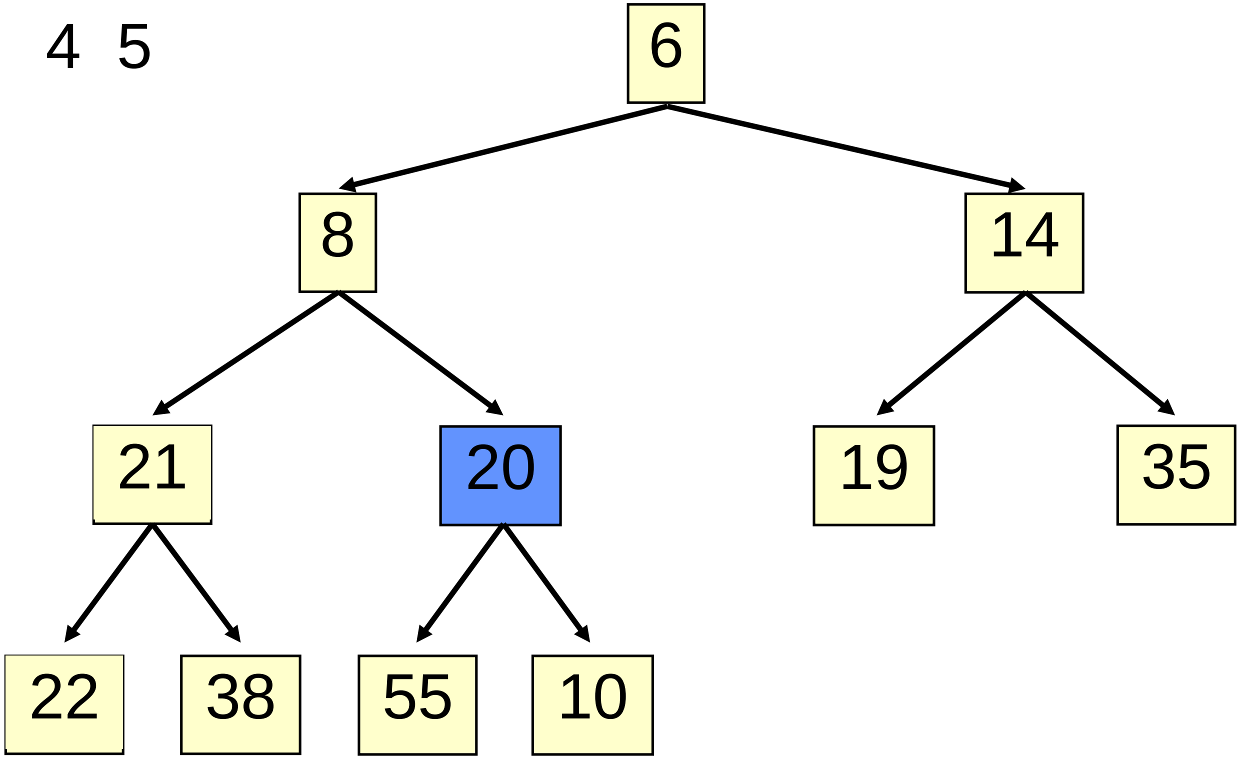

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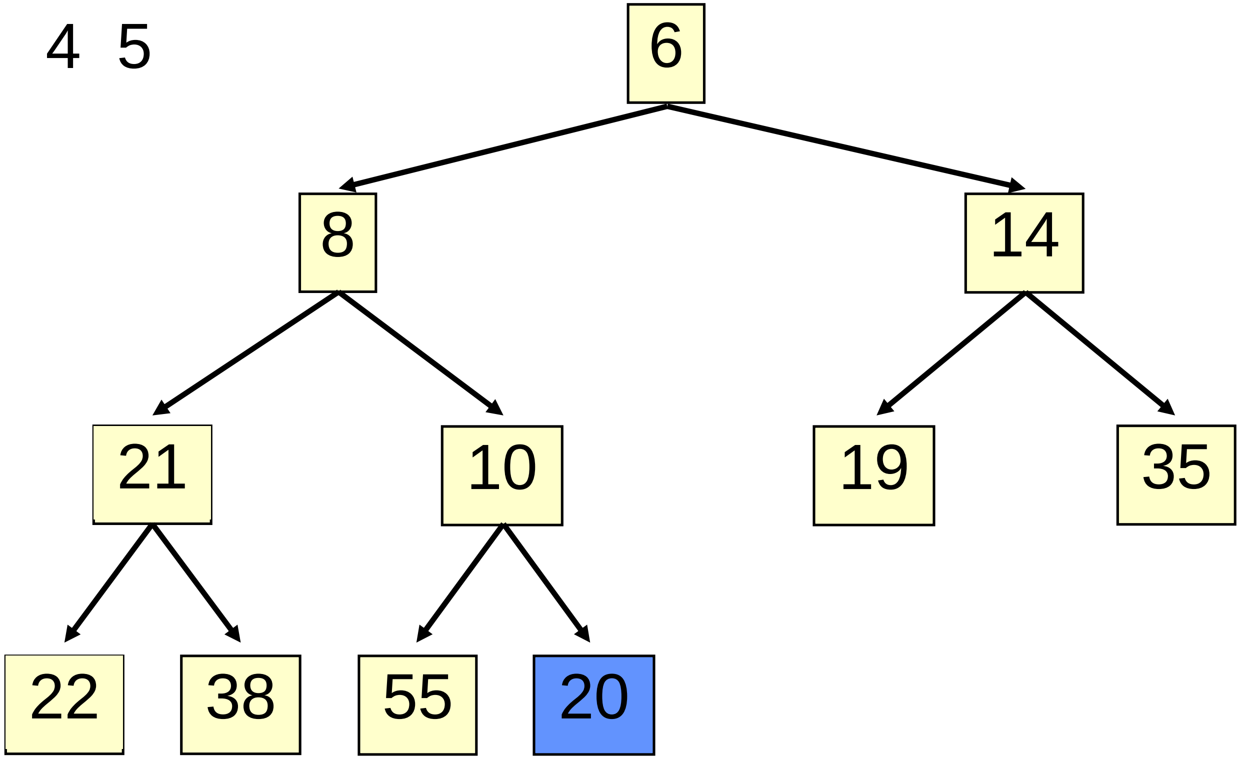

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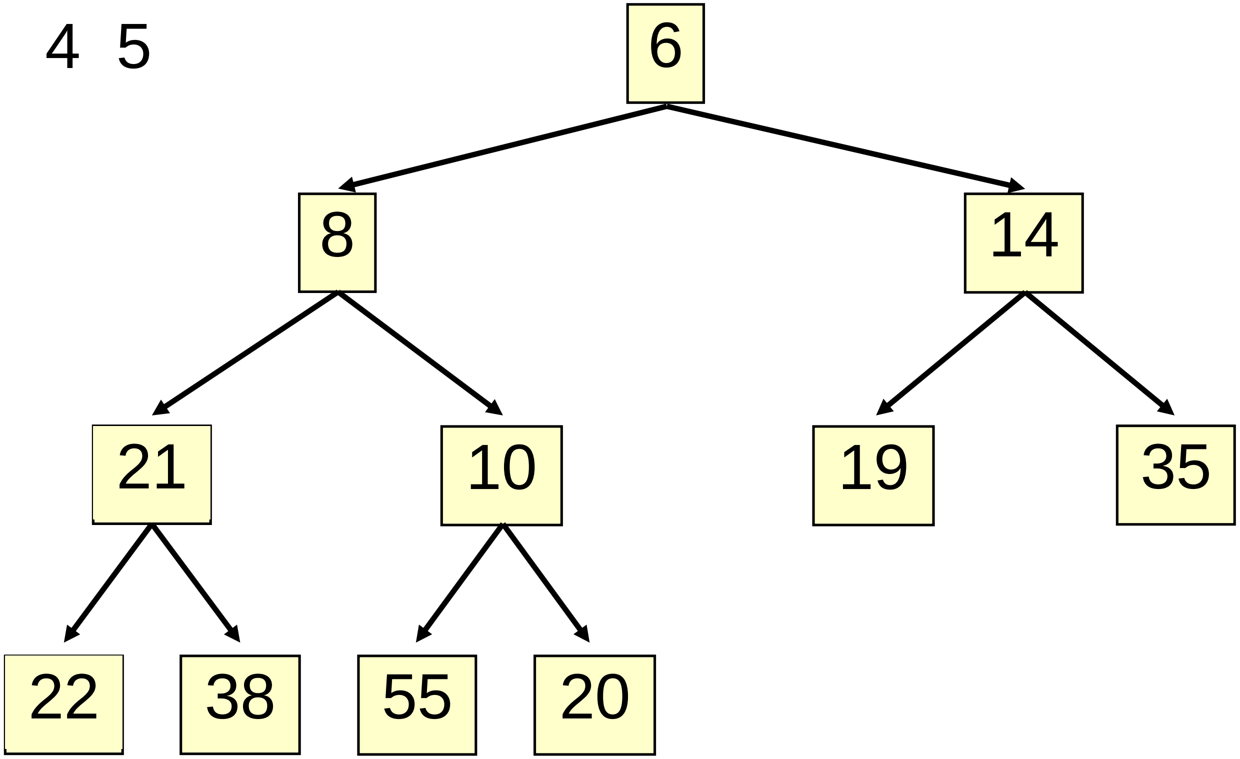

3. Bubble root value down

`poll()`

- Save the least element (the root)
- Assign last element of the heap to the root.
- Remove last element of the heap.
- Bubble element down –always with smaller child, until heap invariant is true again.

The heap invariant is maintained!

- Return the saved element

Time is O(log n), since the tree is balanced

`poll()`. Remember, heap is in b[0..n-1]


```
/** Remove and return the smallest element
  * (return null if list is empty) */
public E poll() {
    if (n == 0) return null;
    E v=  b[0];    // smallest value at root.
    n= n – 1;      // move last
    b[0]= b[n];    // element to root
    bubbleDown(0);
    return v;
}
```

c's smaller child


```
/** Tree has n node.
 *  Return index of smaller child of node k
    (2k+2 if k >= n) */
public int smallerChild(int k, int n) {
    int c= 2*k + 2;     // k's right child
    if (c >= n || b[c-1].compareTo(b[c]) < 0)
       c= c-1;
    return c;
}
```

```
/** Bubble root down to its heap position.
    Pre: b[0..n-1] is a heap except maybe b[0] */
private void bubbleDown() {
    int k= 0;
    int c= smallerChild(k, n);
   // inv: b[0..n-1] is a heap except maybe b[k] AND
   //      b[c] is b[k]'s smallest child
   while ( c < n &&  b[k].compareTo(b[c]) > 0) {

      swap(b[k], b[c]);
      k= c;
      c= smallerChild(k, n);
   }
}
```

Change heap behaviour a bit

Separate priority from value and do this:

add(e, p); //add element e with priority p (a double)

THIS IS EASY!

Be able to change priority

change(e, p); //change priority of e to p

THIS IS HARD!

Big question: How do we find e in the heap?
Searching heap takes time proportional to its size! No good!
Once found, change priority and bubble up or down. OKAY

Assignment A6: implement this heap! Use a second data structure to make change-priority expected log n time

HeapSort(b, n) —Sort b[0..n-1]

Whet your appetite –use heap to get exactly n log n in-place sorting algorithm. 2 steps, each is O(n log n)

1. Make b[0..n-1] into a max-heap (in place)

1. for (k= n-1; k > 0; k= k-1) {
   b[k]= poll –i.e. take max element out of heap.
   }

This algorithm is on course website

A max-heap has max value at root