

## Application of Recursion

$\square$ So far, we have discussed recursion on integers
$\square$ Factorial, fibonacci, $a^{n}$, combinatorials
$\square$ Let us now consider a new application that shows off the full power of recursion: parsing
$\square$ Parsing has numerous applications: compilers, data retrieval, data mining,...

Pointers. DO visit the java spec website

Parse trees: Text page 592 (23.34), Figure 23-31

- Definition of Java Language, sometimes useful: http:// docs.oracle.com/iavase/specs/ils/se7/html/index.htm
- Grammar for most of Java, for those who are curious: http://csci.csusb.edu/dick/samples/java.syntax.html


## Homework:

$\square$ Learn to use these Java string methods:
s.length, s.charAt(), s.indexOf(), s.substring(), s.toCharArray(), $\mathrm{s}=$ new string(char[] array).
$\square$ Hint: These methods will be useful on prelim1! (They can be useful for parsing too...)

## Motivation

$\square$ The cat ate the rat.
$\square$ The cat ate the rat slowly.
$\square$ The small cat ate the big rat slowly.
$\square$ The small cat ate the big rat on the mat slowly.
$\square$ The small cat that sat in the hat ate the big rat on the mat slowly, then got sick.

- ...
http://docs.oracle.com/javase/specs/jls/se7/html/index.html

A Grammar

| Sentence $\rightarrow$ Noun Verb Noun | Grammar: set of rules for |
| :--- | :--- |
| Noun $\rightarrow$ boys | generating sentences of a |
| Noun $\rightarrow$ girls | language. |
| Noun $\rightarrow$ bunnies |  |
| Verb $\rightarrow$ like | Examples of Sentence: |
| Verb boys see bunnies |  |
|  | - bunnies like girls |

## A Grammar

| Sentence $\rightarrow$ Noun Verb Noun |  |
| :--- | :--- |
| Noun $\rightarrow$ boys | - White space between words does |
| Noun $\rightarrow$ girls | not matter |
| Noun $\rightarrow$ bunnies | - This is a very boring grammar |
| Verb $\rightarrow$ like | because the set of Sentences is finite |
| Verb $\rightarrow$ see |  |

Our sample grammar has these rules:
A Sentence can be a Noun followed by a Verb followed by a Noun
A Noun can be 'boys' or 'girls' or 'bunnies'
A Verb can be 'like' or 'see'

| A Recursive Grammar |  |
| :--- | :--- |
| Sentence $\rightarrow$ Sentence and Sentence |  |
| Sentence $\rightarrow$ Sentence or Sentence |  |
| Sentence $\rightarrow$ Noun Verb Noun |  |
| Noun $\rightarrow$ boys |  |
| Noun $\rightarrow$ girls | Grammar is more interesting than |
| Noun $\rightarrow$ bunnies | Sentences is infinite |
| Verb $\rightarrow$ like | What makes this set infinite? <br> Verb $\rightarrow$ see |
|  | Answer: |
|  | Recursive definition of |
|  | Sentence |

## Detour

What if we want to add a period at the end of every sentence?
Sentence $\rightarrow$ Sentence and Sentence.
Sentence $\rightarrow$ Sentence or Sentence.
Sentence $\rightarrow$ Noun Verb Noun.
Noun $\rightarrow \ldots$
Does this work?
$\underbrace{\text { No! This produces sentences like: }}_{\text {Sentence }}$ girls like boys. and $\underbrace{\text { Sentence }}_{\text {Sentence like bunnies }}$.

Grammars for programming languages

## Sentences with Periods



Grammar describes every possible legal expression You could use the grammar for Java to list every possible Java program. (It would take forever)

Grammar tells the Java compiler how to understand a Java program

Grammar for Simple Expressions (not the best)

| $\mathrm{E} \rightarrow$ integer | Some legal expressions: |
| :---: | :---: |
| $\mathrm{E} \rightarrow(\mathrm{E}+\mathrm{E})$ | - 2 |
| Simple expressions: | - $(3+34)$ |
| $\square \mathrm{An} \mathrm{E}$ can be an integer. | - $((4+23)+89)$ |
| - An E can be '(' followed by an E followed by '+' followed by an E followed by ')' | Some illegal expressions: - (3 |
| Set of expressions defined by this grammar is a recursively-defined set | - $3+4$ |
| $\square$ Is language finite or infinite? | Tokens of this grammar: |
| $\square$ Do recursive grammars always yield infinite languages? | $(+)$ and any integer |

## Parsing

Use a grammar in two ways:

- A grammar defines a
language (i.e., the set of properly structured sentences)
- A grammar can be used to
parse a sentence (thus,
checking if the sentence is in the language)
To parse a sentence is to build a parse tree: much like diagramming a sentence
- Example: Show that

$$
((4+23)+89)
$$

$$
\text { is a valid expression } \mathrm{E} \text { by }
$$

building a parse tree


|  |
| :--- |
| Recursive Descent Parsing |
| Write a set of mutually recursive methods to check if a sentence |
| is in the language (show how to generate parse tree later) |
| One method for each nonterminal of the grammar. The method is |
| completely determined by the rules for that nonterminal. On the |
| next pages, we give a high-level version of the method for |
| nonterminal $\mathrm{E}:$ |
| $\mathrm{E} \rightarrow$ integer |
| $\mathrm{E} \rightarrow$ ( $\mathrm{E}+\mathrm{E}$ ) |



## The scanner constructs tokens

An object scanner of class Scanner is in charge of the input String. It constructs the tokens from the String as necessary.
e.g. from the string " $1464+634$ " build the token " 1464 ", the token " + ", and the token " 634 ".
It is ready to work with the part of the input string that has not yet been processed and has thrown away the part that is already processed, in left-to-right fashion.

$$
\frac{\text { already processed }}{(2+(4+8)} \frac{\text { unprocessed }}{+9)}
$$



Illustration of parsing to check syntax



## Does Recursive Descent Always Work?

Some grammars cannot be used for recursive descent
Trivial example (causes infinite recursion):

$$
\mathrm{S} \rightarrow \mathrm{~b}
$$

$$
\mathrm{S} \rightarrow \mathrm{Sa}
$$

For some constructs, recursive descent is hard to use
Can rewrite grammar

| $\mathrm{S} \rightarrow \mathrm{b}$ | Other parsing techniques |
| :--- | :--- |
| $\mathrm{S} \rightarrow \mathrm{bA}$ | exists - take the compiler |
| $\mathrm{A} \rightarrow \mathrm{a}$ | writing course |
| $\mathrm{A} \rightarrow \mathrm{aA}$ |  |

Grammar that gives precedence to * over +

| Grammar that gives precedence to * over + |  |
| :---: | :---: |
| $\begin{aligned} & \mathrm{E}->\mathrm{T}\{+\mathrm{T}\} \\ & \mathrm{T}->\mathrm{F}\{* \mathrm{~F}\} \\ & \mathrm{F}->\text { integer } \\ & \mathrm{F} \rightarrow(\mathrm{E}) \end{aligned}$  | Notation: \{ xxx \} means 0 or more occurrences of xxx . <br> E: Expression <br> T: Term <br> F: Factor <br> Try to do + first, can't complete tree |

Using a Parser to Generate Code

| $\square$ Code for $2+(3+4)$ | parseE can generate code |
| :---: | :---: |
| PUSH 2 | as follows: |
| PUSH 3 |  |
| PUSH 4 | - For integer i, return string "PUSH" + $\mathrm{i}+$ " n " |
| ADD |  |
| ADD | - For (E1 + E2), return a string containing |
|  | - Code for E1 |
| ADD removes the two top | - Code for E2 |
| values from the stack, adds | *"ADD\n" |
| them, and placed the result on the stack |  |

## Syntactic Ambiguity

Sometimes a sentence has more than one parse tree

$$
\mathrm{S} \rightarrow \mathrm{~A} \mid \operatorname{aaxB}
$$

$\mathrm{A} \rightarrow \mathrm{x} \mid \mathrm{aAb} \quad$ aaxbb can
$\mathrm{B} \rightarrow \mathrm{b} \mid \mathrm{bB}$
axbb can
be parsed in two ways
This kind of ambiguity sometimes shows up in programming languages. In the following, which then does the else go with?
if E1 then if E2 then S1 else S2

## Syntactic Ambiguity

This kind of ambiguity sometimes shows up in programming languages. In the following, which then does the else go with?
if E 1 then if E 2 then S 1 else S 2
This ambiguity actually affects the program's meaning
Resolve it by either
(1) Modify the grammar to eliminate the ambiguity (best)
(2) Provide an extra non-grammar rule (e.g. else goes with closest if)

Can also think of modifying the language (require end delimiters)
Exercises

| Think about recursive calls made to parse and generate code for |
| :--- |
| simple expressions |
| 2 |
| $(2+3)$ |
| $((2+45)+(34+-9))$ |

Derive an expression for the total number of calls made to
parseE for parsing an expression Hint: think inductively
Derive an expression for the maximum number of recursive calls
that are active at any time during the parsing of an expression
(i.e. max depth of call stack)

## Exercises

## 26 Write a grammar and recursive program for sentence

palindromes that ignores white spaces \& punctuation

| Was it Eliot's toilet I saw? | No trace; not one carton |
| :--- | :--- |
| Go deliver a dare, vile dog! | Madam, in Eden I'm Adam |

Write a grammar and recursive program for strings $A^{n} B^{n}$ AB AABB AAAAAAABBBBBBB
Write a grammar and recursive program for Java identifiers $<$ letter> $[<\text { letter> or }<\text { digit> }>]^{0 \ldots \mathrm{~N}}$ j27, but not 2 j 7 that are active at any time during the parsing of an expression

