Median Finding Algorithm

Submitted By:

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Problem Definition

- **Given** a set of "n" unordered numbers we want to find the "k th" smallest number. (k is an integer between 1 and n).

A Simple Solution

- A simple sorting algorithm like heapsort will take Order of $O(nlg_2n)$ time.

Step	Running Time
Sort n elements using heapsort	$O(nlog_2n)$
Return the k^{th} smallest element	$O(1)$
Total running time	$O(nlog_2n)$

Linear Time selection algorithm

- Also called Median Finding Algorithm.
- Find k th smallest element in O (n) time in worst case.
- Uses Divide and Conquer strategy.
- Uses elimination in order to cut down the running time substantially.

Steps to solve the problem

- Step 1: If n is small, for example n<6, just sort and return the k^{th} smallest number in constant time i.e; O(1) time.

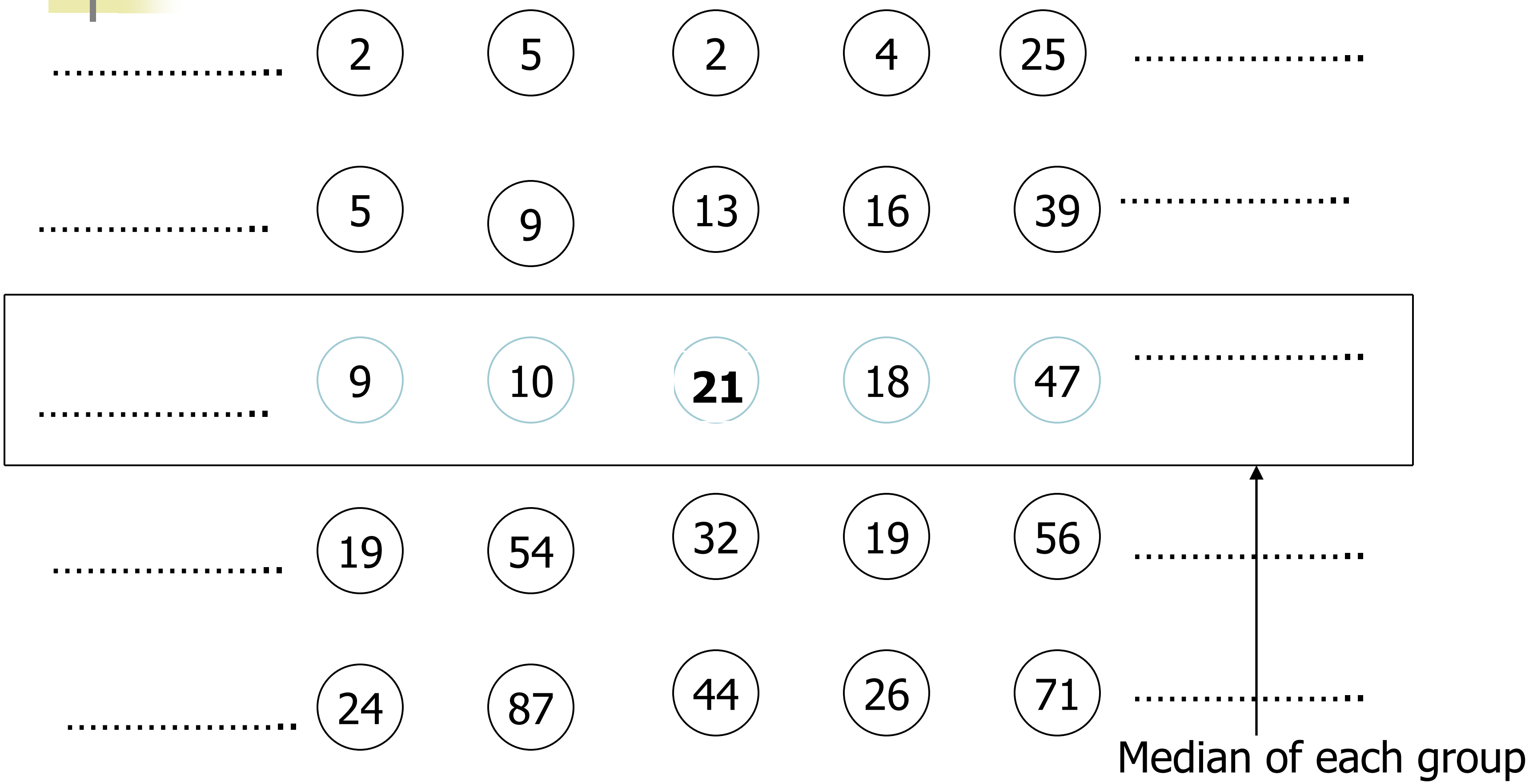
- Step 2: Group the given number in subsets of 5 in O(n) time.

- Step3: Sort each of the group in O (n) time. Find median of each group.
- Given a set (……..2,5,9,19,24,54,5,87,9,10,44,32,21,13,24,18,26,16,19,25,39,47,56,71,91,61,44,28………) having n elements.

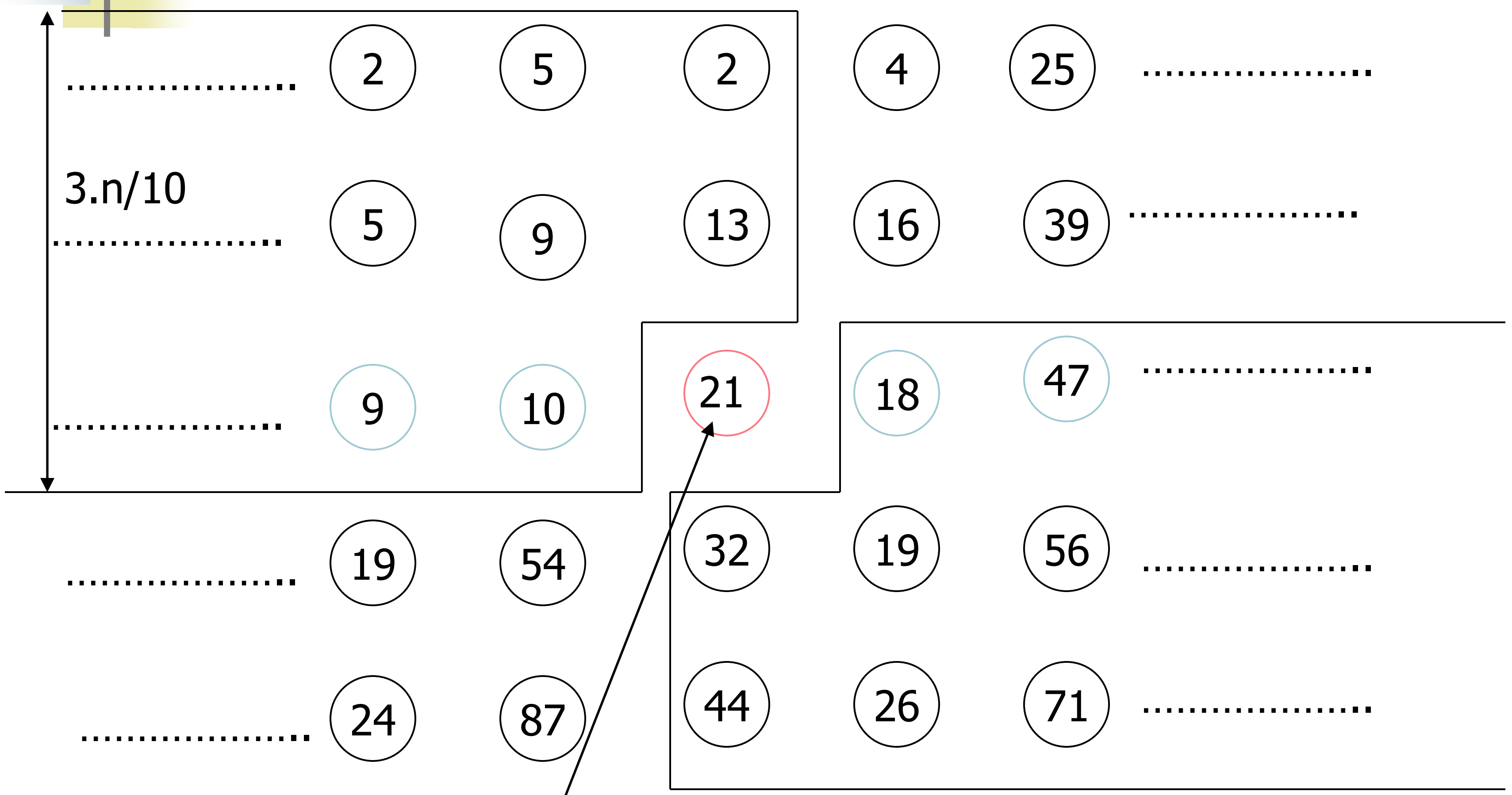
Arrange the numbers in groups of five

………………..	2	54	44	4	25	………………..
………………..	5	5	32	18	39	………………..
………………..	9	87	21	26	47	………………..
………………..	19	9	13	16	56	………………..
………………..	24	10	2	19	71	………………..

Find median of N/5 groups



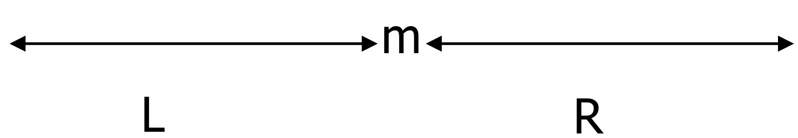
Find the Median of each group



Find m ,the median of medians

Find the sets L and R

- Compare each n-1 elements with the median m and find two sets L and R such that every element in L is smaller than M and every element in R is greater than m.



3n/10<L<7n/10

3n/10<R<7n/10

Description of the Algorithm step

- If n is small, for example n<6, just sort and return the k the smallest number.(Bound time- 7)
- If n>5, then partition the numbers into groups of 5.(Bound time n/5)
- Sort the numbers within each group. Select the middle elements (the medians). (Bound time- 7n/5)
- Call your "Selection" routine recursively to find the median of n/5 medians and call it m. (Bound time-$T_{n/5}$)
- Compare all n-1 elements with the median of medians m and determine the sets L and R, where L contains all elements <m, and R contains all elements >m. Clearly, the rank of m is r=|L|+1 (|L| is the size or cardinality of L). (Bound time- n)

Contd….

- If k=r, then return m
- If k<r, then return k th smallest of the set L .(Bound time $T_{7n/10}$)
- If k>r, then return k-r th smallest of the set R.

Recursive formula

- $T(n)=O(n) + T(n/5) + T(7n/10)$

  We will solve this equation in order to get the complexity.

  We assume that $T(n) < C*n$

  $T(n) = a*n + T(n/5) + T(7n/10)$

  $C*n >= T(n/5) + T(7n/10) + a*n$

  $C*n >= C*n/5 + C*7*n/10 + a*n$

  $C >= 9*C/10 + a$

  $C/10 >= a$

  $C >= 10*a$

  There is such a constant that exists....so $T(n) = O(n)$

Why group of 5 why not some other term??

- If we divide elements into groups of 3 then we will have

  $T(n) = O(n) + T(n/3) + T(2n/3)$ so $T(n) > O(n)$.....
- If we divide elements into groups of more than 5, the value of constant 5 will be more, so grouping elements in to 5 is the optimal situation.