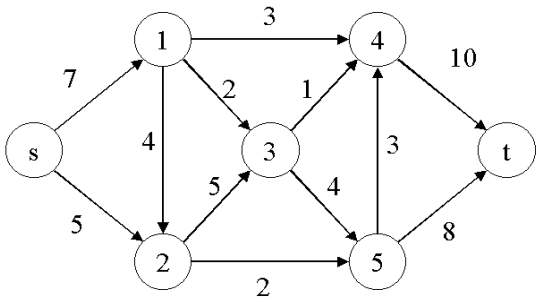
CS 211 Final Exam Spring 2008

All questions are equally weighted – don't spend too long on any one question.

1. Show the steps in finding the maximum flow in the following network:



2. You are given a set of N sticks, which are lying on top of each other in some configuration. A stick may be picked up only if there is no stick on top of it. Explain in words and pseudo-code an algorithm to determine whether the sticks can all be picked up and to give a sequence of stick pickups that does this if possible. You may assume that you can access the sticks as in an array and that given two sticks A and B you can determine if A is on top of B. Illustrate your algorithm on the following sticks: A, B, C, D, E, F, G, where A>C, G>C, B>A, B>G, F>G, F>E, D>F, D>E (where A>C means A is on top of C).

3. Consider the following greedy strategy for finding a shortest weighted path from vertex `start` to vertex `goal` in a given connected graph, then answer the question at the end.

a. Initialize `path` to `start`.
b. Initialize `VisitedVertices` to {`start`}.
c. If `start`==`goal`, return `path` and exit. Otherwise continue.
d. Find the edge (`start`, `v`) of minimum weight such that `v` is adjacent to `start` and `v` is not in `VisitedVertices`.
e. Add `v` to `path` and add `v` to `VisitedVertices`.
f. Set `start` equal to `v` and go to step c.

Does this greedy strategy always find a shortest path from start to goal? Either explain intuitively why it works or give a counter example (i.e., give an example of a weighted graph, a start vertex and a goal vertex, where the given algorithm does not give the shortest path, and explain briefly what goes wrong).

4. Explain how to solve the following problem using a shortest-path algorithm: The input is a two-dimensional array `a[][]` of doubles indicating the exchange rates between currencies. So the entry `a[i][j]` is the amount of currency j obtained for one unit of currency `i.` If `i` corresponds to the US Dollar and `j` corresponds to the Euro, then `a[i][j]` would be about .65 (one dollar buys roughly 0.65 euros) and `a[j][i]` would be about 1/.65 or 1.55. Determine if there is a sequence of exchanges that makes money. i.e., if `a[0][1] = 2`, `a[1][2] =2`, and `a[2][0] = .3`, then exchanging 0's for 1's, then 1's for 2's, then 2's for 0's results in 1.2 0's for each 0 exchanged! You must describe how to set up the problem and how to solve it, including describing the algorithm you use to solve it.

5. Write a method that takes as input a binary search tree, T, and two keys k1 and k2, which are ordered so that k1 <= k2, and prints all elements X in the tree such that k1 <= key(X) <= k2. You may assume that the keys are of type Comparable. Your program should run in time O(K + log N), where K is the number of keys printed.

6. Write code for a class of general binary trees (not so specific as a binary search tree) and for a class of tree iterators. Now consider the following. Suppose that you are given two binary heaps A and B. Assume that both are full, complete trees containing $2^a - 1$ and $2^b - 1$ nodes respectively.

a. Give an O(log N) algorithm to merge the two heaps if a = b.
b. Give an O(log N) algorithm to merge the two heaps if |a – b| = 1.
c. Give an O($\log^2$ N) algorithm to merge the two heaps regardless of a and b.

7. If there's a topic you'd prepped for but which wasn't on this exam, give a short exposition of it with illustrative examples.