Standard ADTs

Lecture 16
CS211 – Fall 2006

Abstract Data Types (ADTs)

- A method for achieving abstraction for data structures and algorithms
- ADT = model + operations
- Describes what each operation does, but not how it does it
- An ADT is independent of its implementation
- In Java, an interface corresponds well to an ADT
  - The interface describes the operations, but says nothing at all about how they are implemented
- Example: Stack interface/ADT

```
public interface Stack {
   public void push (Object x);
   public Object pop ();
   public Object peek ();
   public boolean isEmpty ();
   public void makeEmpty ();
}
```

Queues & Priority Queues

- ADT Queue
  - Operations:

```
void enQueue (Object x);
Object deQueue ( );
Object peek ( );
boolean isEmpty ( );
void makeEmpty ( );
```

- Where used:
  - Simple job scheduler (e.g., print queue)
  - Wide use within other algorithms
- ADT PriorityQueue
  - Operations:

```
void insert (Object x);
Object getMax ( );
Object peekAtMax ( );
boolean isEmpty ( );
void makeEmpty ( );
```

- Where used:
  - Job scheduler for OS
  - Event-driven simulation
  - Can be used for sorting
  - Wide use within other algorithms

Sets

- ADT Set
  - Operations:

```
void insert (Object element);
boolean contains (Object element);
void remove (Object element);
boolean isEmpty ( );
void makeEmpty ( );
```

- Where used:
  - Wide use within other algorithms
- Note: no duplicates allowed
  - A "set" with duplicates is usually called a *bag*

Dictionaries

- ADT Dictionary
  - Operations:

```
void insert (Object key, Object value);
void update (Object key, Object value);
Object find (Object key);
void remove (Object key);
boolean isEmpty ( );
void makeEmpty ( );
```

- Think of: key = word; value = definition
- Where used:
  - Symbol tables
  - Wide use within other algorithms

Data Structure Building Blocks

- These are *implementation* "building blocks" that are often used to build more-complicated data structures
  - Arrays
  - Linked Lists
    - Singly linked
    - Doubly linked
  - Binary Trees
  - Graphs
    - Adjacency matrix
    - Adjacency list

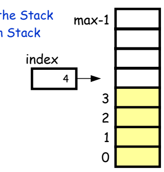
Array Implementation of Stack

```
class ArrayStack implements Stack {

   private Object[] array;      // Array that holds the Stack
   private int index = 0;       // First empty slot in Stack

   public ArrayStack (int maxSize)
         {array = new Object[maxSize];}

   public void push (Object x) {array[index++] = x;}
   public Object pop () {return array[--index];}
   public Object peek () {return array[index-1];}
   public boolean isEmpty () {return index == 0;}
   public void makeEmpty () {index = 0;}
}
// Better for garbage collection if makeEmpty( ) also cleared the array
```



O(1) worst-case time for each operation

Linked List Implementation of Stack

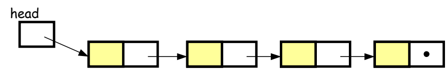
```
class ListStack implements Stack {

   private Node head = null;  // Head of list that holds the Stack

   public void push (Object x) {head = new Node(x, head);}
   public Object pop ()
     {Node temp = head; head = head.next; return temp.data;}
   public Object peek () {return head.data;}
   public boolean isEmpty () {return head == null;}
   public void makeEmpty () {head = null;}
}
```

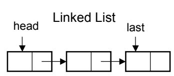
O(1) worst-case time for each operation

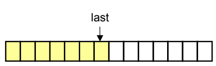
Note that array implementation can overflow, but the linked list version can't



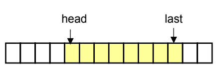
Queue Implementations

- Possible implementations





Array with head always at A[0]
(deQueue( ) becomes expensive)
(can overflow)



Array with wraparound
(can overflow)

- Recall: operations are enQueue, deQueue, peek,…
  - For linked-list
    - All operations are O(1)
  - For array with head at A[0]
    - deQueue takes time O(n)
    - Other ops are O(1)
    - Can overflow
  - For array with wraparound
    - All operations are O(1)
    - Can overflow

Choosing an Implementation

- What operations do I need to perform on the data?
  - Insertion, deletion, searching, reset to initial state?
- How efficient do the operations need to be?
- Are there any additional constraints on the operations or on the data structure?
  - Can there be duplicates?
  - When extracting elements, does order matter?
- Is there a known upper bound on the amount of data? Or can it grow unboundedly large?

Goal: Design a *Dictionary*

- Operations
  - void insert (key, value)
  - void update (key, value)
  - Object find (key)
  - void remove (key)
  - boolean isEmpty ()
  - void makeEmpty ()

Array implementation:

Using an array of (key,value) pairs

	Unsorted	Sorted
insert	O(1)	O(n)
update	O(n)	O(log n)
find	O(n)	O(log n)
remove	O(n)	O(n)

n is the number of items currently held in the dictionary

Direct Address Table

- Assumes the key set is from a small *Universe*
- Example: Addresses on my street
  - Started at 1, go to 40
  - A few lots don't have houses
- For a *Direct Address Table*, we make an array as large as the *Universe*
- To find an entry, we just index to that entry of the array
- Dictionary operations all take O(1) time

What if the Universe is large?

- Idea is to re-use table entries via a *hash function* h
- h: U → [0,…,m-1] where m = table size
- h must
  - Be easy to compute
  - Cause few *collisions*
  - Have equal probability for each table position

Typical situation:

U = all legal identifiers

Typical hash function:

h converts each letter to a number and we compute a function of these numbers

A Hashing Example

- Suppose each word below has the following hashCode

jan	7
feb	0
mar	5
apr	2
may	4
jun	7
jul	3
aug	7
sep	2
oct	5

- How do we resolve collisions?
  - We'll use chaining: each table position is the head of a list
  - For any particular problem, this *might* work terribly
- In practice, using a good hash function, we can assume each position is equally likely

Analysis for Hashing with Chaining

- Analyzed in terms of *load factor* λ = n/m = (items in table)/(table size)
- We count the expected number of *probes* (key comparisons)
- Goal: Determine U = number of probes for an *unsuccessful* search

- Claim U is the same as the average number of items per table position = n/m = λ
- Claim S = number of probes for a *successful* search = 1 + $\lambda/2$

Table Doubling

- We know each operation takes time O(λ) where λ=n/m
- But isn't $\lambda = \Theta(n)$?
- What's the deal here? It's still linear time!

Table Doubling:

- Set a bound for λ (call it λ_0)
- Whenever λ reaches this bound we
  - Create a new table, twice as big and
  - Re-insert all the data
- Easy to see operations *usually* take time O(1)
  - But sometimes we copy the whole table

Analysis of Table Doubling

- Suppose we reach a state with n items in a table of size m and that we have just completed a table doubling

	Copying Work
Everything has just been copied	n inserts
Half were copied previously	n/2 inserts
Half of those were copied previously	n/4 inserts
…	…
Total work	n + n/2 + n/4 + … = 2n

Analysis of Table Doubling, Cont'd

- Total number of insert operations needed to reach current table = copying work + initial insertions of items = 2n + n = 3n inserts
- Each insert takes expected time O(λ_0) or O(1), so total expected time to build entire table is O(n)
- Thus, expected time per operation is O(1)

- Disadvantages of table doubling:
  - Worst-case insertion time of O(n) is definitely achieved (but rarely)
  - Thus, not appropriate for time critical operations

Java Hash Functions

- Most Java classes implement the hashCode() method
- hashCode() returns an int
- Java's HashMap class uses $h(X)$ = X.hashCode() mod m
- h(X) in detail:

```
int hash = X.hashCode();
int index = (hash & 0x7FFFFFFF) % m;
```

- What hashCode() returns:
  - Integer:
    - uses the int value
  - Float:
    - converts to a bit representation and treats it as an int
  - Short Strings:
    - 37*previous + value of next character
  - Long Strings:
    - sample of 8 characters; 39*previous + next value

hashCode( ) Requirements

- Contract for hashCode() method:
  - Whenever it is invoked in the same object, it must return the same result
  - Two objects that are equal must have the same hash code
  - Two objects that are not equal should return different hash codes, but are not required to do so

Hash Tables in Java

- java.util.HashMap
- java.util.HashSet
- java.util.Hashtable (legacy)
- Use chaining
- Initial (default) size = 101
- Load factor = $\lambda_0 = 0.75$
- Uses table doubling (2*previous+1)

- A node in each chain looks like this:

hashCode	key	value	next

original hashCode (before mod m)
Allows faster rehashing and (possibly) faster key comparison

Linear & Quadratic Probing

- These are techniques in which all data is stored directly within the hash table array
- Linear Probing
  - Probe at h(X), then at
    - h(X) + 1
    - h(X) + 2
    - …
    - h(X) + i
  - Leads to *primary clustering*
    - Long sequences of filled cells
- Quadratic Probing
  - Similar to Linear Probing in that data is stored within the table
  - Probe at h(X), then at
    - h(X)+1
    - h(X)+4
    - h(X)+9
    - …
    - $h(X)+ i^2$
  - Works well when
    - $\lambda < 0.5$
    - Table size is prime

Hash Table Pitfalls

- Good hash function is required
- Watch the load factor (λ), especially for Linear & Quadratic Probing

Dictionary Implementations

- Ordered Array
  - Better than unordered array because Binary Search can be used
- Unordered Linked-List
  - Ordering doesn't help
- Direct Address Table
  - Small universe $\Rightarrow$ limited usage
- Hashtables
  - O(1) expected time for Dictionary operations

- Goal: Want ability to *report-in-order*, but can't afford inefficiency of ordered array
- Idea: Use a Binary Search Tree (BST)
- BST Property: