More Lists & Trees

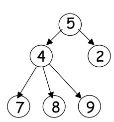
Lecture 7
CS211 – Fall 2006

Announcements

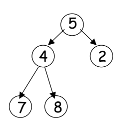
- Academic Integrity Reminder
  - We treat AI violations seriously
  - The AI Hearing process is unpleasant
    - Please help us avoid this process by maintaining Academic Integrity
  - We test all pairs of submitted programming assignments for similarity
    - Similarities are caught even if variables are renamed

Tree Overview

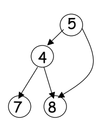
- *Tree*: recursive data structure (similar to list)
  - Each cell may have two or more successors (children)
  - Each cell has at most one predecessor (parent)
    - Distinguished cell called *root* has no parent
  - All cells are reachable from *root*
- *Binary tree*: tree in which each cell can have at most two children: a left child and a right child



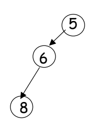
General tree



Binary tree



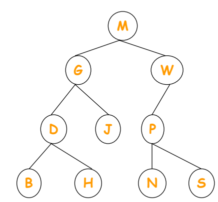
Not a tree



List-like tree

Tree Terminology

- M is the *root* of this tree
- G is the *root* of the *left subtree* of M
- B, H, J, N, and S are *leaves*
- N is the *left child* of P; S is the *right child*
- P is the *parent* of N
- M and G are *ancestors* of D
- P, N, and S are *descendents* of W
- Node J is at *depth* 2 (i.e., *depth* = length of path from root)
- Node W is at *height* 2 (i.e., *height* = length of longest path from leaf)
- A collection of several trees is called a ??



Class for Binary Tree Cells

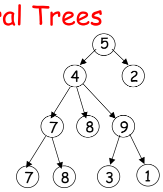
```
class TreeCell {
    private Object datum;
    private TreeCell left;
    private TreeCell right;

    public TreeCell (Object x) {datum = x;}
    public TreeCell (Object x, TreeCell l, TreeCell r) {
       datum = x;
       left = l;
       right = r;
    }
   Continues with methods called getDatum, setDatum, getLeft,
   setLeft, getRight, setRight with obvious code
}
```

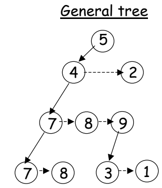
Class for General Trees

```
class GTreeCell{
  private Object datum;
  private GTreeCell left;
  private GTreeCell sibling;
  ....appropriate getter and setter
   methods
}
```

- Parent node points directly only to its leftmost child
- Leftmost child has pointer to next sibling which points to next sibling, etc.



General tree



Tree represented using GTreeCell

Applications of Trees

- Most languages (natural and computer) have a recursive, hierarchical structure
- This structure is implicit in ordinary textual representation
- Recursive structure can be made explicit by representing sentences in the language as trees: Abstract Syntax Trees (ASTs)
- ASTs are easier to optimize, generate code from, etc. than textual representation
- Converting textual representations to AST: job of parser!

Example

- Expression grammar:
  - E → integer
  - E → (E + E)
- In textual representation
  - Parentheses show hierarchical structure
- In tree representation
  - Hierarchy is explicit in the structure of the tree.

Text | AST Representation

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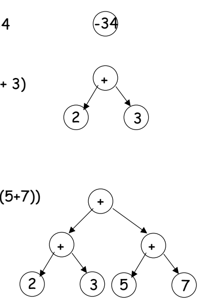
(2 + 3)

((2+3) + (5+7))



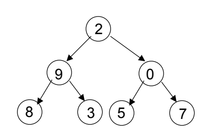
Recursion on trees

- Recursive methods can be written to operate on trees in the obvious way
- In most problems
  - Base case: empty tree
    - Sometimes base case is leaf node
  - Recursive case: solve problem on left and right sub-trees then put solutions together to compute solution for full tree

Searching in a Binary Tree

- Analog of linear search in lists: Given tree and an object, find out if object is stored in tree
- Trivial to write recursively; harder to write iteratively

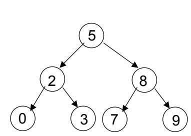
```
public static boolean treeSearch (Object x, TreeNode node) {
    if (node  == null) return false;
    return node.datum.equals(x) ||
                treeSearch(x, node.lchild) ||
                treeSearch(x, node.rchild);
}
```



Binary Search Tree (BST)

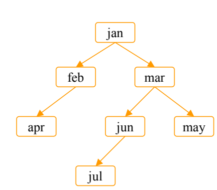
- Idea: tree nodes are ordered
  - All *left* descendents of node come *before* node
  - All *right* descendents of node come *after* node
- This makes it *much* faster to search

```
public static boolean treeSearch (Object x, TreeNode node) {
    if (node  == null) return false;
    if (node.datum.equals(x)) return true;
    if (node.datum.compareTo(x) < 0)
       return treeSearch(x, node.lchild);
    return treeSearch(x, node.rchild);
}
```



Building a BST

- To insert a new item
  - Pretend to look for the item
  - Put the new node in the place where you fall off the tree
- This can be done using either recursion or iteration
- Example
  - Tree uses *alphabetical order*
  - Months appear for insertion in *calendar order*
    - This way the months are in "random" order alphabetically



TreeNode

- This version is for a tree of Strings

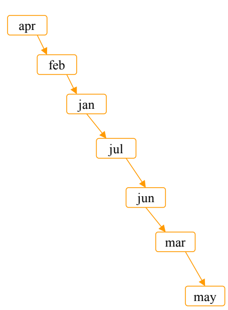
```
class TreeNode {

    String datum;                          // Data for a node
    TreeNode lchild, rchild;               // Left and right children

    public TreeNode (String datum) {       // Constructor
        this.datum = datum;
        lchild = null; rchild = null;
    }
}
```

BST Code

```
public class BST {

    TreeNode root;    // The root of the BST

    public BST () {
        root = null;
    }

    public void insert (String string) {
        root = insert(string, root);
    }

    private static TreeNode insert (String string, TreeNode node) {
        if (node == null) return new TreeNode(string);
        int compare = string.compareTo(node.datum);
        if (compare < 0) node.lchild = insert(string, node.lchild);
        else if (compare > 0) node.rchild = insert(string, node.rchild);
        return node;
    }
}
```

What Can Go Wrong?

- A BST makes searches very fast *unless*...
  - Nodes are inserted in alphabetical order
  - In this case, we're basically building a linked list (with some extra wasted space for the lchild fields that aren't being used)
- BST works great if data arrives in random order



Printing Contents of BST

- Because of the ordering rules for a BST, it's easy to print the items in alphabetical order
  - Recursively print everything in the left subtree
  - Print the node
  - Recursively print everything in the right subtree

```
/**
 * Show the contents of the BST in
 * alphabetical order.
 */
public void show () {
    show(root); System.out.println();
}

private static void show (TreeNode node) {
    if (node == null) return;
    show(node.lchild);
    System.out.print(node.datum + " ");
    show(node.rchild);
}
```

Tree Traversals

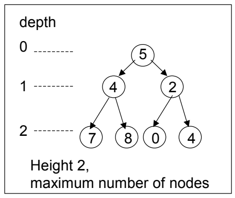
- "Walking" over the whole tree is a tree traversal
  - This is done often enough that there are standard names
  - The previous example is an *inorder traversal*
    - Process left subtree
    - Process node
    - Process right subtree
- Note: we're using this for printing, but any kind of processing can be done
- There are other standard kinds of traversals
  - Preorder traversal
    - Process node
    - Process left subtree
    - Process right subtree
  - Postorder traversal
    - Process left subtree
    - Process right subtree
    - Process node
  - Level-order traversal
    - Not recursive
    - Uses a Queue

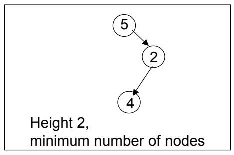
Some Useful Methods

```
// Determine if a TreeNode is a leaf node
public static boolean isLeaf (TreeNode node) {
   return (node != null) && (node.lchild == null) && (node.rchild == null);
}

// Compute height of tree using postorder traversal
public static int height (TreeNode node) {
    if (node == null) return -1; // Height is undefined for empty tree
    if (isLeaf(node)) return 0;
    return 1 + Math.max(height(node.lchild), height(node.rchild));
}

// Compute number of nodes in tree using postorder traversal
public static int nNodes (TreeNode node) {
   if (node == null) return 0;
   return 1 + nNodes(node.lchild) + nNodes(node.rchild);
}
```

Useful Facts about Binary Trees

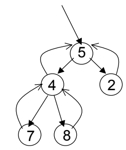
- Maximum number of nodes at depth d = 2^d
- If height of tree is h
  - Minimum number of nodes it can have = h+1
  - Maximum number of nodes it can have = $2^0 + 2^1 + \ldots + 2^h = 2^{h+1} - 1$
- *Full binary tree* (or *complete binary tree*)
  - All levels of tree are completely filled





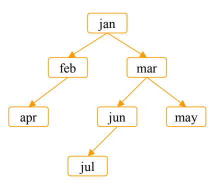
Tree with Parent Pointers

- In some applications, it is useful to have trees in which nodes can reference their parents
  - Tree analog of doubly-linked lists



Things to Think About

- What if we want to delete data from a BST?
- A BST works great as long as it's *balanced*
  - How can we keep it balanced?



List Summary

- A *list* is a sequence of elements
  - Grow and shrink on demand
  - Not random-access, but sequential access
- List operations
  - Create a list
  - Access a list and update data
  - Change structure of list by inserting/deleting cells
- Recursion makes perfect sense on lists; usually have
  - Base case: empty list
  - Recursive case: non-empty list
- Subspecies of lists
  - List with header
  - Doubly-linked lists

Tree Summary

- A *tree* is a recursive data structure
  - Each cell has 0 or more successors (*children*)
  - Each cell except the *root* has at exactly one predecessor (*parent*)
  - All cells are reachable from the *root*
  - A cell with no children is called a *leaf*
- Special case: *binary tree*
  - Binary tree cells have both a left and a right child
  - Either or both children can be null
- Trees are useful for exposing the recursive structure of natural language and computer programs

LISP

- List languages first developed for AI
- LISP: List Processing Language
  - Developed in 50-60's by John McCarthy, et al.
- Lists and list processing are a fundamental part of LISP language
  - Lists are primitive data type
  - Functions operate directly on lists
  - A LISP program is expressed as list of lists
- "car": contents address register (getDatum())
- "cdr": contents decrement register (getNext())
- "caddr" = (car (cdr (cdr list))) = object in 3rd element