

CS 211

Computers and Programming

<http://www.cs.cornell.edu/courses/cs211/2005su>

**Lecture 4: Grammars and Parsing
Summer 2005**

Announcements

- Read Chapter 11 for more about parsing and stacks
- Quiz on Friday: probably 1 Induction problem and two shortish concept problems.
- Check to see if you are on CMS! If you aren't, email us immediately.
- Assignment 1 due Tuesday. No class Monday.
- If you need a programming partner still, see me after class.

Application of Recursion

- So far, we have written recursive programs on integers: factorial, fibonacci, permutations, a^n
- Let us now consider a new application, **grammars and parsing**, that shows off the full power of recursion.
- Parsing has numerous applications: compilers, data retrieval, data mining,....

Motivation

The cat ate the rat.

The cat ate the rat slowly.

The small cat ate the big rat slowly.

The small cat ate the big rat on the mat slowly.

The small cat that sat in the hat ate the big rat on the mat slowly.

The small cat that sat in the hat ate the big rat on the mat slowly, then got sick.

...

- Not all sequences of words are legal sentences
 - *The ate cat rat the*
- How many legal sentences are there?
- How many legal programs are there?
- Are all Java programs that compile legal programs?
- How do we know what programs are legal?
- http://java.sun.com/docs/books/jls/first_edition/html/19.doc.html

Grammars

Sentence => Noun Verb Noun

Noun => boys

Noun => girls

Noun => dogs

Verb => like

Verb => see

- **Grammar:** set of rules for generating sentences in a language.
- Our sample grammar has these rules:
 - a Sentence can be a Noun followed by a Verb followed by a Noun
 - a Noun can be ‘boys’ or ‘girls’ or ‘dogs’
 - a Verb can be ‘like’ or ‘see’
- Examples of Sentence:
 - *boys see dogs*
 - *dogs like girls*
 -
- Note: white space between words does not matter
- This is a very boring grammar because the set of Sentences is finite (exactly 18 sentences).

Recursive Grammar

Sentence => Sentence and Sentence

Sentence => Sentence or Sentence

Sentence => Noun Verb Noun

Noun => boys

Noun => girls

Noun => dogs

Verb => like

Verb => see

- Examples of Sentences in this language:
 - *boys like girls*
 - *boys like girls and girls like dogs*
 - *boys like girls and girls like dogs and girls like dogs*
 - *boys like girls and girls like dogs and girls like dogs and girls like dogs*
 -
- This grammar is more interesting than the one in the last slide because the set of Sentences is infinite.
- What makes this set infinite? Answer: recursive definition of Sentence

Grammar Subtleties

- What if we want to add a period at the end of every sentence?

Sentence => Sentence and Sentence .

Sentence => Sentence or Sentence .

Sentence => Noun Verb Noun .

Noun =>

- Does this work?
- No! This produces sentences like:

girls like boys . and boys like dogs . .

Sentences with Periods

TopLevelSentence => Sentence •

Sentence => Sentence and Sentence

Sentence => Sentence or Sentence

Sentence => Noun Verb Noun

Noun => boys

Noun => girls

Noun => dogs

Verb => like

Verb => see

- Add a new rule that adds a period only at the end of the sentence.
- Thought exercise: how does this work?

Grammar for Simple Expressions

Expression $\Rightarrow$ integer

Expression $\Rightarrow$ (Expression + Expression)

- This is a grammar for simple expressions:
 - An E can be an integer.
 - An E can be ‘(‘ followed by an E followed by ‘+’ followed by an E followed by ‘)’
- Set of Expressions defined by this grammar is a recursively-defined set.
- Is language finite or infinite?
- Do recursive grammars always yield infinite languages?

$$E \Rightarrow \text{integer}$$
$$E \Rightarrow (E + E)$$

- Here are some legal expressions:

$$2$$
$$(3 + 34)$$
$$((4+23) + 89)$$
$$((89 + 23) + (23 + (34+12)))$$

- Here are some illegal expressions:

$$(3$$
$$3 + 4$$

- Each legal expression can be **parsed** into a **parse tree**.

Parsing

- **Parsing:** given a grammar and some text, determine if text is a legal sentence in the language defined by that grammar
- For many grammars (e.g. the simple expression grammar), we can write efficient programs to answer this question.
- Next slides: parser for our small expression language
 - Caveat: code uses CS211In object for doing input from a file.
 - Goal: understand the structure of the code to see the parallel between the language definition (recursive set) and the parser (recursive function)

Helper class: CS211In

- On-line code for the CS211In class
- Code lets you
 - open file for input:
 - `CS211In f = new CS211In(String-for-file-name)`
 - examine what the next thing in file is: `f.peekAtKind()`
 - Integer?: such as 3, -34, 46
 - Word?: such as x, r45, y78z (variable name in Java)
 - Operator?: such as +, -, *, (,), etc.
 - read next thing from file:
 - integer: `f.getInt()`
 - Word: `f.getWord()`
 - Operator: `f.getOp()`

- Useful methods in CS211In class:
 - `f.check(char c)`:
 - Example: `f.check('*');` //true if next thing in input is *
 - Check if next thing in input is c
 - If so, eat it up and return true
 - Otherwise, return false
 - `f.check(String s)`:
 - Example of its use: `f.check("if");`
 - Return true if next thing in input is word `if`

Parser for expression language

```
static boolean expParser(String fileName) { //returns true if file has single expression
    CS211In f = new CS211In(fileName);
    boolean gotIt = getExp(f);
    if (f.peekAtKind() == CS211In.EOF) //no junk in file after expression
        return gotIt;
    else //file contains some junk after expression, so return false
        return false;
}
```

```
static boolean getExp(CS211In f) { //reads one expression from file
//defined on next slide
}
```

Parser for Expression Language

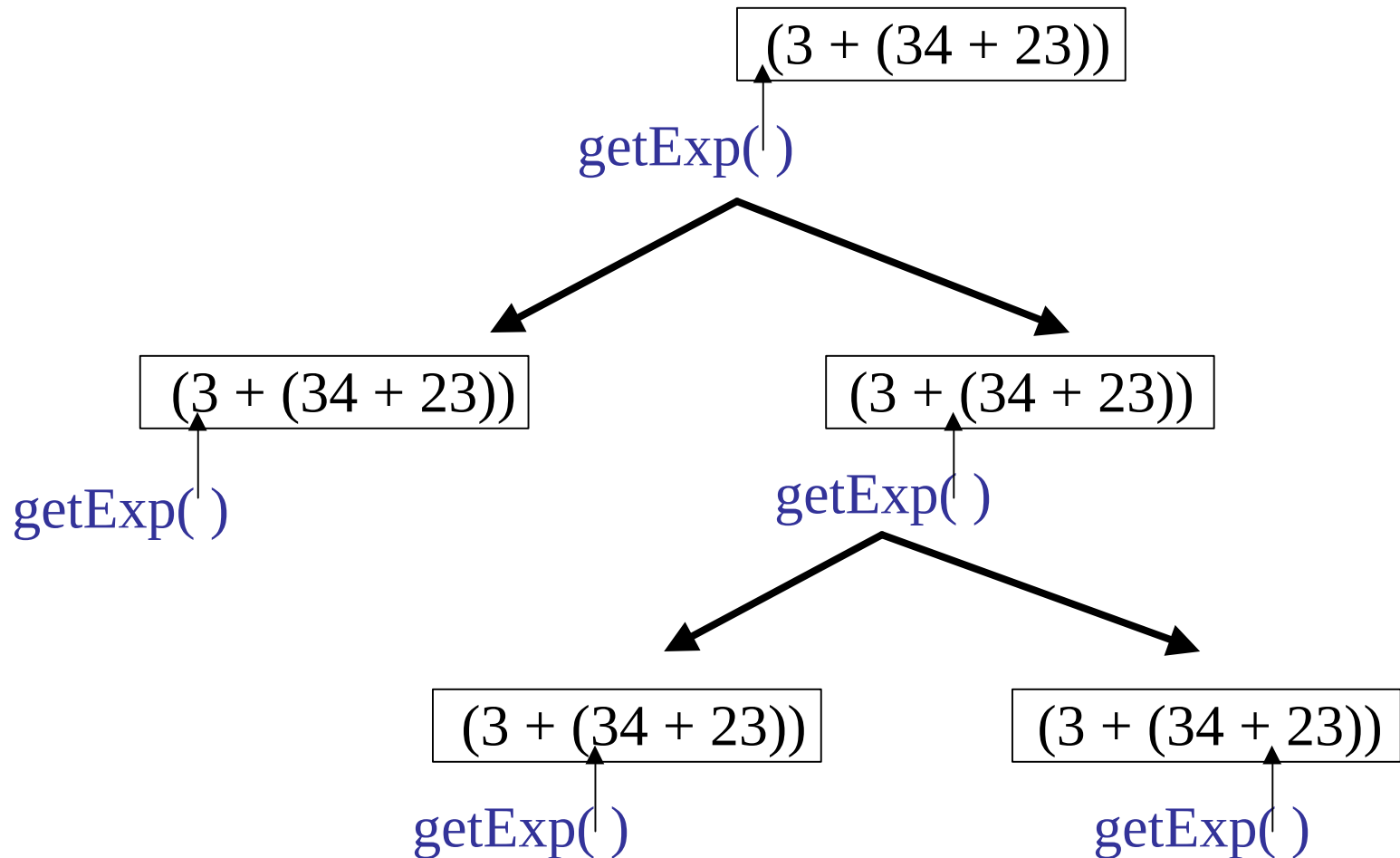
```
static boolean expParser(String fileName) { //returns true if file has single expression
//defined on previous slide
}
```

```
static boolean getExp(CS211In f) { //reads one expression from file
    switch (f.peekAtKind()) {
        case CS211In.INTEGER: { //E => integer
            f.getInt();
            return true;
        }
        case CS211In.OPERATOR: { //E => (E+E)
            return f.check('(') &&
                getExp(f) &&
                f.check('+') &&
                getExp(f) &&
                f.check(')'));
        }
        default: return false;
    } //ends switch f.peekAtKind
}
```

Note on Boolean Operators

- Java supports two kinds of Boolean operators:
 - E1 & E2:
 - Evaluate both E1 and E2 and compute their conjunction (i.e., “and”)
 - E1 && E2:
 - Evaluate E1. If E1 is false, E2 is not evaluated, and value of expression is false. If E1 is true, E2 is evaluated, and value of expression is the conjunction of the values of E1 and E2.
- In our parser code, we use &&
 - if “f.check('(') returns false, we simply return false without trying to read anything more from input file. This gives a graceful way to handling errors.
 - don't worry about this detail if it seems too abstruse...

Trace of Recursive Calls to getExp



Modifying Parser to Generate Code for a Stack Machine

- Let us modify the parser so that it generates stack code (for a hypothetical stack machine) to evaluate arithmetic expressions
- Recall that stacks only give you access to the top element. We need to be clever about operations.

2 : PUSH 2
 STOP

(2 + 3) : PUSH 2
 PUSH 3
 ADD
 STOP

Idea

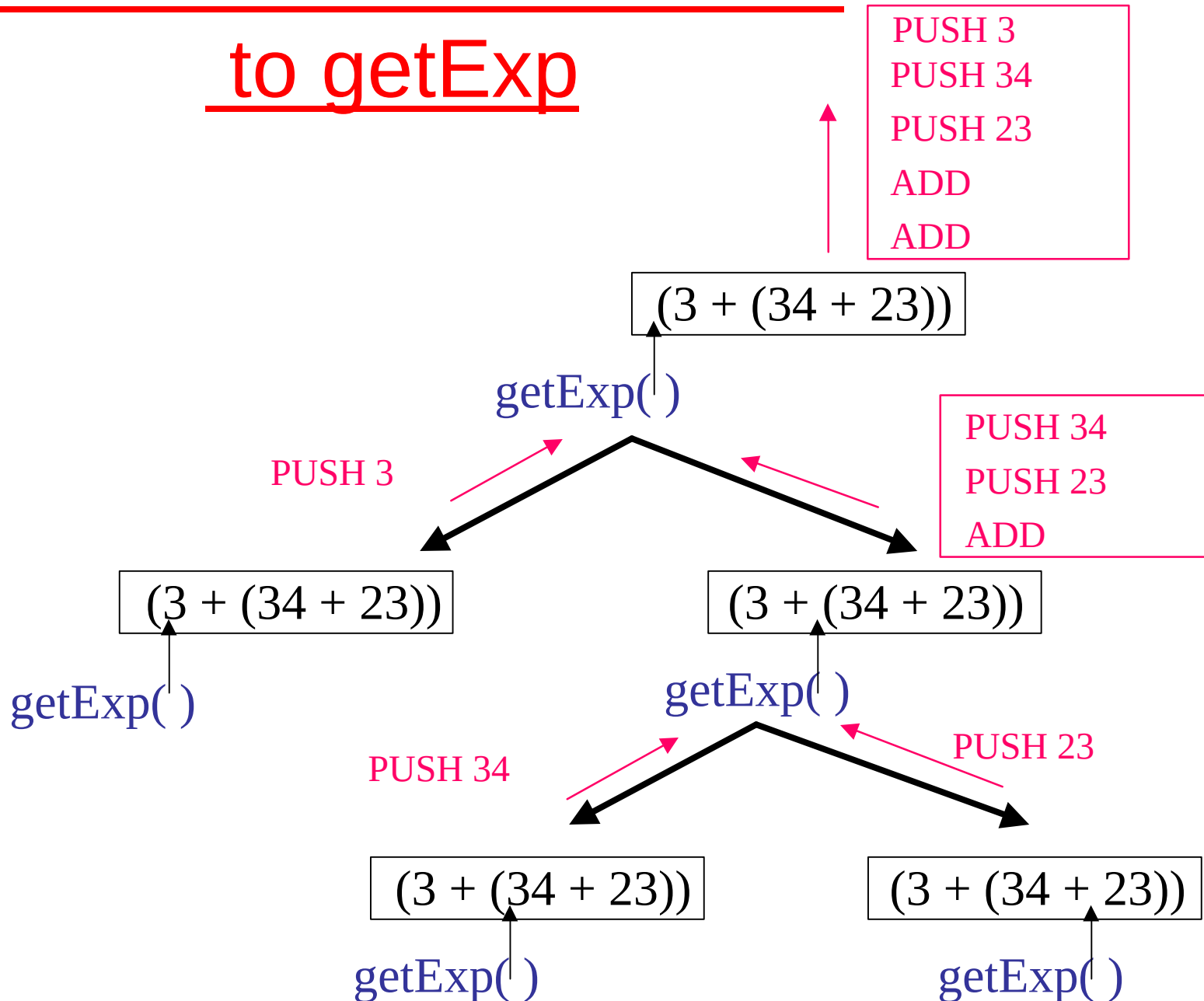
- Modify recursive method `getExp` to return a string containing stack code for expression it has parsed.
- Top-level method `expParser` should now tack on a `STOP` command after code received from `getExp`.
- The modified method `getExp` will generate code in a recursive way:
 - For integer i , it returns string `"PUSH" + i + "\n"`
 - For $(E1 + E2)$,
 - recursive calls return code for $E1$ and $E2$
 - say these are strings $S1$ and $S2$
 - method returns $S1 + S2 + \text{"ADD"}$

CodeGen for Expression language

```
static String expCodeGen(String fileName) { //returns stack code for expression in file
    CS211In f = new CS211In(fileName);
    String pgm = getExp(f);
    return pgm + "STOP\n"; //not doing error checking to keep it simple
}

static String getExp(CS211In f) { //no error checking to keep it simple
    switch (f.peekAtKind()) {
        case CS211In.INTEGER: //E => integer
            return "PUSH" + f.getInt() + "\n";
        case CS211In.OPERATOR: //E => (E+E)
            {
                f.check('(');
                String s1 = getExp(f);
                f.check('+');
                String s2 = getExp(f);
                f.check(')');
                return s1 + s2 + "ADD\n";
            }
        default: return "ERROR\n";
    }
}
```

Trace of Recursive Calls to getExp



Exercises

- Think about recursive calls made to parse and generate code for simple expressions
 - 2
 - $(2 + 3)$
 - $((2 + 45) + (34 + -9))$
- Can you derive an expression for the total number of calls made to `getExp` for parsing an expression?
 - Hint: think inductively
- Can you derive an expression for the maximum number of recursive calls that are active at any time during the parsing of an expression?

Exercises

- Write a grammar and recursive program for palindromes?
 - mom
 - dad
 - i prefer pi
 - race car
 - red rum sir is murder
 - murder for a jar of red rum
 - sex at noon taxes
- Write a grammar and recursive program for strings $A^N B^N$
 - AB
 - AABB
 - AAAAAAABBBBBBBB
- Write a grammar and recursive program for Java identifiers
 - $\langle \text{letter} \rangle [\langle \text{letter} \rangle \text{ or } \langle \text{digit} \rangle]^{0 \dots N}$
 - j27, but not 2j7

Number of recursive calls

- Claim:

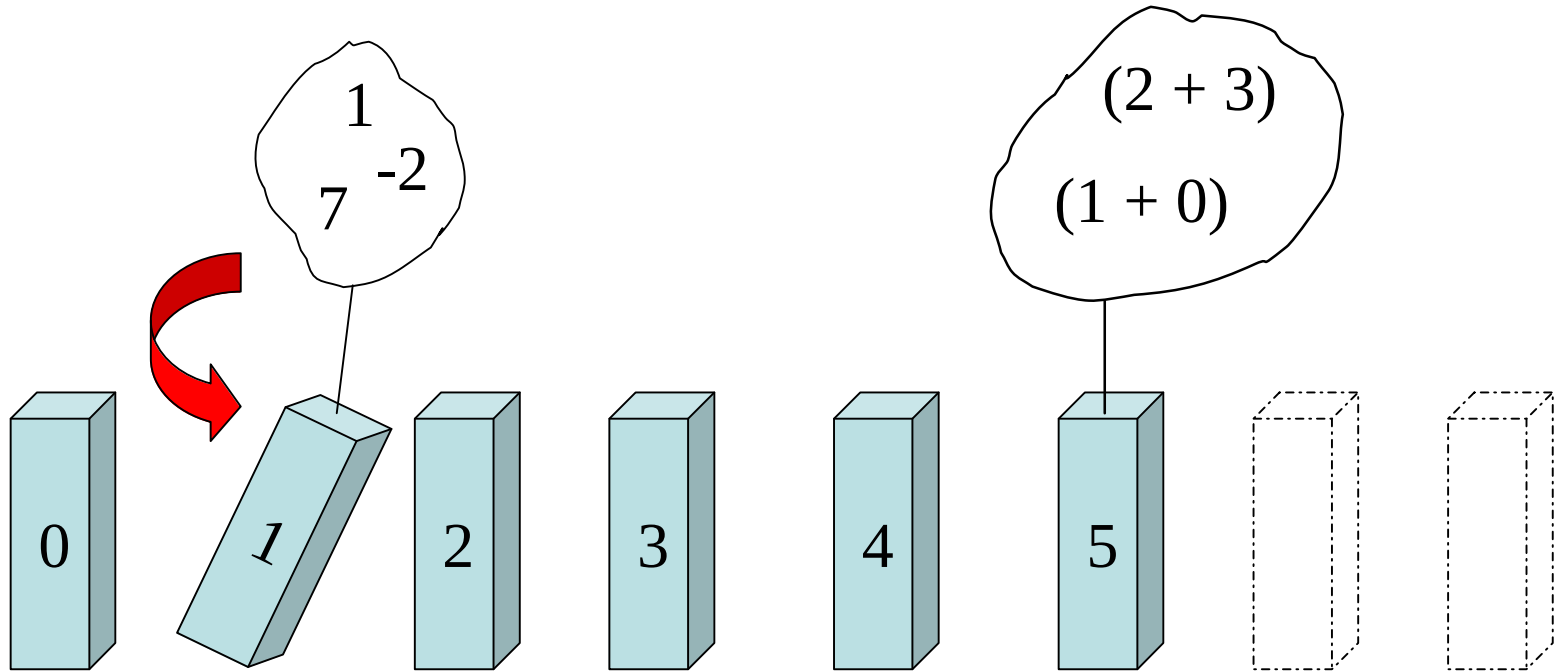
of calls to getExp for expression E =
of integers in E +
of addition symbols in E.

Example: $((2 + 3) + 5)$

of calls to getExp = $3 + 2 = 5$

Inductive Proof

- Order expressions by their length (# of tokens)
- $E1 < E2$ if $\text{length}(E1) < \text{length}(E2)$.



Proof of # of recursive calls

- *Base case:* (length = 1) Expression must be an integer. getExp will be called exactly once as predicted by formula.
- *Inductive case:* Assume formula is true for all expressions with n or fewer tokens.
 - If there are no expressions with $n+1$ tokens, result is trivially true for $n+1$.
 - Otherwise, consider expression E of length $n+1$. E cannot be an integer; therefore it must be of the form $(E1 + E2)$ where $E1$ and $E2$ have n or fewer tokens. By inductive assumption, result is true for $E1$ and $E2$.
(contd. on next slide)

Proof(contd.)

$$\begin{aligned}\text{\#-of-calls-for-E} &= \\ &= 1 + \text{\#-of-calls-for-E1} + \text{\#-of-calls-for-E2} \\ &= 1 + \text{\#-of-integers-in-E1} + \text{\#-of-'+'-in-E1} \\ &\quad + \text{\#-of-integers-in-E2} + \text{\#-of-'+'-in-E2} \\ &= \text{\#-of-integers-in-E} + \text{\#-of-'+'-in-E} \\ &\textit{as required.}\end{aligned}$$

Conclusion

- Recursion is a very powerful technique for writing compact programs that do complex things.
- Common mistakes:
 - Incorrect or missing base cases
 - Sub-problems must be simpler than top-level problem
- Try to write description of recursive algorithm and reason about base cases etc. *before* writing code.
 - Why?
 - Syntactic junk such as type declarations ... can create mental fog that obscures the underlying recursive algorithm.
 - Try to separate logic of program from coding details.