Abstract Datatypes and their Implementation

Some Data Structures

- Elementary Data Structures
  - Arrays, Lists, Trees
- Search Structures
  - Binary Search Trees, Hashtables
- Sequence Structures
  - Stacks, Queues, Priority Queues, Heaps, Extensible Arrays (Java Vectors)
- Graphs

Choosing a Data Structure

Issues:

- What operations do I need to perform on the data?
  - Insertion, deletion, searching, reset to initial state?
- How efficient do the operations need to be?
- Are there any additional constraints on the operations or on the data structure?
  - Can there be duplicates?
  - When extracting elements, does order matter?
- Is there a known upper bound on the amount of data? Or can it grow unboundedly large?

First Things First

- What operations do you need to perform?
- in Java, these are usually specified by an interface (e.g. **`Iterator, Collection, Set`**)
- independent of the implementation
- avoid overspecification!

Abstract Datatypes (ADTs)

- A collection of abstract operations and constraints specified independently of the implementation
- Examples: bag, priority queue, dictionary

Two Examples

```
interface Searchable<E> {
  void insert(E obj);
  void delete(E obj); //remove all objects equal to obj
  boolean search(E obj);
}

interface Bag<E> {
  void put(E obj);
  E get(); //extract some object
  boolean isEmpty();
}
```

One ADT, Many Implementations

```
interface Bag<E> {
  void put(E obj);
  E get(); //extract some object
  boolean isEmpty();
}
```

```
class Stack<E> extends java.util.Stack<E>
               implements Bag<E> {
   public boolean isEmpty() { return empty(); }
   public E get() { return pop(); }
   public void put(E obj) { push(obj); }
}
```

One ADT, Many Implementations

```
interface Bag<E> {
  void put(E obj);
  E get(); //extract some object
  boolean isEmpty();
}
```

```
class Queue<E> extends java.util.LinkedList<E>
               implements Bag<E> {
   //isEmpty() inherited from LinkedList<E>
   public E get() { return remove(); }
   public void put(E object) { add(object); }
}
```

One ADT, Many Implementations

```
interface Bag<E> {
  void put(E obj);
  E get(); //extract some object
  boolean isEmpty();
}
```

```
class RandomBag<E> extends Queue<E> implements Bag<E> {
   //isEmpty, put inherited from Queue<E>
   Random rand = new java.util.Random();
   public E get() {
      return remove(rand.nextInt(size()));
   }
}
```

Searching

```
interface Searchable<E> {
  void insert(E obj);
  void delete(E obj); //remove all objects equal to obj
  boolean search(E obj);
}
```

Searching -- Arrays vs Lists

- Arrays
  - Advantage: Random access, fast searching -- O(log n) if sorted
  - Disadvantage: fixed size, insertion & deletion are linear if sorted
- Lists
  - Advantage: Extensible, insertion & deletion are constant time
  - Disadvantage: No random access, searching is linear (even if sorted)

Extensible Arrays (Vectors)

- A good compromise
  - random access, but extensible
  - reallocates if **add** would cause array bound to be exceeded

```
public class Vector<E> {
  boolean add(E o);
  void add(int index, E element);
  boolean addAll(Collection<? extends E> c);
  boolean contains(Object elem);
  E elementAt(int index);
  Enumeration<E> elements();
  int indexOf(Object elem);
  int lastIndexOf(Object elem);
  boolean remove(Object o);
  int size();
}
```

Hashing

- An excellent solution if duplicates not allowed
  - In practice, constant time insert, delete, search
- Based on a hash function that converts data to an index into a large array of lists
  - unlikely that two randomly chosen data items would hash to the same value (this is called a collision)
  - usually implemented in native code -- extremely fast

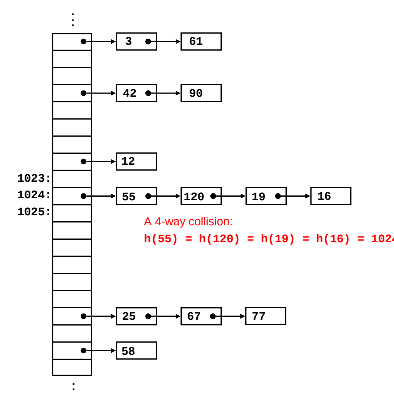
Java HashSet

```java
public class HashSet<E> {
  boolean add(E o);
  void clear();
  Object clone();
  boolean contains(Object o);
  boolean isEmpty();
  Iterator<E> iterator();
  boolean remove(Object o);
  int size();
}
```

Hashing

Data structure consists of an array of lists

- Insertion:
  - Hash data to get array index
  - Append data to a list at that index
- Search:
  - Hash data to get array index
  - Look for data by walking down list at that index
- Deletion:
  - Hash data to get array index
  - Walk down list at that index and remove data



Performance

Affected by many factors:

- Size of array relative to number of data items
  - Consider limit where there is only 1 bucket
  - as bad as simple linked lists!
- Quality of hash function
  - Good hash functions do not lead to clustering of data → low collision rate

Examples of Hash Functions

`int → {0,1,...,99}`

- Bad:
  - constant functions: hash(x) = 7
  - two most significant digits: hash(379988) = 37
- Better:
  - two least significant digits: hash(379988) = 88
  - sum of digit pairs mod 100: hash(379988) = 37+99+88 (mod 100) = 24
  - square number and take middle digits

Universal Hashing

- Parametrized family of numeric functions
  - e.g., $f_{abc}(x) = ax^2 + bx + c \pmod{100}$
- Pick a,b,c at random!
- Works as well or better than hand-crafted hash functions in most cases!
- Disadvantage: no persistence

Test of Hash Function

- Multiplicative hash function
- size of hashtable = 1024
- key k is in range 0..32677
- hash function h(k) = (((32768*0.6125423371*k)%32768)%1024)

Testing a Hash Function

```
class HashTest {
  public static void main(String[] args) {
    int[] histogram = new int[1024];
    for (int i = 0; i < 32768; i++) {
    int bucket = ((int)((32768*0.6125423*i)%32768))%1024;
      histogram[bucket]++;
    }
    //print histogram
    System.out.println("Histogram:");
    for (int i = 0; i < 1024; i++) {
      System.out.print(i + " " + histogram[i] + "   ");
      if (i%10 == 0) System.out.println();
    }
  }
}
```

Testing a Hash Function

Distribution of keys among buckets

- Number of keys = 32768
- Number of buckets = 1024
- Average number of keys/bucket = 32
- Number of keys in each bucket was always in range 29-34
- Conclusion: this is a good hash function

Hashing Objects

So far, we have stored only integers in hash tables. In general, we want to store objects.

- Give each object an **`int`** hash code. Java method: **`hashCode()`**
- Contract for **`hashCode()`** method:
  - Whenever it is invoked in the same object, it must return the same result
  - Two objects that are equal must have the same hash code
  - Two objects that are not equal should return different hash codes, but are not required to do so

Observations

- Hashing is popular in practice because code is easy to write and maintain and performance is typically excellent
- Performance depends on two key factors:
  - load factor λ = number of entries/size of array
  - choice of hash function
  - if $\lambda \leq 3/4$ and hash function is chosen well, get expected O(1) complexity for all operations
- Our version is called hashing with separate lists or chained hashing -- used in Java Collections
- Other methods such as open-address hashing

Dictionaries

- In many applications, we want a more general search structure that stores (key, value) pairs
  - Given a key, find the associated value
- Examples:
  - language dictionaries: key is word, value is meaning
  - telephone directory: key is name, value is telephone number
  - grade sheet for CS211: key is netID, value is grade
- This type of ADT is called a dictionary

Dictionaries

```
public abstract class Dictionary<K,V> {
  abstract Enumeration<V> elements();
  abstract V get(Object key);
  abstract boolean isEmpty();
  abstract Enumeration<K> keys();
  abstract V put(K key, V value);
  abstract V remove(Object key);
  abstract int size();
}
```

```
public class Hashtable<K,V>
  extends Dictionary<K,V> {
  ...
}
```

Java Hashtables

```
class HashTest {
  static Hashtable<String,Integer> h = new Hashtable<String,Integer>();
  static {
    h.put("two", new Integer(2));
    h.put("three", new Integer(3));
    h.put("five", new Integer(5));
    h.put("seven", new Integer(7));
  }
  public static void main(String[] args) {
    System.out.println(h.get("three"));
    Enumeration e = h.elements();
    while (e.hasMoreElements()) {
      System.out.print(e.nextElement());
    }
  }
}
```

```
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```

Next Time

- Priority Queues
- Heaps
- Graphs