CS211 Section 3

More Induction

Untitled

- Prove by induction that:
  - for n>=3: $2n+1 < 2^n$.
- **Base Case:** n = 3: 2(3) +1 < 2^3

  7 < 8

Answer…

- Inductive Case: Assume Inductive hypothesis P(k), where k ⩾ 3; prove P(k+1):
  - P(k+1): 2(k+1)+1 < 2^(k+1)
- 2(k+1) + 1 ?< 2^(k+1)
- <Expose IH> (2k+1) + 2 ?< 2(2^k)
- <Algebra: RHS> (2k+1) + 2 ?< 2^k + 2^k
- **IH:** We know that 2k+1 is < 2^k, hence, we r left with:
- [<2^k] + 2 ?< 2^k + 2^k
- Since 2 < 2^k for any k >= 3, we can conclude that, in fact:
- (2k+1) + 2 < 2(2^k)

STAMPS

- Show that you can make any amount of stamps greater than 8-cents from any amount of 3-cent and 5-cent stamps.
- Theorem: P(n) that for n>=8, a set of 3-cent, 5-cent and 8-cent stamps can be summed to equal n.
- Base Cases:
  - 8-cents: 8 = 3 + 5
  - 9-cents: 9 = 3 + 3 + 3
  - 10-cents: 10 = 5 + 5

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- **Inductive hypothesis:** Assume you can make (k - 3)-cents from 3-cents and 5-cents.
- **Proof:** Show that you can make k-cent stamps.
- k-cents = k + 3 – 3 = (k – 3) + 3
- A k-3 can be made from 3 and 5 cent stamps by inductive assumption. This implies that if a (k-3)-cent stamp can be made of 5 and 3 cent stamps then so can a k-cent stamp.
- In conclusion, since 8, 9, and 10 cent stamps can all be manufactured then so can all n-cent stamps since if (k-3)-cent stamps can be created then so can k-cent stamps and the result holds for three sequential base cases.

So many Questions…

- A2 Questions?
- Induction Questions?
- Recursion Questions?