CS211, Lecture 18 Trees

Readings: Weiss, chapter 18, sections 18.1--18.3.

I think that I shall never see
A poem as lovely as a tree
A tree whose hungry mouth is prest
Against the earth's sweet flowing breast
A tree that looks to God all day,
And lifts her leafy arms to pray;
A tree that may in summer wear
A nest of robins in her hair;
Upon whose bosom snow has lain;
Who intimately lives with rain
Poems are made by fools like me,
But only God can make a tree.
Joyce Kilmer

About assignment A5
Functional programming

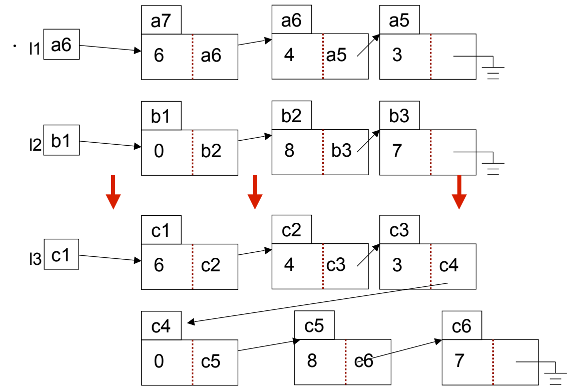
Write an implementation of linked list using only recursion —no assignment statement or loops.

Implement sets using it —again without assignment or loops.

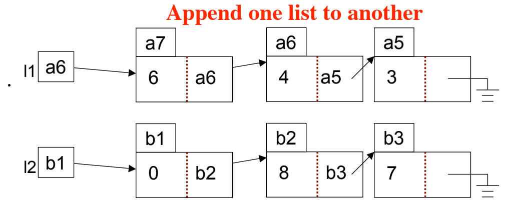
Of course, when *using* the implementation, you can use assignments.

```
public class RList {
        public Object value;
        public RList   next;
}
```

Append one list to another



Append one list to another



```
/** = l1 with l2 appended to it */
public static RList append(Rlist l1, Rlist l2) {
   if (l1 == null)  return l2;
   return new Rlist(l1.value,
                              append(l1.next, l2)
                              );
}
```

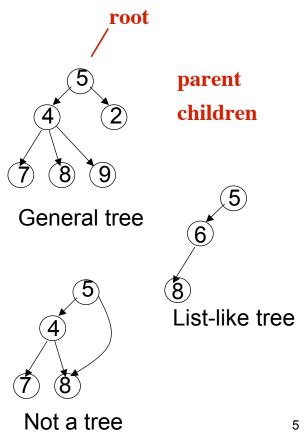
Overview of trees

Tree: recursive data structure.
Definition 1:
A **tree** is a
   (1) a value (the **root** value)

together with

   (2) one or more other trees,
      called its **children**.

Each of the circles is called a **node** of the tree. Definition does not allow for empty trees (trees with 0 nodes).

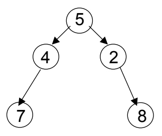


Binary tree

Binary tree: tree in which each node can have at most two children.

Redefinition of binary tree to allow empty tree

A **binary tree** is either
(1) Ø (the empty binary tree)
or
(2) a root node (with a value),
   a left binary tree,
   a right binary tree

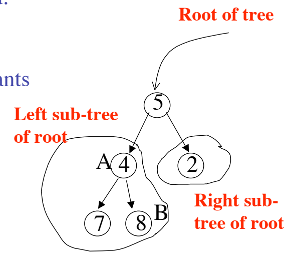


Binary tree

tree with root 4 has an empty right binary tree
tree with root 2 has an empty left binary tree
tree with root 7 has two empty children.

Terminology

- Edge A→B: A parent of B.
  B is child of A.
- Generalization of parent and child: ancestor and descendant
  - root and A are ancestors of B
- Leaf node: node with no descendants (or empty descendents)
- Depth of node: length of path from root to that node
  - depth(A) = 1   depth(B) = 2
- Height of node: length of longest path from node to leaf
  - height(A) = 1   height(B) = 0
- Height of tree = height of root
  - in example, height of tree = 2



Binary tree

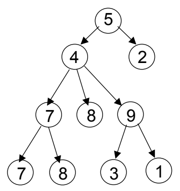
Class for binary tree nodes

```
/** An instance is a nonempty binary tree */
public class TreeNode {
    private Object datum;
    private TreeNode left;
    private TreeNode right;

    /** Constructor: a one-node tree with root value ob */
    public TreeNode(Object ob)
      { datum = ob; }

    /** Constructor: tree with root value ob and left and
        right subtrees l and r */
   public TreeNode(Object ob, TreeNode l, TreeNode r)
      { datum = ob; left = l; right = r; }

    **getter and setter methods for all three fields**
}
```

empty tree is given by value null

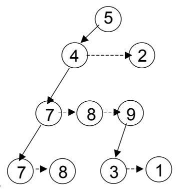
Class for general trees

```
public class GTreeNode{
  private Object datum;
  private GTreeNode left;
  private GTreeNode sibling;
   appropriate constructors and
   getter and setter methods
}
```



General tree

- **Parent node points directly only to its leftmost child.**
- **Leftmost child has pointer to next sibling, which points to next sibling etc.**



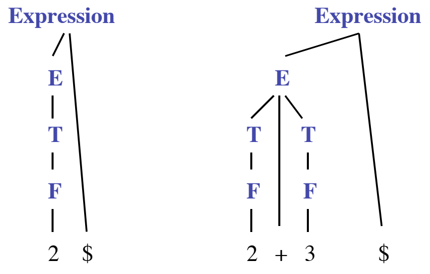
Tree represented using GTreeNode

One application of trees

- Most languages (natural and computer) have a recursive, hierarchical structure.
- This structure is implicit in ordinary textual representation.
- Recursive structure can be made explicit by representing sentences in the language as trees: abstract syntax trees (AST's)
- AST's are easier to optimize, generate code from, etc. than textual representation.
- Converting textual representations to AST: job of parser

Syntax trees

Expression ::= E $	F ::= Integer
E ::= T { <+ | –> T }	F ::= – F
T ::= F { <* | /> F }	F ::= (E)



Writing a parser for the language E ::= T { <+ | –> T }

```
/** Token Scan.getToken() is first token of a sentence for E.
  Parse it, giving error mess. if there are mistakes. After the parse,
  Scan.getToken should be the symbol following the parsed E. */
  public static void parseE() {
         parseT();
         while (Scan.getToken() is + or - ) {
                  Scan.scan();
                  parse(T);
         }
}
```

Writing a parser for the language E ::= T { <+ | –> T }

```
/** Token Scan.getToken() is first token of a sentence for E.
  Parse it, giving error mess. if there are mistakes. After the parse,
  Scan.getToken should be the symbol following the parsed E.
  Return a TreeNode that  describes the parsed E */
  public static TreeNode parseE() {
        TreeNode lop= parseT();
        while (Scan.getToken() is + or - ) {
                 Token op= Scan.getToken());
                 Scan.scan();
                 TreeNode rop= parse(T);
                 lop= new TreeNode(op, lop, rop);
        }
}
```

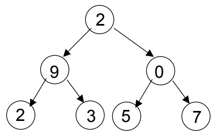
Recursion on trees

- Recursive methods can be written to operate on trees in the obvious way.
- In most problems
  - base case: empty tree
    - sometimes base case is leaf node
  - recursive case: solve problem on left and right subtrees, and then put solutions together to compute solution for tree

Tree search

- Analog of linear search in lists: given tree and an object, find out if object is stored in tree.
- Trivial to write recursively; much harder to write iteratively.

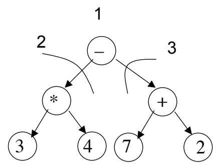
```
/** = "v occurs in t" */
public static boolean treeSearch(Object v, TreeNode t) {
   if (t  == null)
        return false;
   return t.getDatum().equals(v) ||
            treeSearch(v, t.getleft()) ||
            treeSearch(v, t.getRight());
}
```



Walks of tree

A **walk** of a tree processes each node of the tree in some order.

Example on last slide showed pre-order walk of tree:
- process root
- process left sub-tree
- process right sub-tree



- Intuition: think of prefix representation of expressions

infix: 3 * 4 – (7 + 2)

prefix: – * 3 4 + 7 2

postfix: 3 4 * 7 2 + –

In-order and post-order walks

- In-order walk: infix
  - process left sub-tree
  - process root
  - process right sub-tree

```
/** = "v occurs in tree t */
public static boolean treeSearch(Object v,
                              TreeNode t) {
   if (t  == null) return false;
   return   treeSearch(v, t.getleft()) ||
            t.getDatum().equals(v)   ||
            treeSearch(v, t.getRight());
}
```

- Post-order walk: postfix
  - process left sub-tree
  - process right sub-tree
  - process root

```
/** = "v occurs in tree t */
public static boolean treeSearch(Object v,
                              TreeNode t) {
   if (t  == null) return false;
   return   treeSearch(v, t.getleft()) ||
            treeSearch(v, t.getRight()) ||
            t.getDatum().equals(v)   ||
}
```

Some useful routines

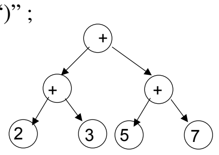
```
/** = " t  is a leaf node" */
public static boolean isLeaf(TreeNode t)
  { return (t != null) && t.getLeft() == null && t.getRight() == null; }

/** = height of tree (calculated using post-order walk) */
public static int height(TreeNode t) {
   if (t == null) return –1; // height is undefined for empty tree
   if (isLeaf(t)) return 0;
   return 1 + Math.max(height(t.getLeft()), height(t.getRight()));
}

/** = number of nodes in tree */
public static int nNodes(TreeNode t) {
  if (t == null) return 0;
  return 1 + nNodes(t.getLeft()) + nNodes(t.getRight());
}
```

Example

- Generate textual representation from AST (Abstract Syntax Tree)

```
/** = textual representation of AST t, with each binary
        operation parenthesized */
public static String flatten(TreeNode t) {
    if (t == null) return “”;
    if (isLeaf(t)) return t.getDatum();
    return “(“ + flatten(t.getLeft()) + t.getDatum() +
            flatten(t.getRight())  + “)” ;
}
```

Note. The code above does not deal with unary operators. Fix it yourself so that it does. A unary prefix operator will have a right subtree but no left subtree (I.e. an empty left subtree).

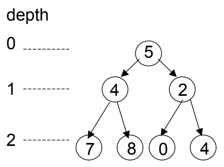


Useful facts about binary trees

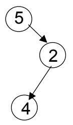
- Maximum number of nodes at depth d = 2^d
- If height of tree is h,
  - minimum number of nodes it can have = h+1
  - maximum number of nodes it can have is =

$2^0 + 2^1 + \ldots + 2^h = 2^{h+1} - 1$

- Full binary tree of height h:
  - all levels of tree up to depth h are completely filled.



Height 2, max number of nodes



Height 2, min number of nodes

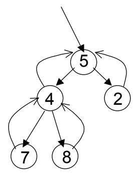
Tree with header element

- As with lists, some prefer to have an explicit class Tree, an instance of which contains a pointer to the root of the tree.
- With this design, methods that operate on trees can be made into instance methods in this class, and the root of the tree does not have to be passed in explicitly to method.
- Feel free to use whatever works for you.

Tree with parent pointers

- In some applications, it is useful to have trees in which nodes other than root have references to their parents.
- Tree analog of doubly-linked lists.

```
class TreeWithPPNode {
   private Object datum;
   private TreeWithPPNode
              left, right, parent;

  …..appropriate constructors and
       getter and setter methods…
}
```



Summary

- Tree is a recursive data structure built from class TreeNode.
  - special case: binary tree
- Binary tree cells have both a left and a right “successor”
  - called children rather than successors
  - similarly, parent rather than predecessor
  - generalization of parent and child to ancestors and descendents
- Trees are useful for exposing the recursive structure of natural language programs and computer programs.
- File system on your hard drive is a tree.
- Table of contents of a book is a tree.