CS211 Prelim 2. 20 Nov. 2001 NAME_____ NETID_____

This prelim has 4 questions. Be sure to answer them all. Please write clearly, and show all your work. It is difficult to give partial credit if all we see is a wrong answer. Also, be sure to place suitable comments --method specifications, variable definitions-- in your programs.

Write your name and netid at the top of each page.

Question 1 _____ out of 25

Question 2 _____ out of 25

Question 3 _____ out of 25

Question 4 _____ out of 25

Total _____ out of 100

Question 1 Sorting (25 points).

Part (a) Write an algorithm to sort array segment b[0..n-1], where n>= 0. Your algorithm must be a selection sort. We expect you to write

(a) the precondition
(b) the postcondition
(c) the invariant (perhaps as a diagram or picture)
(d) the bound function

and then to write the loop. Do not write a complete method. Just write the loop with initialization. You don't have to write an inner loop, if you write the body at a suitable level of abstraction.

Part (b) Give the worst-case order-of-execution time and best-case order-of-execution time of mergesort and of quicksort.

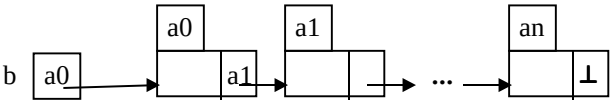
Part (c) Give the worst-case and best-case times of the binary search that we wrote in class (and that appears in the handout on correctness) --Given x and b[0..n-1], it looks for an index k that satisfies b[k] <= x < b[k+1].

Question 2. Loops and linked lists (25 points). Consider a (singly-)linked list b without a header. Its nodes are of class ListNode. ListNode has the usual field *next*. In the diagrams, the second component of each folder is field next.

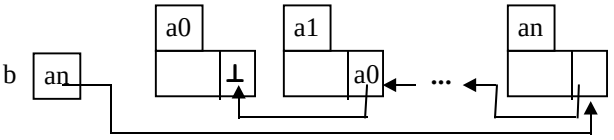
Write a loop with initialization that reverses the linked list by changing b and field next of each node. The linked list may be empty, in which case b contains **null** --in the diagrams, **null** is represented by ⊥ **.** Your algorithm shouldn't use any variables except b and c (but it can declare variables within the loop body). The notations *an*, *ah*, *aj*, *ak*, etc. cannot appear in your algorithm; they are used only to describe the problem.

Your grade depends solely on our being able to check the correctness of your work using the check list for understanding loops --so you should use it in developing the algorithm. It doesn't matter how well your algorithm "works" if it does not use our invariant and bound function.

Precondition Q: The linked list looks like:

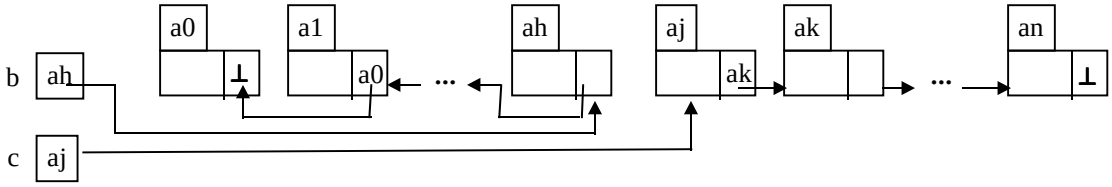


Postcondition R: The linked list looks like:



Bound function: Length of list c (see below).

Invariant P: The linked list looks as shown below (using a new variable c). In words: the first part of the list has been reversed, and c contains the name of the first node of the list that has not been reversed. Note that b or c (or both) could be **null**, meaning that list b or c (or both) is empty. For example, if c is **null**, b contains the name of the reversed.list.



Question 3 (25 points). Algorithmic complexity

Part (a). Let $f(n) = 2n^2 + 3n$.
Let $g(n) = 5n + 10$.
Prove that g(n) is O(f(n)).

Part (b). What is the order of execution time for the algorithm shown at the bottom of the page? Explain your reasoning --just giving an answer is not appropriate and will not receive full credit, even if it is correct.

```
/* Store in x the number of sections of equal elements of array b[0..n-1], for n>=0. For example, for the array b
= (3,5,5,3,3,3,3,6,6,6), the value 4 is stored in x: there is a section of (one) 3's, then a section of 5's, then a
section of 3's, and finally a section of 6's. */
    int x= 0; int k= 0;
    // invariant P: 0 <= k <= n,   and
                x = number of sections of equal elements in b[0..k-1],   and
                the following holds:  k=0  or  k=n  or  b[k-1] != b[k]
    // bound function: n-k
    while (k != n) {
        x= x+1;
        // Let v be the value in b[k] at this point. Increase k until either k = n  or  b[k] != v
            k= k+1;
            // invariant: 1 <= k <= n,   and
                        x = number of sections of equal elements in b[0..k-1],
            // bound function: n-k
            while (k != n  &&  b[k-1] = b[k]) {
                k= k+1;
            }
    }
    // R: x = number of sections of equal elements in b[0..n-1]
```

Question 4. Miscellaneous.

Part (a) Given is a doubly linked list with head and tail. p is the name of a node in the list (not the head or tail node). Write the code to remove p from the list. Remember, each node has fields *prev* and *next*; we assume you know what these are for.

Part (b) Nodes of a binary tree are of the class-type shown to the right. Write function printPreorder:

```
// Print the elements of tree t, in preorder
public void printPreorder(BinaryNode t)
```

```
public class BinaryNode {
    public BinaryNode left;
    public Object element;
    public Binarynode right
}
```

Part (c) What is the load factor of a hash table? In looking for a value in a hash table, what is the expected number of probes if the load factor is 1/2? Why is quadratic probing preferred over linear probing?

1 (a) Precondition: n >= 0
Postcondition: b[0..n-1] is sorted

```
int k= 0;
// {inv:  0 <= k <= n   and
//        b[0..k-1 is sorted  and
//        b[0..k-1] <= b[k..n-1]}
// {bound function: n-k}
while (k != n) {
    Store in j the index of smallest value in b[k..n-1];
    Swap b[k] and b[j];
    k= k+1;
}
```

1 (b) To sort an array of size n, mergesort is always O(n log(n)). So the worst-case and best-case times are the same: O(n log(n)).
Quicksort's worst-case time is O(n2), which happens when method partition always makes one of its two partitions empty. Its best-case time is O(n log(n)), which happens when method partition always makes the two partitions the same size.

2 (c) The binary search loop always cuts the size of the segment still being looked at it half, and it terminates only when the size is 1. Hence, it's worst-case and best-case times are the same: O(log n).

2 (b)
```
c= b; b= null;
// inv: as shown on prelim 2
// bound function: size of list c
while (c != null) {
    ListNode t= c;
    c= c.next;
    t.next= b;
    b= t;
}
```

3 (a) The definition is: g(n) is O(f(n)) if there exists a c>0 and N0 > 0 such that for all n>= N0, g(n) < c*f(n). In this case, we calculate as follows:

$g(n) < c*f(n)$
$=$ <substitute for g and f
$5n + 10 <= 2cn^2 + 3cn$
$=$ <arithmetic>
$0 <= 2cn^2 + 3cn - 5n - 10$
$=$ <choose c = 1; arithmetic>
$0 <= 2n^2 + -2n - 10$

The last formula is true if n>10. So, we choose N0 = 10 and c=1.

3 (b) Variable k is increased by 1 in two places, and n is not changed. Since k <= n always holds, the maximum number of times k is increased is at most n. Therefore, in total, the inner loop body is executed at most n times.Therefore, in spite of the nested loops, the algorithm is O(n).

4 (a)
```
p.prev.next= p.next;
p.next.prev= p.prev;
```

4 (b)
```
// Print the elements of tree t, in preorder
public void printPreorder(BinaryNode t) {
    if (t == null)
        return;
    System.out.println("" + t.element);
    printPreorder(t.left);
    printPreorder(t.right);
}
```

4 (c) The load factor of a hash table is the ratio of the number of elements actually in it to the size of the array. The expected number of probes if the load factor is 1/2 is 2. Linear probing has primary clustering --values that hash to k (say) and k+1 and k+2 tend to cluster together, making the time it takes to find one of them longer. Quadratic probing eliminates such primary clustering.