CS/ENGRI 172, Fall 2003: Computation, Information, and Intelligence
11/10/03: Push-down Automata

Push-down automata (PDAs) are essentially limited versions of Turing machines. We will use them to efficiently determine if a CFG G can generate a parse tree for a sentence x. We only consider *deterministic* PDAs; that is, for any given configuration at most one move, and perhaps no move, is possible.

We will define a PDA P to have:

- a set of m distinct *states* $s_1, \ldots, s_m$

  s_1 is the *initial state*; s_m is the *accept state*

- an *input alphabet* of l distinct symbols $a_1, \ldots, a_l$

  a_1 is the right-end marker $\dashv$

- a *stack alphabet* of k distinct symbols $A_1, \ldots, A_k$

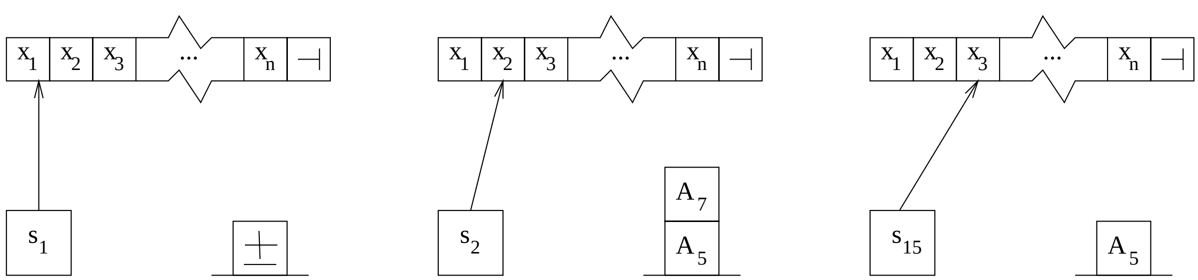
  A_1 is the initial stack symbol $\pm$

- *rules* of the form $(s, a_i, A_j) \to (s', \alpha)$ where s and s' are states, a_i is a single input symbol, A_j is a single stack symbol and α is a sequence of stack symbols or the word "pop".

where $l \geq 2$; $k, m \geq 1$. We impose a restriction that no two rules have the same left-hand side (this is the determinism condition). Every time a rule is applied, the top stack symbol A_j is removed to be read, and based on it, the input symbol a_i under the input head, and the current state s, the PDA will enter state s', push α onto the stack (or add nothing if $\alpha =$ "pop"), and move the input head one space to the right.

A legal input to P would be a finite sequence $x = x_1 x_2 \ldots x_n$, where each x_i is one of $a_2, \ldots, a_l$ (the input alphabet *except* the right-end marker; repeats of input symbols are permitted in x).

If P had rules $(s_1, x_1, \pm) \to (s_2, A_7 A_5)$ and $(s_2, x_2, A_7) \to (s_{15}, \text{pop})$, then the first three configurations of P on input x would be as follows, where the first configuration shows the *initial configuration* in which P would start:



P *accepts* x if it can start in the initial configuration corresponding to x and, following its rules, have the input head fall off the input tape while changing to its accept state. If it would halt in any other configuration, such as getting stuck somewhere on the tape because no rule applies or falling off of the end of the tape but *not* in the accept state, it does not accept x.

Example PDAs

PDA One

- States: count-a, match-b, matched
- Initial state: count-a
- Accept state: matched
- Input symbols: $\dashv$, a, b
- Stack symbols: $\pm$, A, M
- Moves:
  1. (count-a, a, $\pm$) $\to$ (count-a, AM)
  2. (count-a, a, A) $\to$ (count-a, AA)
  3. (count-a, b, A) $\to$ (match-b, pop)
  4. (match-b, b, A) $\to$ (match-b, pop)
  5. (match-b, $\dashv$, M) $\to$ (matched, M)

PDA Two - trying to "optimize" PDA One

- States: count-a
- Initial state: count-a
- Accept state: count-a
- Input symbols: $\dashv$, a, b
- Stack symbols: $\pm$, A
- Moves:
  1. (count-a, a, $\pm$) $\to$ (count-a, $A\pm$)
  2. (count-a, a, A) $\to$ (count-a, AA)
  3. (count-a, b, A) $\to$ (count-a, pop)
  4. (count-a, $\dashv$, $\pm$) $\to$ (count-a, $\pm$)