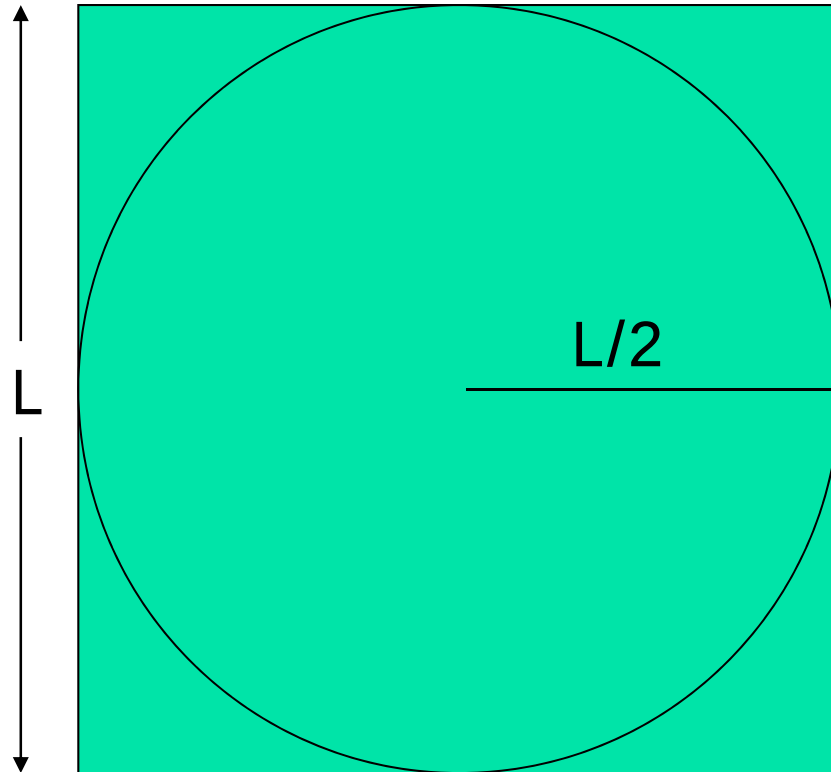


- Today's Lecture:
 - Vectorized computation
 - Introduction to graphics

- Announcements:
 - **Assignment 1a**: due Tues at 11:59pm, at which time submission on CMS will close. Will re-open for re-submission later.
 - **Assignment 1b**: due Tues Feb 12 at 11:59pm

Monte Carlo Approximation of π



Throw N darts

$$\text{Sq. area} = N = L \times L$$

$$\begin{aligned}\text{Circle area} &= N_{in} \\ &= \pi L^2 / 4\end{aligned}$$

$$\pi = 4 N_{in} / N$$

1. Make a plot
2. Use vectors to store all values
3. Use vectorized arithmetic

Vectorized addition

$$\begin{array}{rcl} & \mathbf{x} & \begin{array}{|c|c|c|c|} \hline 2 & 1 & .5 & 8 \\ \hline \end{array} \\ + & \mathbf{y} & \begin{array}{|c|c|c|c|} \hline 1 & 2 & 0 & 1 \\ \hline \end{array} \\ \hline = & \mathbf{z} & \begin{array}{|c|c|c|c|} \hline 3 & 3 & .5 & 9 \\ \hline \end{array} \end{array}$$

Matlab code: **`z = x + y`**

Vectorized subtraction

$$\begin{array}{r} \mathbf{x} \quad \begin{array}{|c|c|c|c|} \hline 2 & 1 & .5 & 8 \\ \hline \end{array} \\ - \quad \mathbf{y} \quad \begin{array}{|c|c|c|c|} \hline 1 & 2 & 0 & 1 \\ \hline \end{array} \\ \hline = \quad \mathbf{z} \quad \begin{array}{|c|c|c|c|} \hline 1 & -1 & .5 & 7 \\ \hline \end{array} \end{array}$$

Matlab code: $\mathbf{z} = \mathbf{x} - \mathbf{y}$

Vectorized code

—a Matlab-specific feature

See Sec 4.1 for list of vectorized arithmetic operations

- Code that performs element-by-element arithmetic/relational/logical operations on array operands in one step
- Scalar operation: $x + y$
where x , y are scalar variables
- **Vectorized code:** $x + y$
where x and/or y are vectors. If x and y are both vectors, they must be of the **same shape and length**

Vectorized multiplication

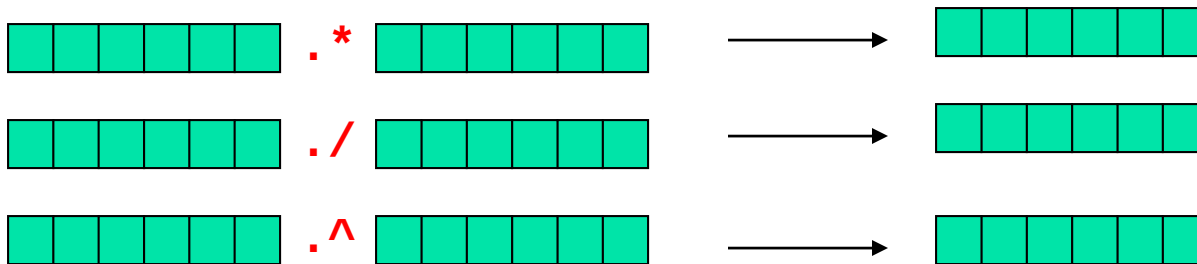
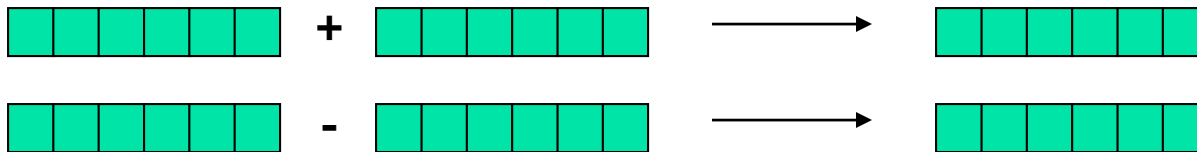
$$\begin{array}{rcl} & \mathbf{a} & \begin{bmatrix} 2 & 1 & .5 & 8 \end{bmatrix} \\ \times & \mathbf{b} & \begin{bmatrix} 1 & 2 & 0 & 1 \end{bmatrix} \\ \hline = & \mathbf{c} & \begin{bmatrix} 2 & 2 & 0 & 8 \end{bmatrix} \end{array}$$

Matlab code: **c = a .* b**



Vectorized

element-by-element arithmetic operations on arrays



A dot (.) is necessary in front of these math operators

Shift

$$\begin{array}{r} \mathbf{x} \quad \boxed{3} \\ + \quad \mathbf{y} \quad \boxed{2 \quad 1 \quad .5 \quad 8} \\ \hline = \quad \mathbf{z} \quad \boxed{5 \quad 4 \quad 3.5 \quad 11} \end{array}$$

Matlab code: **z = x + y**

Reciprocate

$$\begin{array}{rcl} & \mathbf{x} & \boxed{1} \\ / & \mathbf{y} & \boxed{2 \quad 1 \quad .5 \quad 8} \\ \hline = & \mathbf{z} & \boxed{.5 \quad 1 \quad 2 \quad .125} \end{array}$$

Matlab code: $\mathbf{z} = \mathbf{x} \mathbf{. / y}$



Vectorized

element-by-element arithmetic operations between an array and a scalar

$$\begin{array}{|c|c|c|c|c|c|c|} \hline & & & & & & \\ \hline \end{array} + \begin{array}{|c|} \hline \\ \hline \end{array}$$

$$\begin{array}{|c|c|c|c|c|c|c|} \hline & & & & & & \\ \hline \end{array} - \begin{array}{|c|} \hline \\ \hline \end{array}$$

$$\begin{array}{|c|c|c|c|c|c|c|} \hline & & & & & & \\ \hline \end{array} * \begin{array}{|c|} \hline \\ \hline \end{array}$$

$$\begin{array}{|c|c|c|c|c|c|c|} \hline & & & & & & \\ \hline \end{array} / \begin{array}{|c|} \hline \\ \hline \end{array}$$

$$\begin{array}{|c|} \hline \\ \hline \end{array} + \begin{array}{|c|c|c|c|c|c|c|} \hline & & & & & & \\ \hline \end{array}$$

$$\begin{array}{|c|} \hline \\ \hline \end{array} - \begin{array}{|c|c|c|c|c|c|c|} \hline & & & & & & \\ \hline \end{array}$$

$$\begin{array}{|c|} \hline \\ \hline \end{array} * \begin{array}{|c|c|c|c|c|c|c|} \hline & & & & & & \\ \hline \end{array}$$

$$\begin{array}{|c|c|c|c|c|c|c|} \hline & & & & & & \\ \hline \end{array} \text{.}^{\wedge} \begin{array}{|c|} \hline \\ \hline \end{array}$$

$$\begin{array}{|c|} \hline \\ \hline \end{array} \text{.} / \begin{array}{|c|c|c|c|c|c|c|} \hline & & & & & & \\ \hline \end{array}$$

$$\begin{array}{|c|} \hline \\ \hline \end{array} \text{.}^{\wedge} \begin{array}{|c|c|c|c|c|c|c|} \hline & & & & & & \\ \hline \end{array}$$

A dot (.) is necessary in front of these math operators

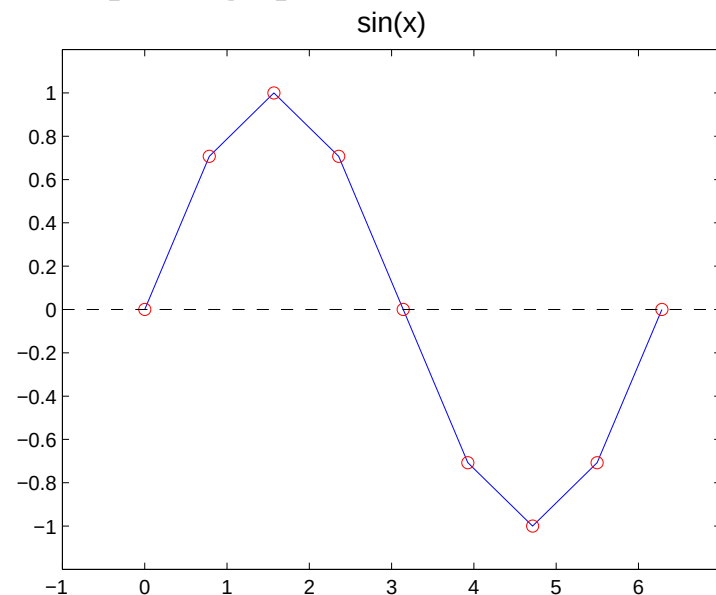
The dot in $\begin{array}{|c|c|} \hline & \\ \hline \end{array} \text{.} * \begin{array}{|c|} \hline \\ \hline \end{array}$, $\begin{array}{|c|} \hline \\ \hline \end{array} \text{.} * \begin{array}{|c|c|} \hline & \\ \hline \end{array}$, $\begin{array}{|c|c|} \hline & \\ \hline \end{array} \text{.} / \begin{array}{|c|} \hline \\ \hline \end{array}$ not necessary but OK

Generating tables and plots

x, y are vectors. A vector is a 1-dimensional list of values

x	sin(x)
0.000	0.000
0.784	0.707
1.571	1.000
2.357	0.707
3.142	0.000
3.927	-0.707
4.712	-1.000
5.498	-0.707
6.283	0.000

```
x= linspace(0, 2*pi, 9);  
y= sin(x);  
plot(x, y)
```



Note: x, y are shown in **columns** due to space limitation; they should be **rows**.

Does this assign to y the values
 $\sin(0^\circ)$, $\sin(1^\circ)$, $\sin(2^\circ)$, ..., $\sin(90^\circ)$?

```
x = linspace(0, pi/2, 90);
```

```
y = sin(x);
```

A: yes

B: no

Plot this!

See `plotComparison.m`

$$f(x) = \frac{\sin(5x) \exp(-x/2)}{1 + x^2}$$

for
 $-2 \leq x \leq 3$

```
x = linspace(-2, 3, 200);  
y = sin(5*x) .* exp(-x/2) ./ (1 + x.^2);  
plot(x, y)
```



Element-by-element arithmetic
operations on arrays

Element-by-element arithmetic operations on arrays...

Also called “vectorized code”

```
x = linspace(-2, 3, 200);
```

```
y = sin(5*x) .* exp(-x/2) ./ (1 + x.^2);
```

x and y are vectors

Contrast with scalar operations that we’ve used previously...

```
a = 2.1;
```

```
b = sin(5*a);
```

a and b are scalars

The **operators** are (mostly) the same; the operands may be scalars or vectors.

When an operand is a vector, you have “vectorized code.”

Some format commands to use with **plot**

```
xlabel('text for labeling x-axis')  
ylabel('text for labeling y-axis')  
title('text for plot title at top center')
```

```
hold on % hold subsequent plot commands to current axes
```

```
hold off % subsequent plot command refreshes axes--
```

```
% erase previous items  
default
```

```
close all % close all graphics windows
```

```
axis equal % same scaling for x, y axes
```

```
axis off % hide axes
```

```
axis on % show axes
```

```
default
```

Start with drawing a single line segment

```
a= 0;    % x-coord of pt 1
```

```
b= 1;    % y-coord of pt 1
```

```
c= 5;    % x-coord of pt 2
```

```
d= 3;    % y-coord of pt 2
```

```
plot([a c], [b d], '-*')
```

x-values
(a vector)

y-values
(a vector)

Line/marker
format

Making an x-y plot

```
a= [0 4 3 8]; % x-coords
```

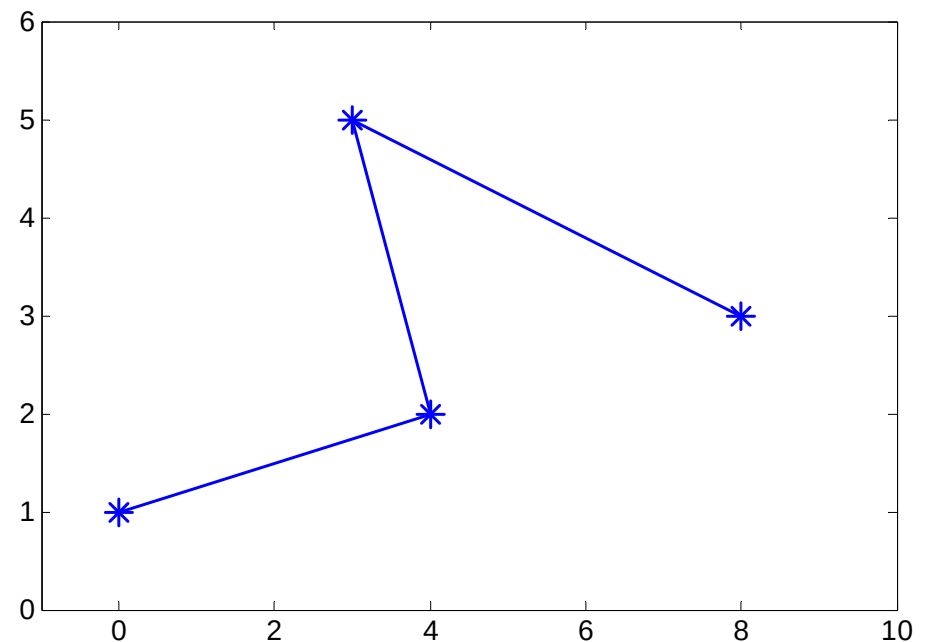
```
b= [1 2 5 3]; % y-coords
```

```
plot(a, b, '-*')
```

x-values
(a vector)

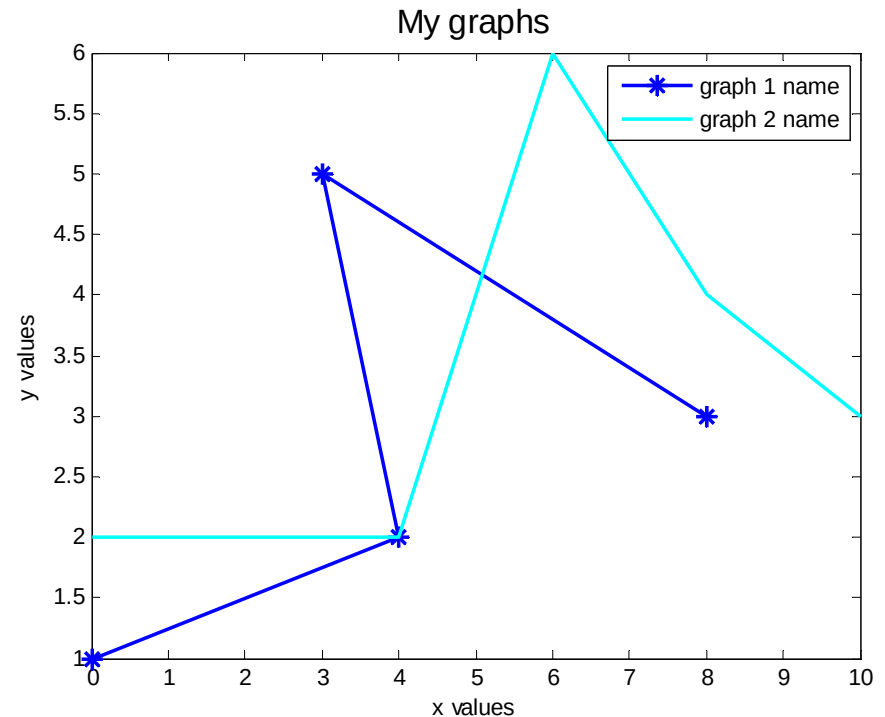
y-values
(a vector)

Line/marker
format



Making an x-y plot with multiple graphs (lines)

```
a= [0 4 5 8];  
b= [1 2 5 3];  
f= [0 4 6 8 10];  
g= [2 2 6 4 3];  
plot(a,b,'-*',f,g,'c')  
legend('graph 1 name', 'graph 2 name')  
xlabel('x values')  
ylabel('y values')  
title('My graphs', 'FontSize',14)
```



See also [plotComparison.m](#)

Drawing a polygon (multiple line segments)

```
% Draw a rectangle with the lower-left
% corner at (a,b), width w, height h.
```

```
x= [      ]; % x data
```

```
y= [      ]; % y data
```

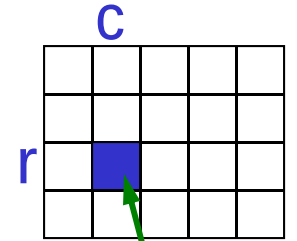
plot(x, y)

Fill in the missing vector values!

Drawing a polygon (multiple line segments)

```
% Draw a rectangle with the lower-left  
% corner at (a,b), width w, height h.  
x= [a    a+w    a+w    a    a ]; % x data  
y= [b    b      b+h    b+h  b ]; % y data  
plot(x, y)
```

2-d array: **matrix**



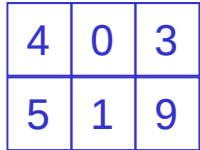
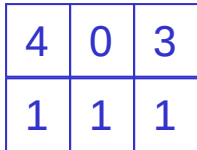
- An array is a **named** collection of **like** data organized into rows and columns
- A 2-d array is a table, called a **matrix**
- Two **indices** identify the position of a value in a matrix, e.g.,

mat(r, c)

refers to component in row **r**, column **c** of matrix **mat**

- Array index starts at **1**
- **Rectangular**: all rows have the same #of columns

Creating a matrix

- Built-in functions: `ones`, `zeros`, `rand(I)`
 - E.g., `zeros(2,3)` gives a 2-by-3 matrix of 0s
- “Build” a matrix using square brackets, `[ ]`, but the dimension must match up:
 - `[x y]` puts `y` to the right of `x`
 - `[x; y]` puts `y` below `x`
 - `[4 0 3; 5 1 9]` creates the matrix 
 - `[4 0 3; ones(1,3)]` gives 
 - `[4 0 3; ones(3,1)]` doesn't work