Lecture 4: Logical operators; nesting

- Previous lecture:
  - Branching (`if`, `elseif`, `else`, `end`)
  - Relational operators (`<`, `>=`, `==`, `~=`, etc.)
- Today:
  - Logical operators (`&&`, `||`, `~`) and "short-circuiting"
  - More branching: nesting
  - Top-down design

- Announcements:
  - Ex2 due Sun, Feb 21
  - P1 due Tue, Feb 23
    - Partners and Academic Integrity

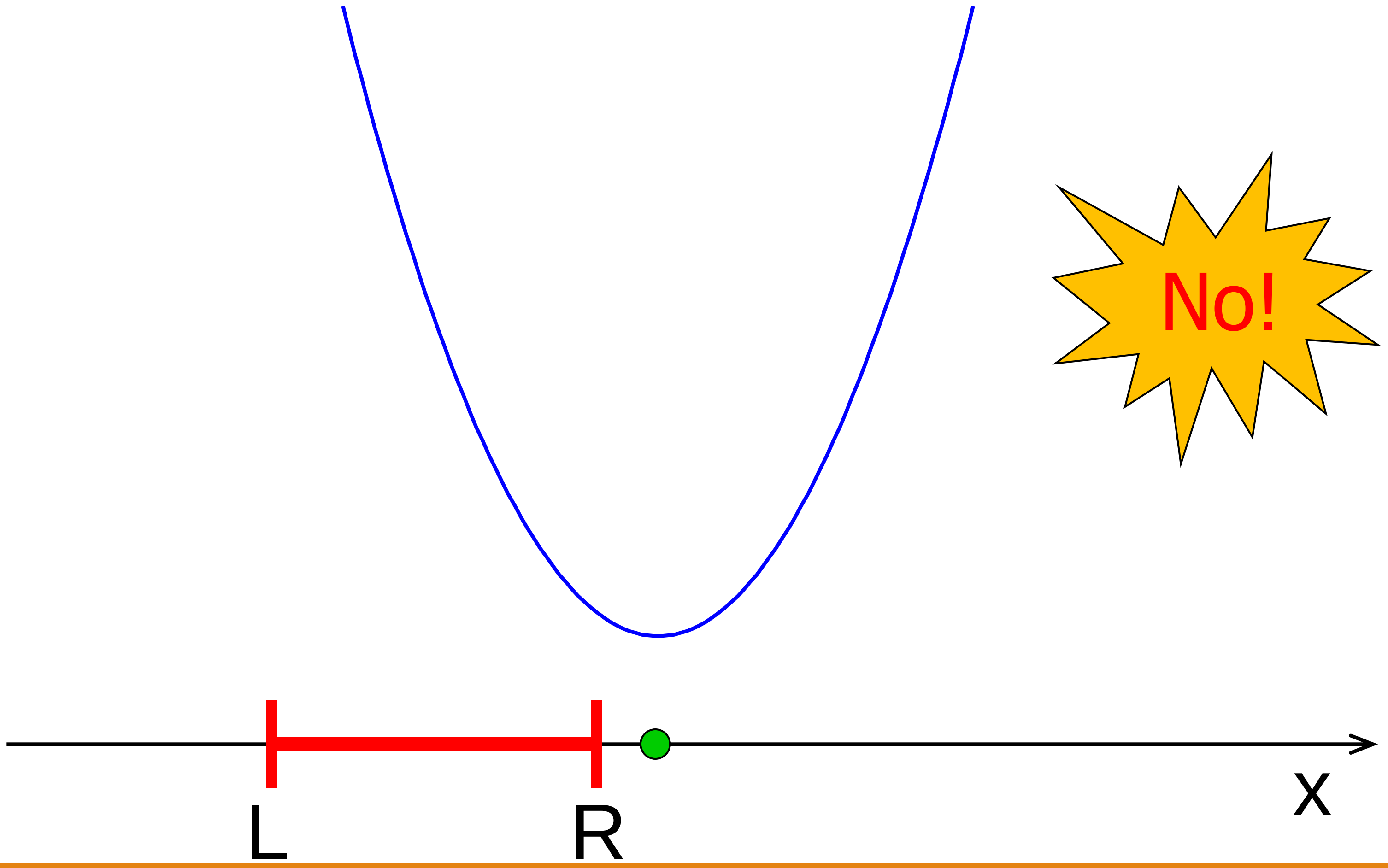
Problem 3

Write a code fragment that prints “Inside” if xc is in the interval and “Outside” if it is not.

Is x_c in the interval [L,R]?

$q(x) = x^2 + bx + c$

$x_c = -b/2$



So what is the requirement?

```
% Determine whether xc is in
% [L,R]
xc = -b/2;

if ________________
    disp('Yes')
else
    disp('No')
end
```

So what is the requirement?

```
% Determine whether xc is in
% [L,R]
xc = -b/2;

if xc>=L && xc<=R
    disp('Yes')
else
    disp('No')
end
```

The `if` construct

`if` $\boxed{\textit{boolean expression1}}$

statements to execute if $\boxed{\textit{expression1}}$ *is true*

`elseif` $\boxed{\textit{boolean expression2}}$

statements to execute if $\boxed{\textit{expression1}}$ *is false*

but $\boxed{\textit{expression2}}$ *is true*

:

`else`

statements to execute if all previous conditions

are false

`end`

The value of a **boolean expression** is either true or false.

```
(L<=xc) && (xc<=R)
```

Above (compound) boolean expression is made up of two (simple) boolean expressions. Each has a value that is either *true* or *false*.

Connect boolean expressions by boolean operators and (&&), or (||)

Also available is the not operator (~)

Logical operators

&& logical <u>and</u>: Are both conditions true?

E.g., we ask "is $L \leq x_c$ and $x_c \leq R$?"

In our code: `L<=xc && xc<=R`

|| logical <u>or</u>: Is at least one condition true?

E.g., we can ask if x_c is outside of $[L,R]$,

i.e., "is $x_c < L$ or $R < x_c$?"

In code: `xc<L || R<xc`

~ logical <u>not</u>: Negation

E.g., we can ask if x_c is not outside $[L,R]$.

In code: `~(xc<L || R<xc)`

“Truth table”

X, Y represent boolean expressions.
E.g., d>3.14

X	Y	X && Y “and”	X || Y “or”	~Y “not”
F	F			
F	T	F	T	F
T	F			
T	T			

Checkpoint

- How many entries in the table are True?

A: 4

B: 5

C: 8

D: other

“Truth table”

X, Y represent boolean expressions.
E.g., d>3.14

X	Y	X && Y “and”	X || Y “or”	~Y “not”
F	F	F	F	T
F	T	F	T	F
T	F	F	T	T
T	T	T	T	F

"Truth table"

Matlab uses 0 to represent false,
1 to represent true

X	Y	X && Y "and"	X || Y "or"	~Y "not"
0	0	0	0	1
0	1	0	1	0
1	0	0	1	1
1	1	1	1	0

Logical operators “short-circuit”

$\underbrace{a > b}_{\text{true}}$ && $c > d$

Go on

$\underbrace{a > b}_{\text{false}}$ && $c > d$

Stop

Entire expression is false since
the first part is false

A && expression short-circuits to false if the left operand evaluates to *false*.

A || expression short-circuits to ____________________ if

____

____

Logical operators "short-circuit"

$\underbrace{a > b}_{\text{false}} \;||\; c > d$

Go on

$\underbrace{a > b}_{\text{true}} \;||\; c > d$

Stop

Entire expression is true since the first part is true

A **&&** expression short-circuits to false if the left operand evaluates to *false*.

A **||** expression short-circuits to true if the left operand evaluates to *true*.

Why short-circuit?

- Right-hand Boolean expression may be *expensive* or potentially *invalid*
- Much clearer than alternatives

```
if (x < 0.5) || (tan(x) < 1)
   % ...
end

if (x ~= 0) && (y/x > 1e-8)
   % ...
end
```

Logical operators are required when connecting multiple Boolean expressions

Why is it wrong to use the expression

~~L <= xc <= R~~

for checking if x_c is in $[L,R]$?

L<=xc && xc<=R

Example: Suppose L is 5, R is 8, and xc is 10. We know that 10 is not in [5,8], but the expression

L <= xc <= R gives…

1

1

Stepping back…

Variables *a*, *b*, and *c* are integers between 1 and 100. Does this fragment correctly identify when lines of length *a*, *b*, and *c* could form a right triangle?

```
if a^2 + b^2 == c^2
   disp('Right tri')
else
   disp('No right tri')
end
```

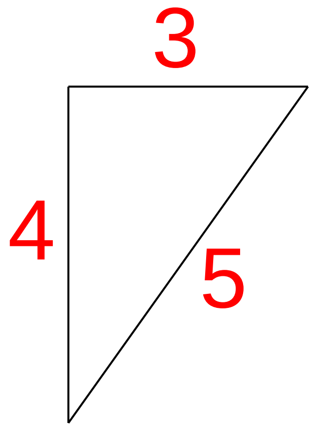
A: correct

B: false positives

C: false negatives

D: both B & C

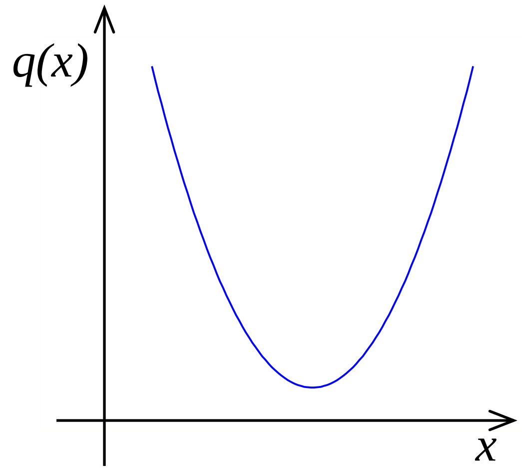
```
a = 5;
b = 3;
c = 4;
if (a^2 + b^2 == c^2)


   disp('Right tri')
else
   disp('No right tri')
end
```



This fragment prints “No” even though we have a right triangle!

```
a = 5;
b = 3;
c = 4;
if (a^2 + b^2 == c^2) || ...
   (a^2 + c^2 == b^2) || ...
   (b^2 + c^2 == a^2)
   disp('Right tri')
else
   disp('No right tri')
end
```

Consider the quadratic function



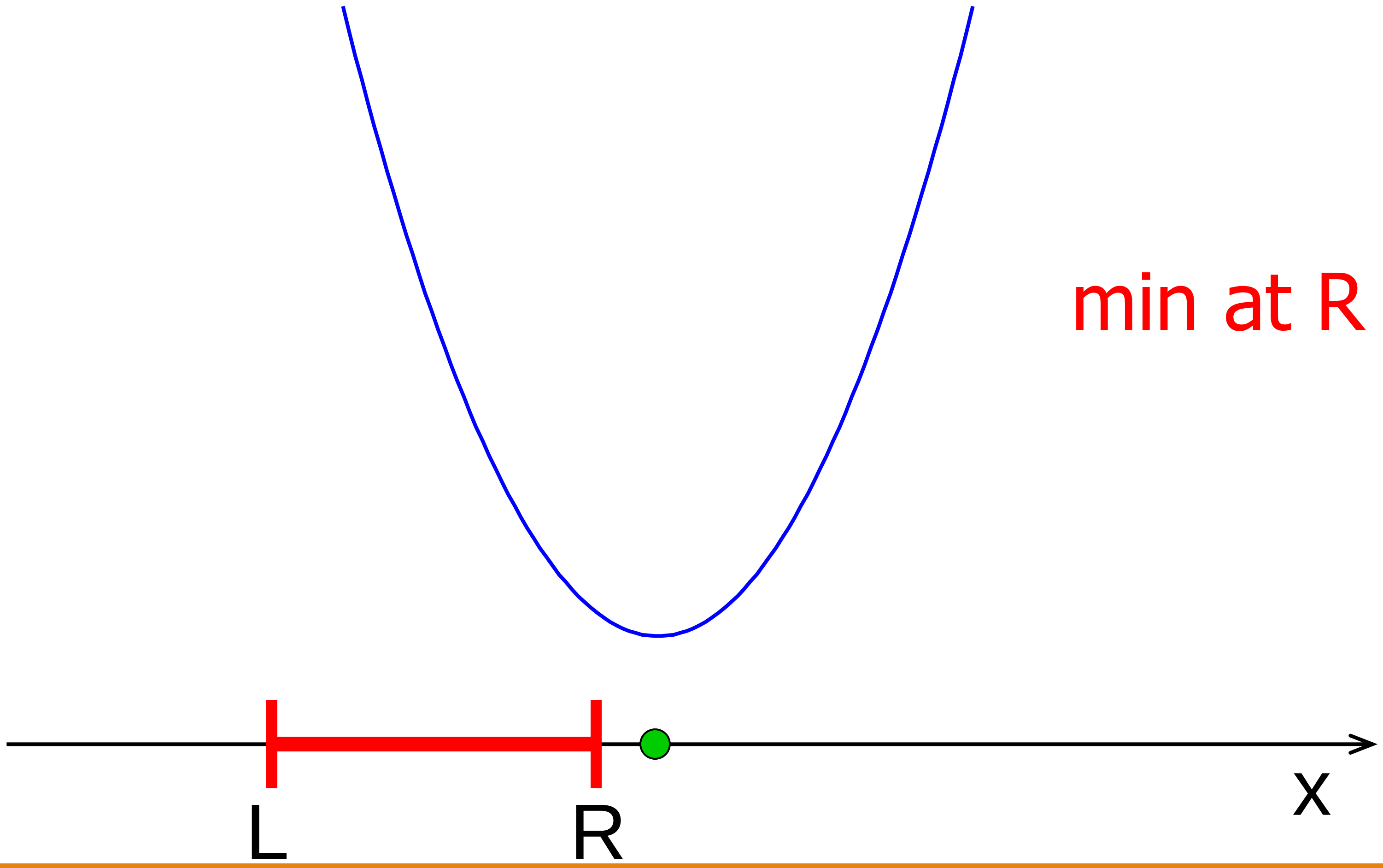
$$q(x) = x^2 + bx + c$$

on the interval $[L\ ,\ R]$:

- Is the function strictly increasing in $[L\ ,\ R]$?
- Which is smaller, $q(L)$ or $q(R)$?
- What is the minimum value of $q(x)$ in $[L\ ,\ R]$?

$$q(x) = x^2 + bx + c$$

$x_c = -b/2$



Conclusion

If x_c is between L and R

Then min is at x_c

Otherwise

Min value is at one of the endpoints

Stepwise refinement

Start with pseudocode

If *xc* is between *L* and *R*

  Min is at *xc*

Otherwise

  Min is at one of the endpoints

We have decomposed the problem into three pieces! Can choose to work with any piece next: the if-else construct/condition, min at xc, or min at an endpoint

Set up structure first: if-else, condition

```
if  L<=xc && xc<=R
```

Then min is at *xc*

```
else
```

Min is at one of the endpoints

```
end
```

Now **refine** our solution-in-progress. I'll choose to work on the if-branch next

Refinement: filled in detail for task “min at xc”

```
if  L<=xc && xc<=R
   % min is at xc
   qMin= xc^2 + b*xc + c;


else
```

Min is at one of the endpoints

```
end
```

Continue with refining the solution… else-branch next

Refinement: detail for task “min at an endpoint”

```
if  L<=xc && xc<=R
   % min is at xc
   qMin= xc^2 + b*xc + c;
else
   % min is at one of the endpoints
   if  % xc left of bracket
      % min is at L
   else  % xc right of bracket
      % min is at R
   end
end
```

Continue with the refinement, i.e., replace comments with code

Refinement: detail for task “min at an endpoint”

```
if  L<=xc && xc<=R
   % min is at xc
   qMin= xc^2 + b*xc + c;
else
   % min is at one of the endpoints
   if  xc < L
      qMin= L^2 + b*L + c;
   else
      qMin= R^2 + b*R + c;
   end
end
```

Final solution (given b,c,L,R,xc)

```
if  L<=xc && xc<=R
   % min is at xc
   qMin= xc^2 + b*xc + c;
else
   % min is at one of the endpoints
   if  xc < L
      qMin= L^2 + b*L + c;
   else
      qMin= R^2 + b*R + c;
   end
end
```

See **quadMin.m**
quadMinGraph.m

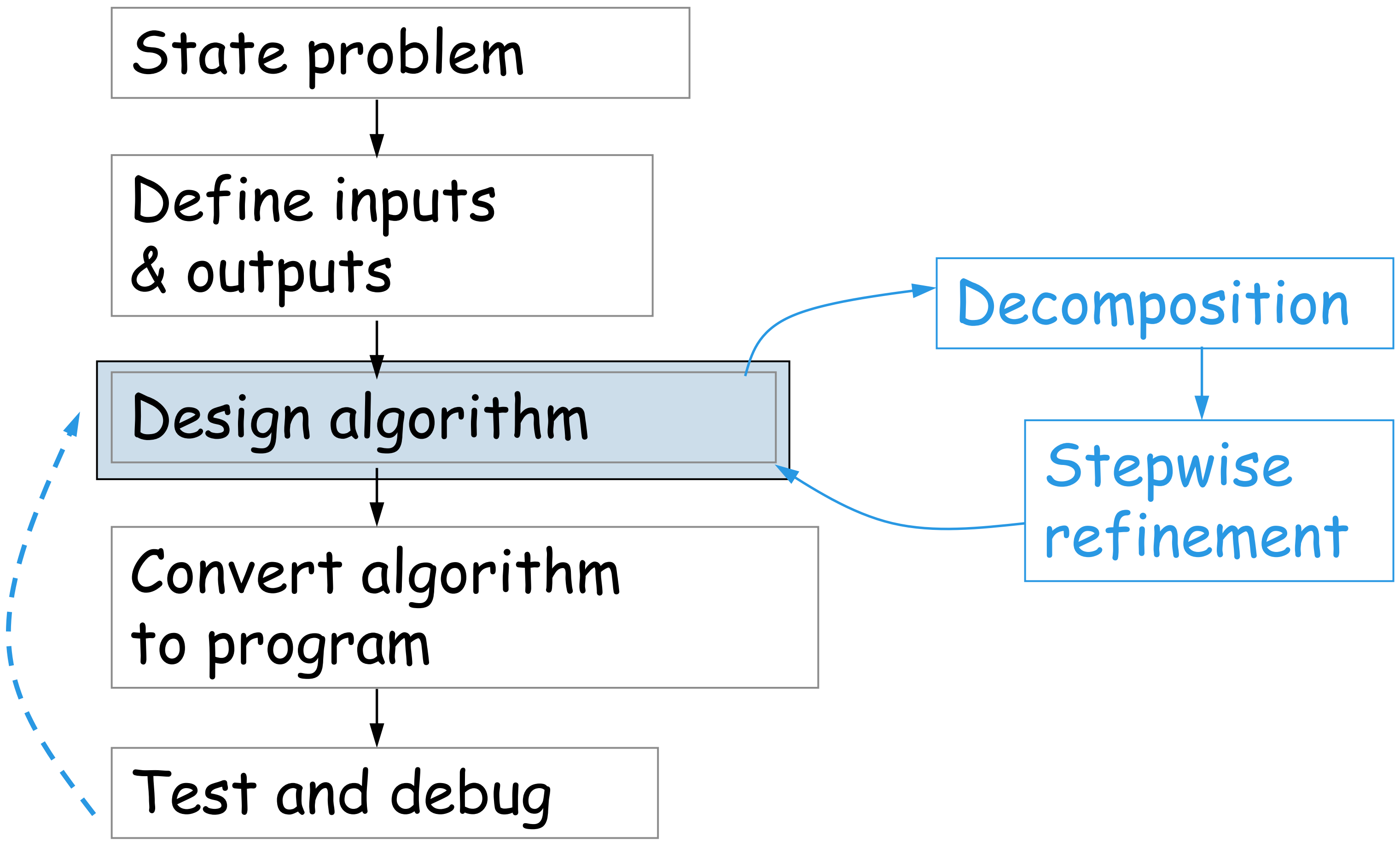
An if-statement can appear within a branch—just like any other kind of statement!

Notice that there are 3 alternatives→can use `elseif`!

```
if  L<=xc && xc<=R
  % min is at xc
  qMin= xc^2 + b*xc + c;
else
  % min at one endpt
  if  xc < L
    qMin= L^2 + b*L + c;
  else
    qMin= R^2 + b*R + c;
  end
end
```

```
if  L<=xc && xc<=R
  % min is at xc
  qMin= xc^2 + b*xc + c;
elseif xc < L
  qMin= L^2 + b*L + c;
else
  qMin= R^2 + b*R + c;
end
```

Top-Down Design



An algorithm is an idea. To use an algorithm you must choose a programming language and implement the algorithm.

If xc is between L and R

Then min value is at xc

Otherwise

Min value is at one of the endpoints

```
if  L<=xc && xc<=R
   % min is at xc

else
   % min is at one of the endpoints




end
```

```
if  L<=xc && xc<=R
   % min is at xc

else
   % min is at one of the endpoints




end
```

```
if  L<=xc && xc<=R
   % min is at xc
   qMin= xc^2 + b*xc + c;
else
   % min is at one of the endpoints




end
```

```
if  L<=xc && xc<=R
   % min is at xc
   qMin= xc^2 + b*xc + c;
else
   % min is at one of the endpoints




end
```

```
if  L<=xc && xc<=R
   % min is at xc
   qMin= xc^2 + b*xc + c;
else
   % min is at one of the endpoints
   if  xc < L

   else

   end
end
```

```
if  L<=xc && xc<=R
   % min is at xc
   qMin= xc^2 + b*xc + c;
else
   % min is at one of the endpoints
   if  xc < L
      qMin= L^2 + b*L + c;
   else
      qMin= R^2 + b*R + c;
   end
end
```

Testing and debugging

- An integral part of the design loop
- The programmer's job, not someone else's
  - Don't ask TAs "is this right?"; *Run your own tests*, then ask for guidance on failures
- Doesn't need to be formal, but does need to be thought through

- Testing tips
  - Know what your immediate goal is
  - Look for simple cases, compare with hand-calcs
  - Think about corner cases – try to break things while still respecting input constraints

Checkpoint: Should we use this code to decide your grade?

```
score= input('Enter score: ');
if score>55
      disp('D')
elseif score>65
      disp('C')
elseif score>80
      disp('B')
elseif score>93
      disp('A')
else
      disp('Try again')
end
```

A: yes

B: no – high scores might get low grade

C: no – low scores might get high grade

D: no – some scores might get no grade

Question

A stick of unit length is split into two pieces. The breakpoint is randomly selected. On average, how long is the shorter piece?

Physical experiment?

Thought experiment? → analysis

Computational experiment! → simulation

Need to <u>repeat</u> many trials!