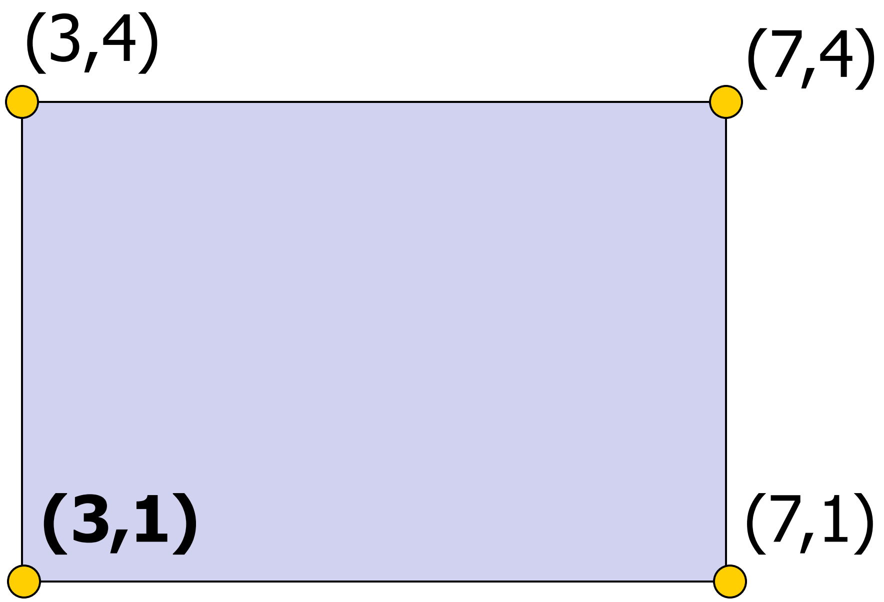
Warmup

Assume vectors x, y contain the coordinates of the vertices of a rectangle:

```
x= [3 3 7 7];
y= [1 4 4 1];
```

Will the following code draw the four sides of the rectangle?

```
plot(x, y, 'k-')
```



A: Yes

B: No

Concatenation

- Concatenate two scalars into a (row-)vector:
  ```
  u= [3 1]
  ```
- Concatenate a scalar onto a (row-)vector:
  ```
  v= [u 4]  % v = [3 1 4]
  ```
- Application: repeat the first element of a vector at its end:
  ```
  w= [v v(1)]  % w = [3 1 4 3]
  ```
- Application: append to a vector:
  ```
  w= [w 5]  % w = [3 1 4 3 5]
  ```

- Previous Lecture:
  - Discrete vs. continuous; finite vs. infinite
  - Linear interpolation
  - RGB color
  - Floating-point arithmetic
  - Introduction to vectorized computation

lots of new topics!

- Today's Lecture:
  - Vectorized operations
  - Introduction to 2-d array—matrix

$$\begin{bmatrix} 1 & 3 \\ 4 & 2 \end{bmatrix}$$

- Announcements:
  - Survey season!
    - Please fill out "Mid-Semester Survey" on CMS
    - Please respond to ENG eval requests (course, TAs, etc.)
  - See website for review materials. Optional review session on Sunday, 1:00-2:30pm in Phillips 203.
  - Prelim 1 Tuesday 3/10 at 7:30pm, Olin Hall
    - Alt exam: 5:45pm; check e-mail

Studying for exams

1. **Write** your own solutions to examples from lecture
2. **Re-do** discussion problems un-aided
3. **Answer** review questions, using notes as needed
4. **Do** one old exam, using notes as needed
5. **Do** a second old exam un-aided – this is your best diagnostic
6. Review specific topics as necessary

Just reading code, solutions will *not* help!

Initialize arrays if dimensions are known ("pre-allocation")

… instead of "building" the array one component at a time

```
% Initialize y
x= linspace(a,b,n);
y= zeros(1,n);
for k = 1:n
   y(k)= myF(x(k));
end
```

```
% Build y on the fly
x=linspace(a,b,n);
for k = 1:n
   y(k)= myF(x(k));
   % OR
   %y= [y myF(x(k));
end
```

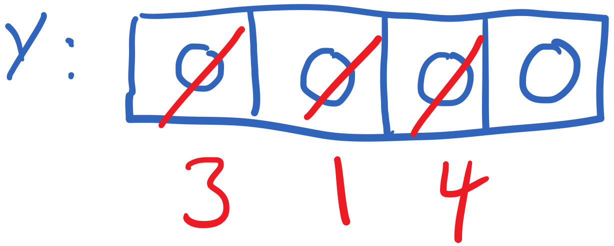
Faster for large n!
BUT you need to know n

Totally fine if you don't know 'n' (ignore Matlab's warning)

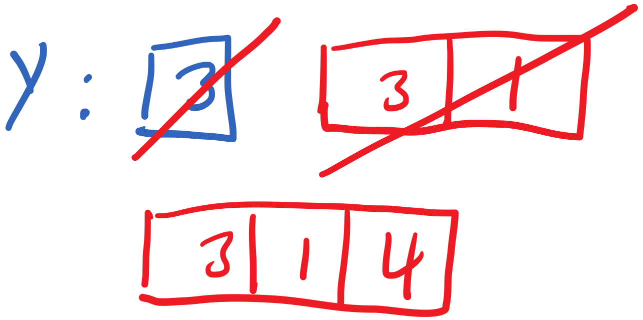
Initialize arrays if dimensions are known ("pre-allocation")

… instead of "building" the array one component at a time

```
% Initialize y
x= linspace(a,b,n);
y= zeros(1,n);
for k = 1:n
   y(k)= myF(x(k));
end
```




```
% Build y on the fly
x=linspace(a,b,n);
for k = 1:n
   y(k)= myF(x(k));
   % OR
   %y= [y myF(x(k));
end
```



Vectorized code

—a Matlab-specific feature

See Sec 4.1 for list of vectorized arithmetic operations

- Code that performs element-by-element arithmetic/relational/logical operations on array operands in one step

- Scalar operation: x + y

  where x, y are scalar variables

  Single value (not containing multiple elements)

- Vectorized code: **x** + **y**

  where **x** and/or **y** are vectors. Generally, vectors **x** and **y** should have the same length and shape

  rows/columns

Vectorized addition

		1	2 ...	k	...
	x	2	1	.5	9
+	**y**	1	2	0	2
=	**z**	3	3	.5	11

Observe:

$$z(1) = x(1) + y(1)$$
$$z(2) = x(2) + y(2)$$
$$\vdots$$
$$z(k) = x(k) + y(k)$$

Matlab code: `z= x + y`

Vectorized multiplication (vector-vector)

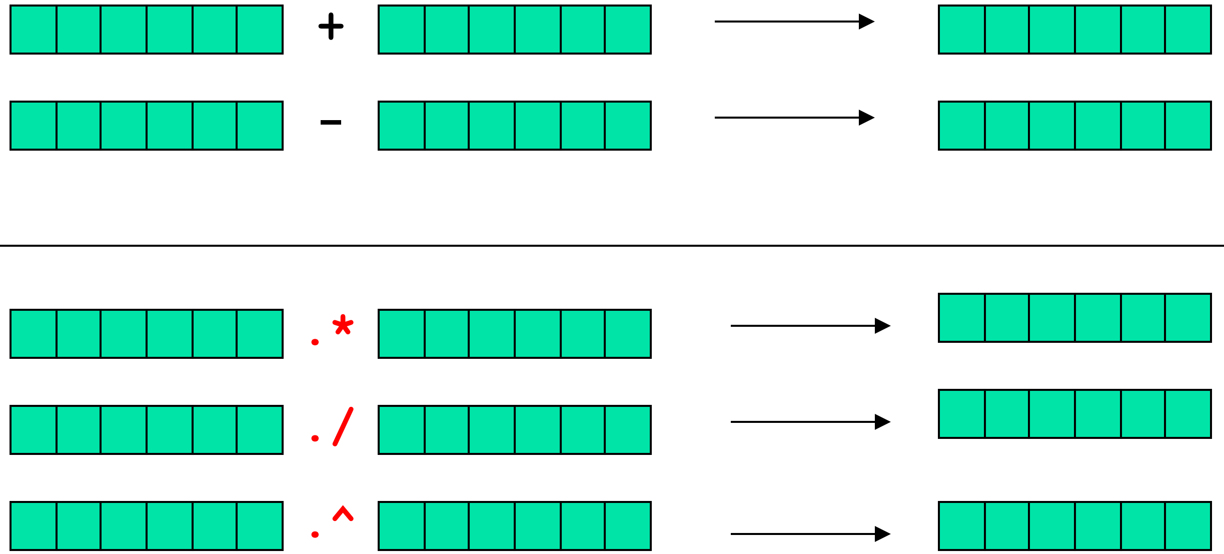
	a	2	1	.5	9
×	b	1	2	0	2
=	c	2	2	0	18

Matlab code: `c= a .* b`

See full list of ops in §4.1

Vectorized

element-by-element arithmetic operations on arrays



A dot (.) is necessary in front of these math operators

Shift (scalar-vector addition)

	x	3				
+	**y**	2	1	.5	9	
=	**z**	5	4	3.5	12	

Matlab code: `z= x + y`

Reciprocate (scalar-vector division)

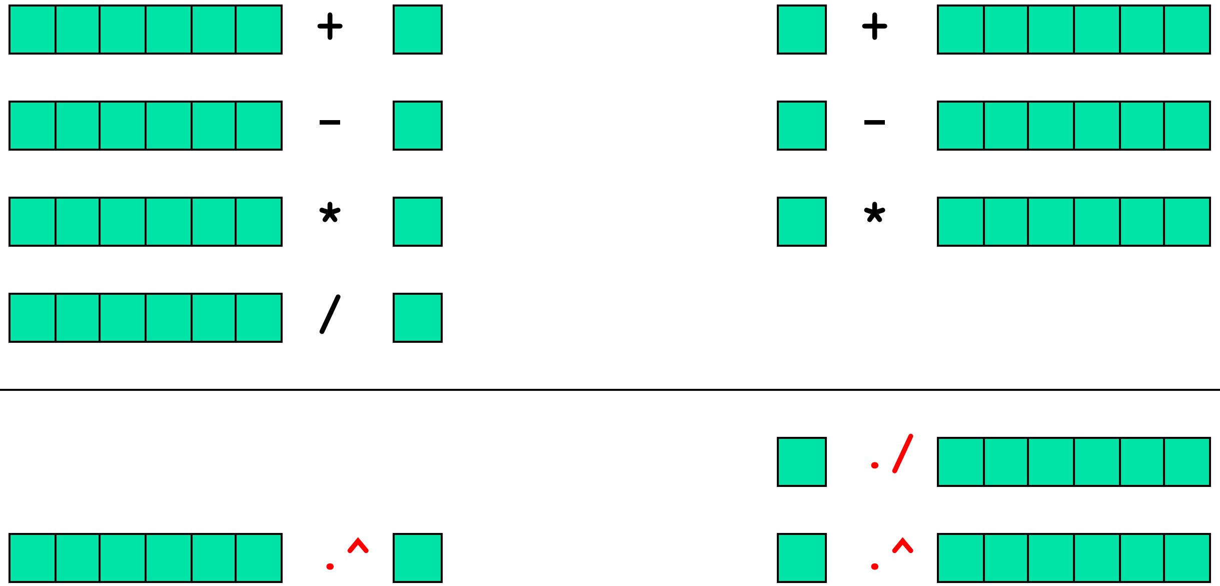
	x	1			
/	**y**	2	1	.5	8
=	**z**	.5	1	2	.125

Matlab code: `z= x ./ y`

See full list of ops in §4.1

Vectorized

element-by-element arithmetic operations between an array and a scalar



A dot (.) is necessary in front of these math operators

Simplified rule: Use dot for these element-by-element ops: * / ^

When are functions vectorized?

- Many built-in functions (`sin()`, `abs()`, …)
- When you only use vectorized operations to implement it
- When you loop over the length of the input
  - Note: Matlab treats scalars like length-1 vectors

```
x = 3.1;
length(x) == 1
x(1) == 3.1
```

Not all functions make sense to vectorize (users can always write their own loops, after all)

See `plotComparison.m`

Can we plot this?

$$f(x) = \frac{\sin(5x)\exp(-x/2)}{1+x^2}$$

for

-2 <= x <= 3

Yes!

```
x = linspace(-2,3,200);
y = sin(5*x).*exp(-x/2)./(1 + x.^2);
plot(x,y)
```

Element-by-element arithmetic operations on arrays

Element-by-element arithmetic operations on arrays…
Also called "vectorized code"

```
x = linspace(-2,3,200);
y = sin(5*x).*exp(-x/2)./(1 + x.^2);
```

x and **y** are vectors

Contrast with scalar operations that we've used previously…

```
a = 2.1;
b = sin(5*a);
```

a and **b** are scalars

The operators are (mostly) the same; the operands may be scalars or vectors.

When an operand is a vector, you have "vectorized code."

End of
Prelim 1 material

Storing and using data in *tables*

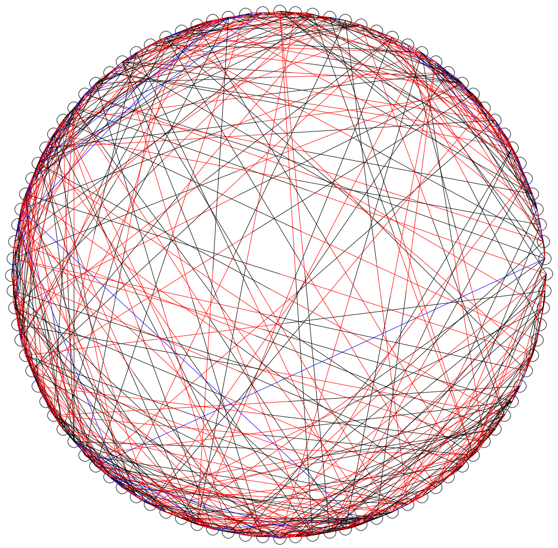
A company has 3 factories that make 5 products with these costs:

Products

factories

10	36	22	15	62
12	35	20	12	66
13	37	21	16	59

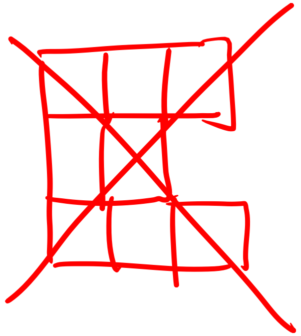
What is the best way to fill a given purchase order?

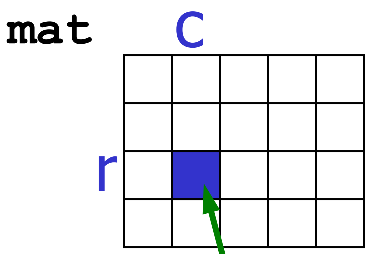


Connections between webpages

0	0	1	0	1	0	0
1	0	0	1	1	1	0
0	1	0	1	1	1	1
1	0	1	1	0	1	0
0	0	1	1	0	1	1
0	0	1	0	1	0	1
0	1	1	0	1	1	0

2-d array: **matrix**





- An array is a named collection of like data organized into rows and columns
- A 2-d array is a table, called a *matrix*
- Two *indices* identify the position of a value in a matrix, e.g.,

  `mat(r,c)`

  refers to component in row r, column c of matrix mat
- Array indices still start at 1
- Rectangular: all rows have the same #of columns

Indexing example

M

M(1,1)	M(1,2)	M(1,3)	M(1,4)
M(2,1)	M(2,2)	M(2,3)	M(2,4)
M(3,1)	M(3,2)	M(3,3)	M(3,4)

M: refers to the whole matrix

M(3,2): refers to the *element* (value) in the third row, second column

Creating a matrix

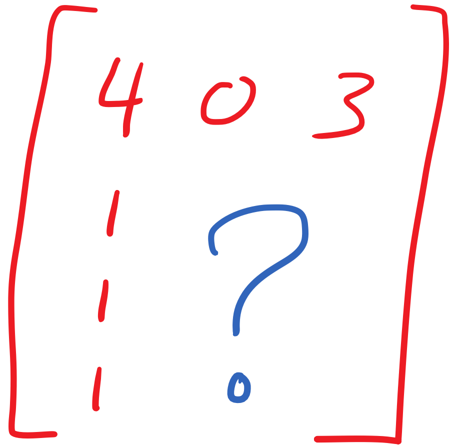
- Built-in functions: `ones()`, `zeros()`, `rand()`
  - E.g., `zeros(2,3)` gives a 2-by-3 matrix of 0s
  - E.g., `zeros(2)` gives a 2-by-2 matrix of 0s
- "Build" a matrix using square brackets, `[ ]`, but the dimension must match up:
  - `[x y]` puts `y` to the right of `x`
  - `[x; y]` puts `y` below `x`
  - `[4 0 3; 5 1 9]` creates the matrix

4	0	3
5	1	9

  - `[4 0 3; ones(1,3)]` gives

4	0	3
1	1	1

  - `[4 0 3; ones(3,1)]` doesn't work



Working with a matrix:
`size()` and individual components

Given a matrix M

2	-1	.5	0	-3
3	8	6	~~7~~ 1	7
5	-3	8.5	9	10
52	81	.5	7	2

```
[nr, nc]= size(M)   % nr is #of rows
                    % nc is #of columns

nr= size(M, 1)   % #of rows
nc= size(M, 2)   % #of columns
n= size(M)
    % n is length 2 vector since M is 2-d:
    %   n(1) is #of rows, n(2) is #of cols
M(2,4)= 1;
disp(M(3,1))
```

'size()' is weird like this

Working with a matrix:
`size()` and individual components

M

2	-1	.5	0	~~7~~ 4
3	8	6	.7	-2
5	-3	8.5	9	10
52	81	.5	1	-8

Given a matrix **M** and the script below

Which statement(s) could make the update shown in **purple** on the diagram?

```
[nr, nc]= size(M);
n= size(M);
M(1,nc)= 4;
M(1,n(2))= 4;
M(0,4)= 4;
```

← A (`M(1,nc)= 4;`)

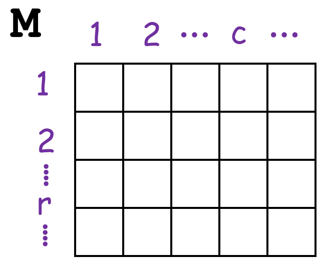
← B (`M(1,n(2))= 4;`)

← C (`M(0,4)= 4;`)

D: None of A, B, C

E: More than one of A, B, C

Example: minimum value in a matrix



function val = minInMatrix(M)

% val is the smallest value in matrix M

Best-in-set pattern:

- Initialize best-so-far
- loop over set:
  - If current value is better than best-so-far:
    - Update best-so-far

Pattern for traversing a matrix M

```
[nr, nc] = size(M)
for  r= 1:nr
     % At row r
     for  c= 1:nc
          % At column c (in row r)
          %
          % Do something with M(r,c) …
     end
end
```