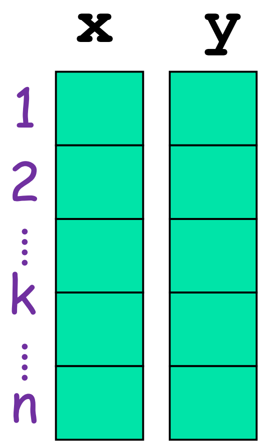
- Previous Lecture:
  - Functions and expressions
  - 1-d array—vector

- Today, Lecture 11:
  - Probability and random numbers
  - Examples of vectors and simulation
  - Loop patterns for processing a vector (watch video)

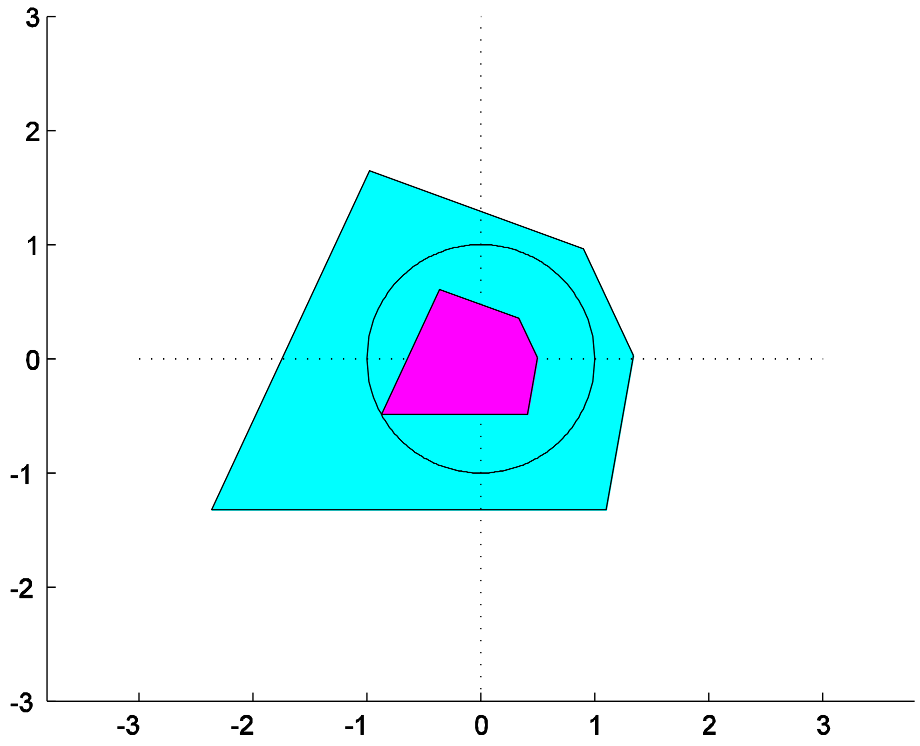
- Announcements:
  - Exercise 6 (Matlab Grader) due Mon, March 2
  - Project 3 due Wed, March 4, at 11pm
  - Social lunch Friday 12:20pm Okenshields (sign up on website)

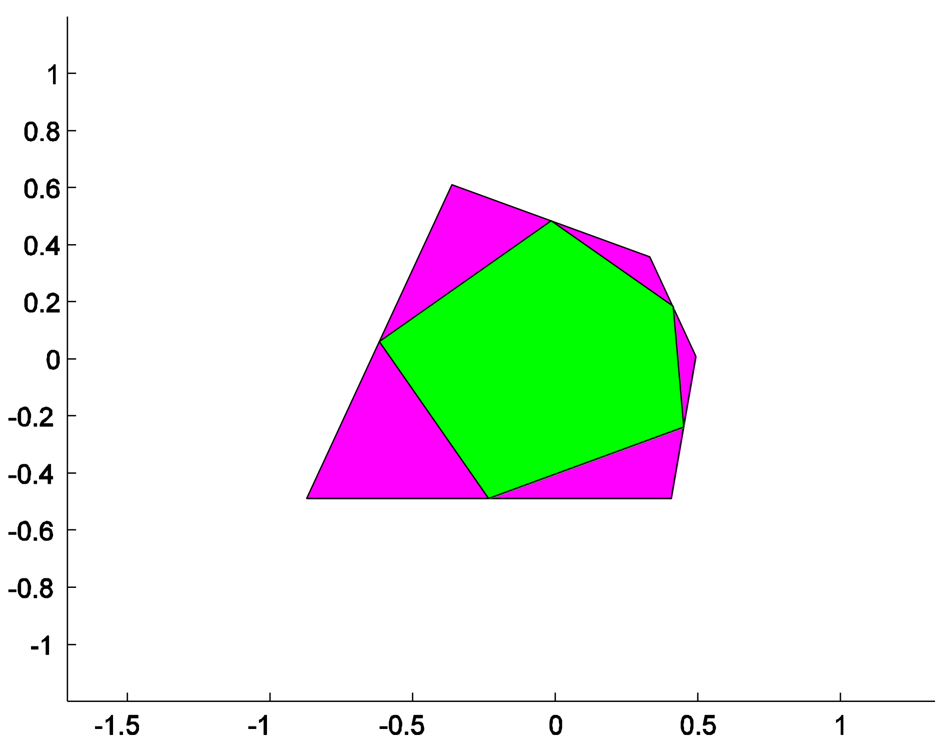
```
function [xNew,yNew] = Centralize(x,y)
% Translate polygon defined by vectors
% x,y such that the centroid is on the
% origin. New polygon defined by vectors
% xNew,yNew.

n= length(x);
xNew= zeros(n,1); yNew= zeros(n,1);
xBar= sum(x)/n;    yBar= sum(y)/n;
for k = 1:n
    xNew(k)= x(k) - xBar;
    yNew(k)= y(k) - yBar;
end
```

sum returns the sum of all values in the vector



Read *Insight* 6.3 for the rest of the story





For-loop pattern for working with a vector

```
% Given a vector v

for k = 1:length(v)

   % Work with v(k)
   % E.g., disp(v(k))

end
```

```
% Count odd values in vector v
count= 0;
for k = 1:length(v)
   if rem(v(k),2)==1
      count= count + 1;
   end
end
```

```
% Pair sums of vector v
s= zeros(1,length(v)-1)
for k = 1:length(v)-1
   s(k)= v(k) + v(k+1);
end
```

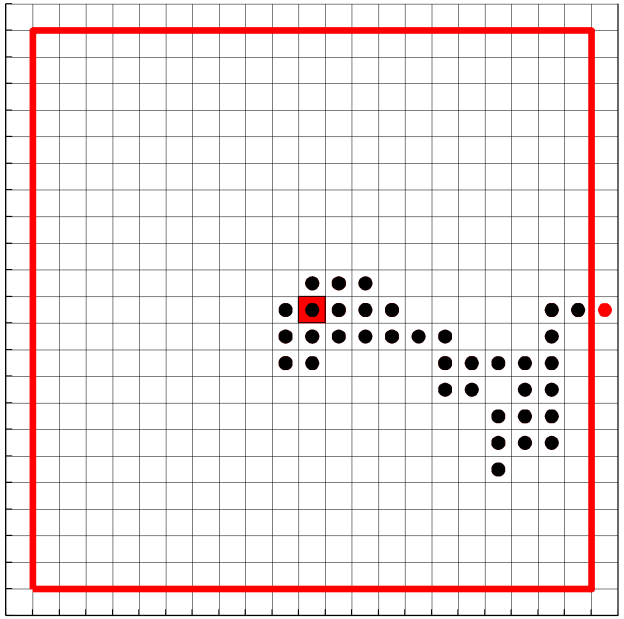
Also good: `1:length(s)`

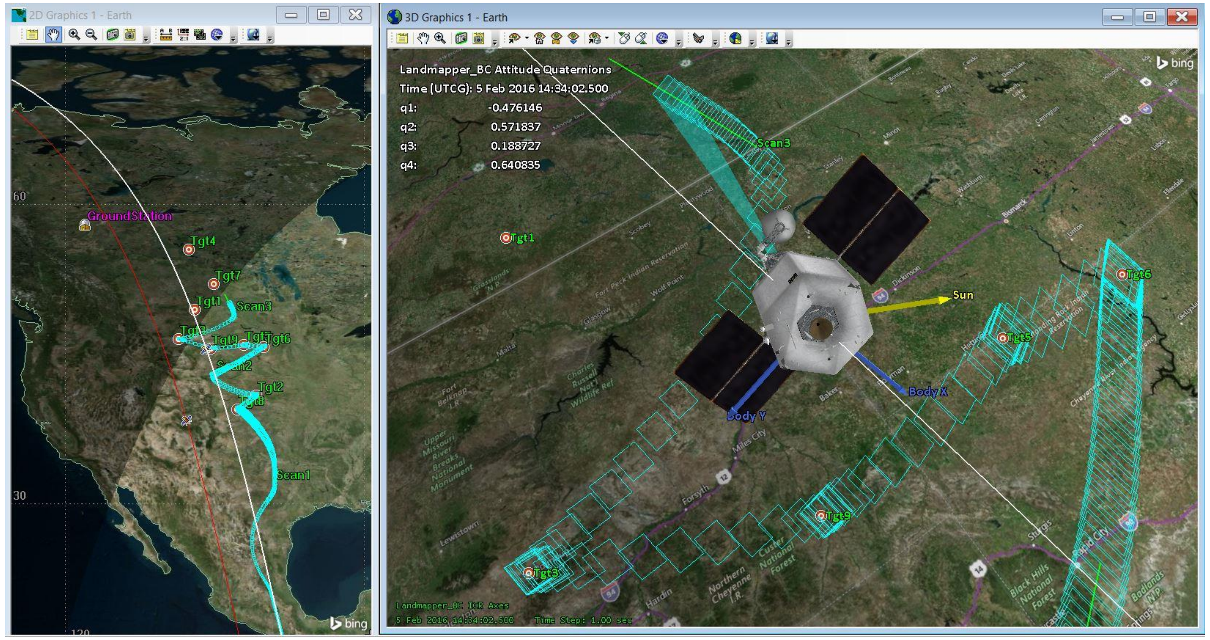
v	5	.4	.9	-4

s	5.4	1.3	-3.1

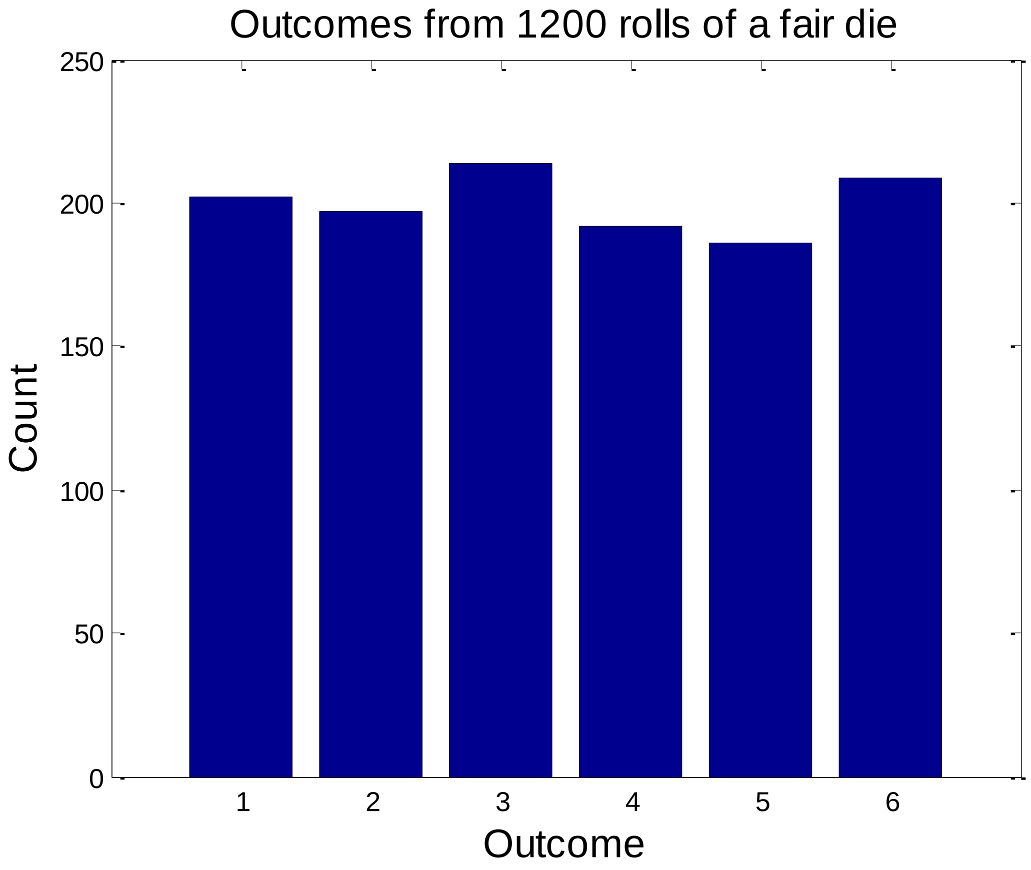
Simulation

- Imitates real system
- Requires judicious use of random numbers
- Requires many trials, or multiple points in time
  - → opportunity to practice working with vectors!

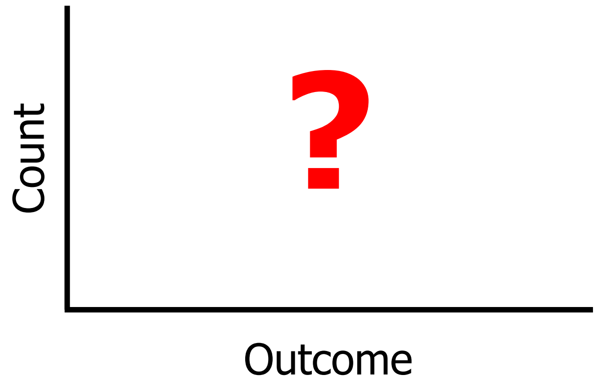




Example: rolling dice



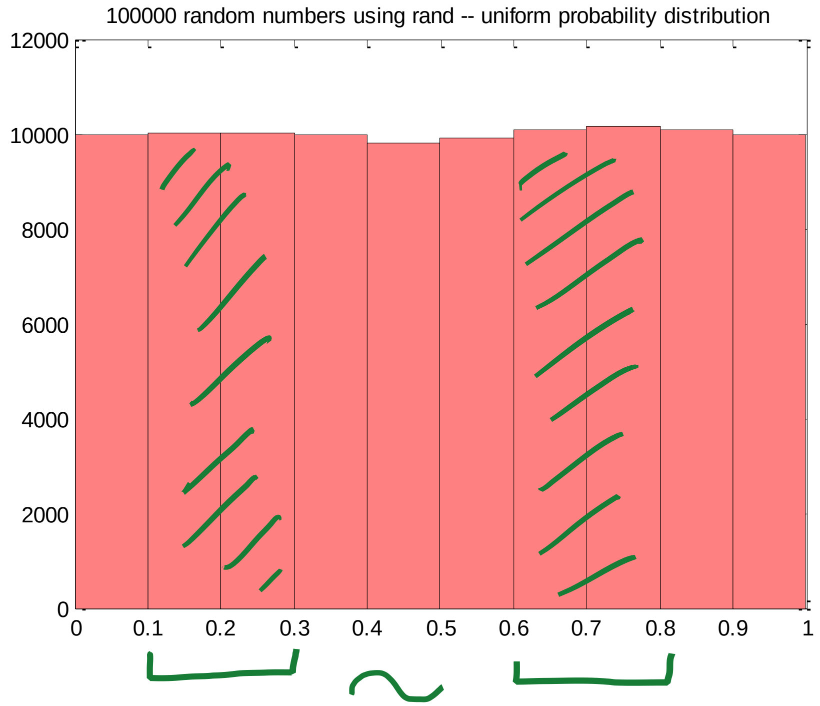
Roll 2 fair dice



Random numbers

- *Pseudorandom* numbers in programming
  - Sequence is reproducible if seeded (e.g., `rng(42)` at start of script)
- Function **`rand()`** generates random real numbers in the interval (0,1). All numbers in the interval (0,1) are equally likely to occur—uniform probability distribution.
- Examples:

`rand()`	one random # in (0,1)
`6*rand()`	one random # in (0,6)
`6*rand()+1`	one random # in (1,7)

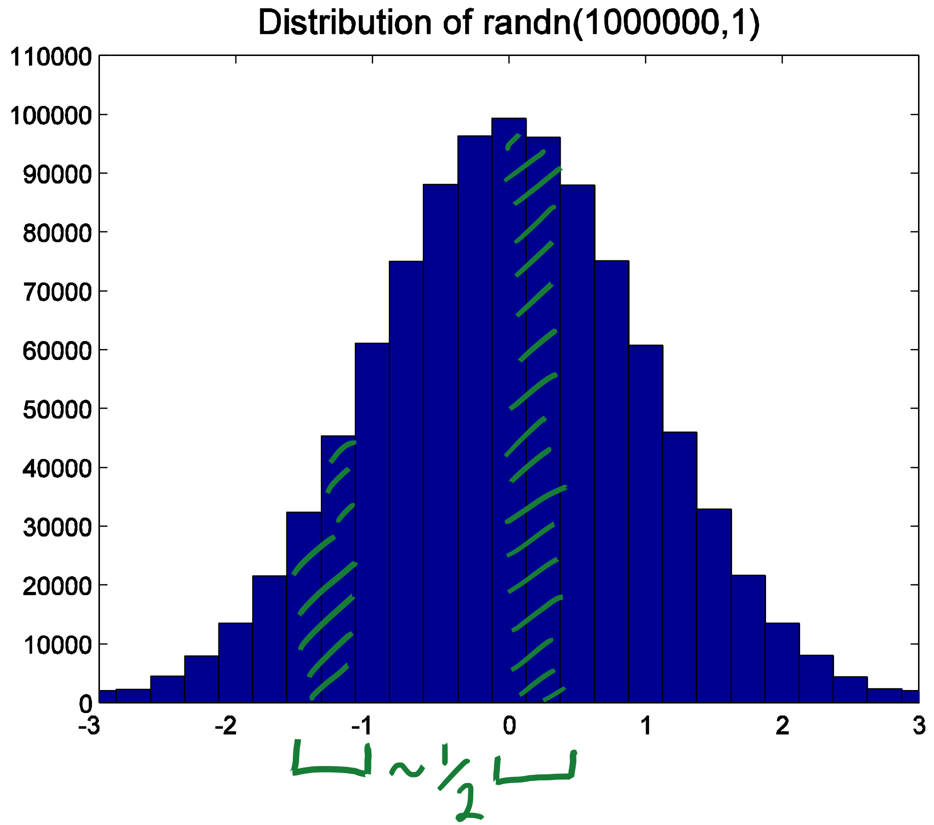


Uniform probability distribution in (0,1)

`rand()`

"Normal" distribution with zero mean and unit standard deviation

`randn()`

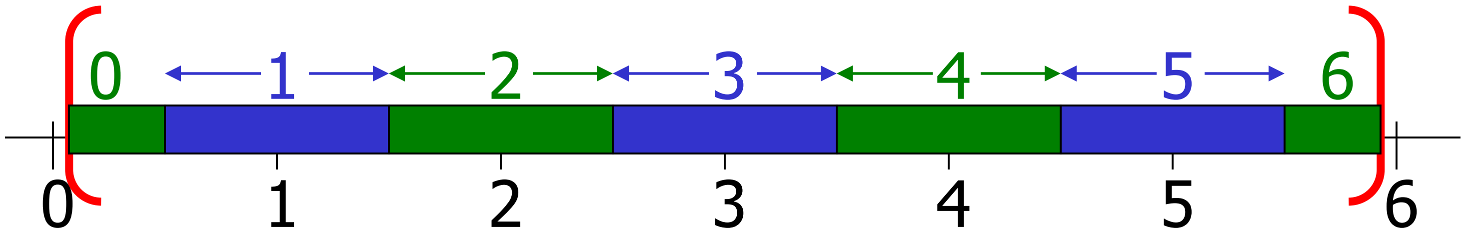


Step 1: Simulate a fair 6-sided die

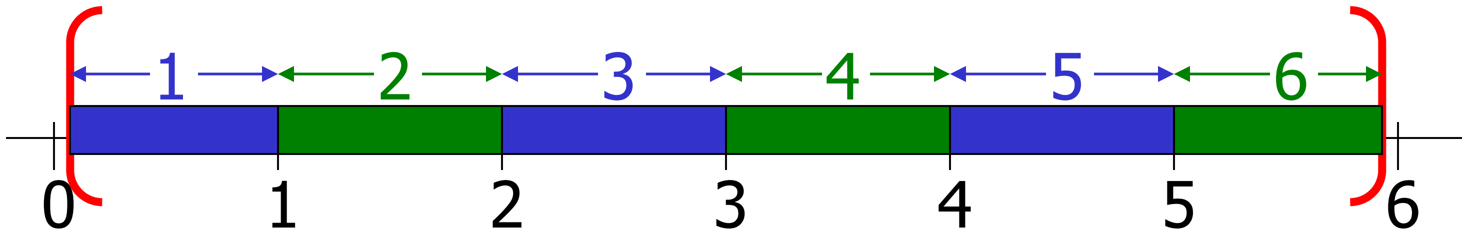
Which expression(s) below will give a random integer in [1..6] with equal likelihood?

- A `round(rand()*6)`
- B `ceil(rand()*6)`
- C *Both expressions above*

round(rand()*6)

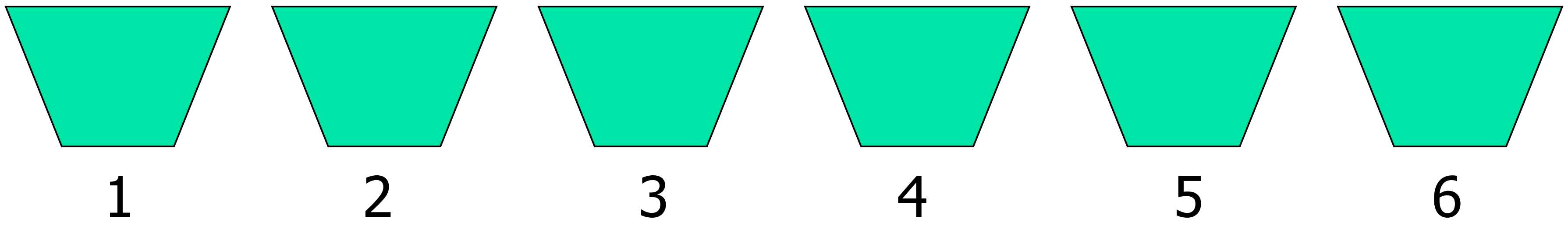


ceil(rand()*6)

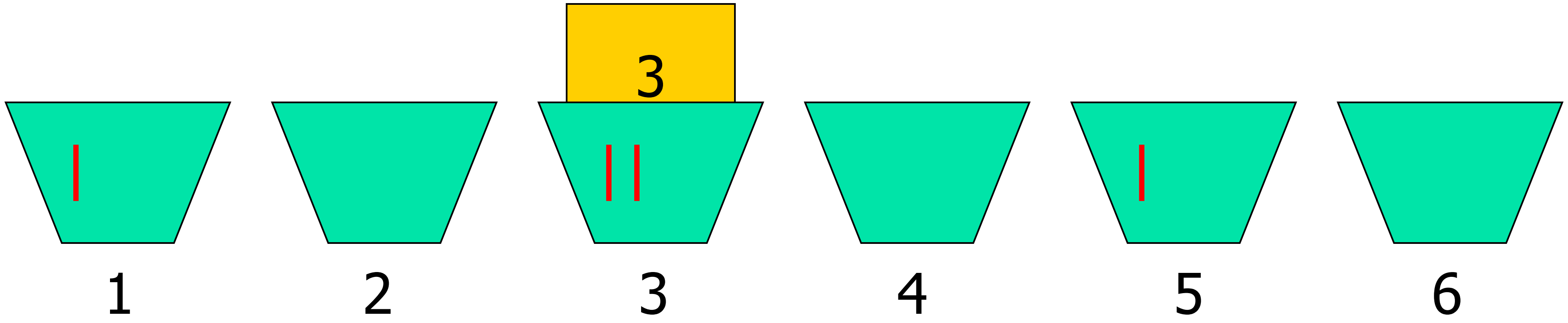


Step 2: Keep track of results

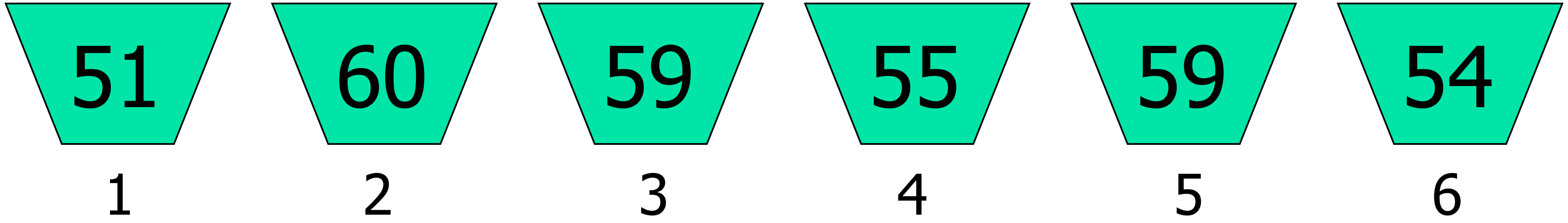
Possible outcomes from rolling a fair 6-sided die



Simulation



Simulation result

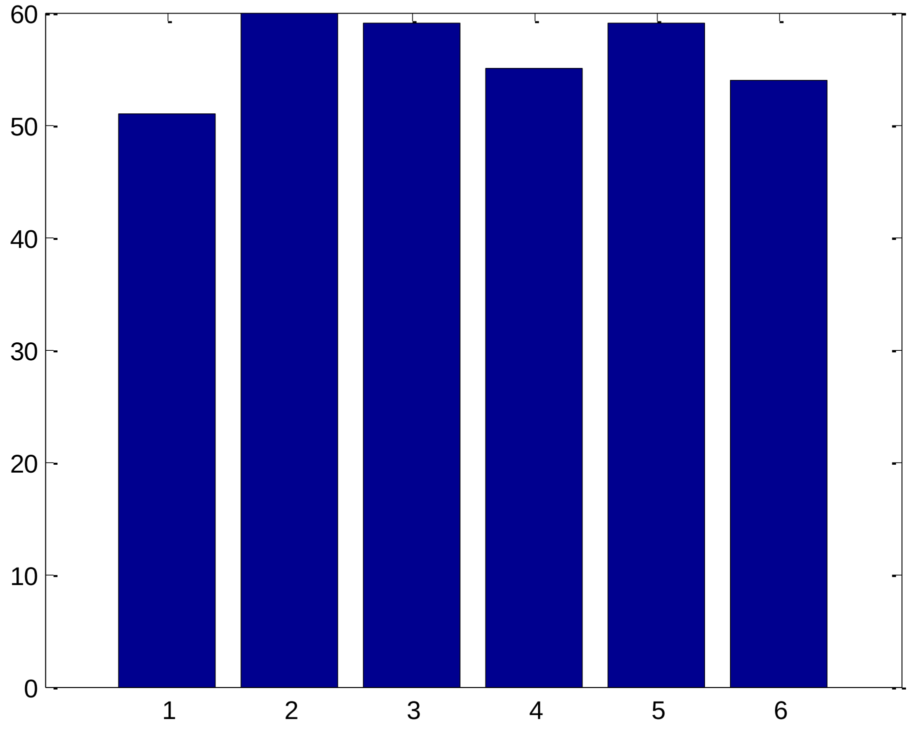


Simulation result

Data in bins

```
bar(1:6, counts)
```

Bin numbers

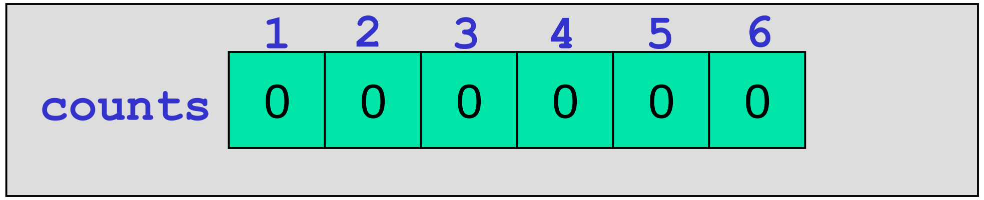


counts	51	60	59	55	59	54
	1	2	3	4	5	6

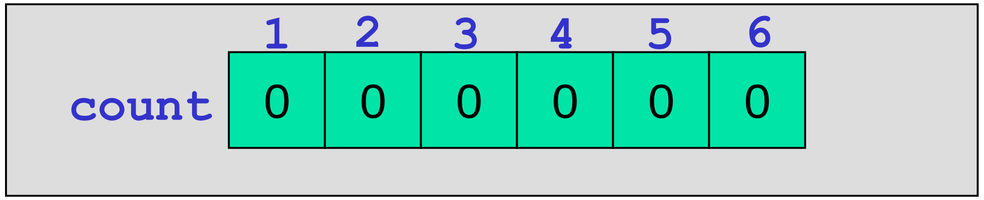
Keep tally on repeated rolls of a fair die

```
Repeat the following:

    % roll the die

    % increment correct “bin”
```

```
function counts = rollDie(rolls)

FACES= 6;                    % #faces on die
counts= zeros(1,FACES);

% Count outcomes of rolling a FAIR die
for k = 1:rolls
    % Roll the die
    face= ceil(rand()*FACES);
    % Increment the appropriate bin

end

% Show histogram of outcome
```

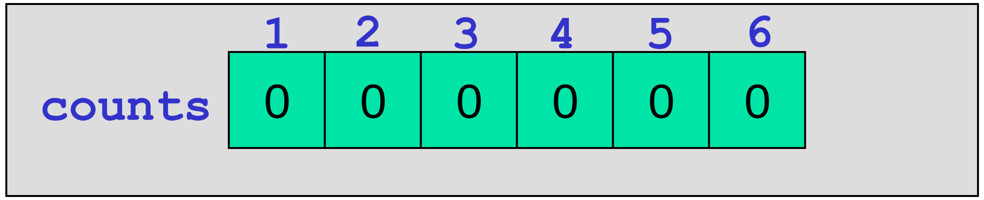


```
% Count outcomes of rolling a FAIR die
counts= zeros(1,6);
for k = 1:100
    face= ceil(rand()*6);
    if face==1
        counts(1)= counts(1) + 1;
    elseif face==2
        counts(2)= counts(2) + 1;
    :
    elseif face==5
        counts(5)= counts(5) + 1;
    else
        counts(6)= counts(6) + 1;
    end
end
```



rollDieV1.m

```
function counts = rollDie(rolls)

FACES= 6;                    % #faces on die
counts= zeros(1,FACES);

% Count outcomes of rolling a FAIR die
for k = 1:rolls
    % Roll the die
    face= ceil(rand()*FACES);
    % Increment the appropriate bin
    counts(face)= counts(face) + 1;
end

% Show histogram of outcome
```



rollDie.m

% Simulate the rolling of 2 fair dice

totalOutcome= ???

A `ceil(rand()*12)`

B `ceil(rand()*11)+1`

C `floor(rand()*11)+2`

D *2 of the above*

E *None of the above*

Single rand() call ⇒ Uniform distribution

ceil(rand()*6) + ceil(rand()*6)

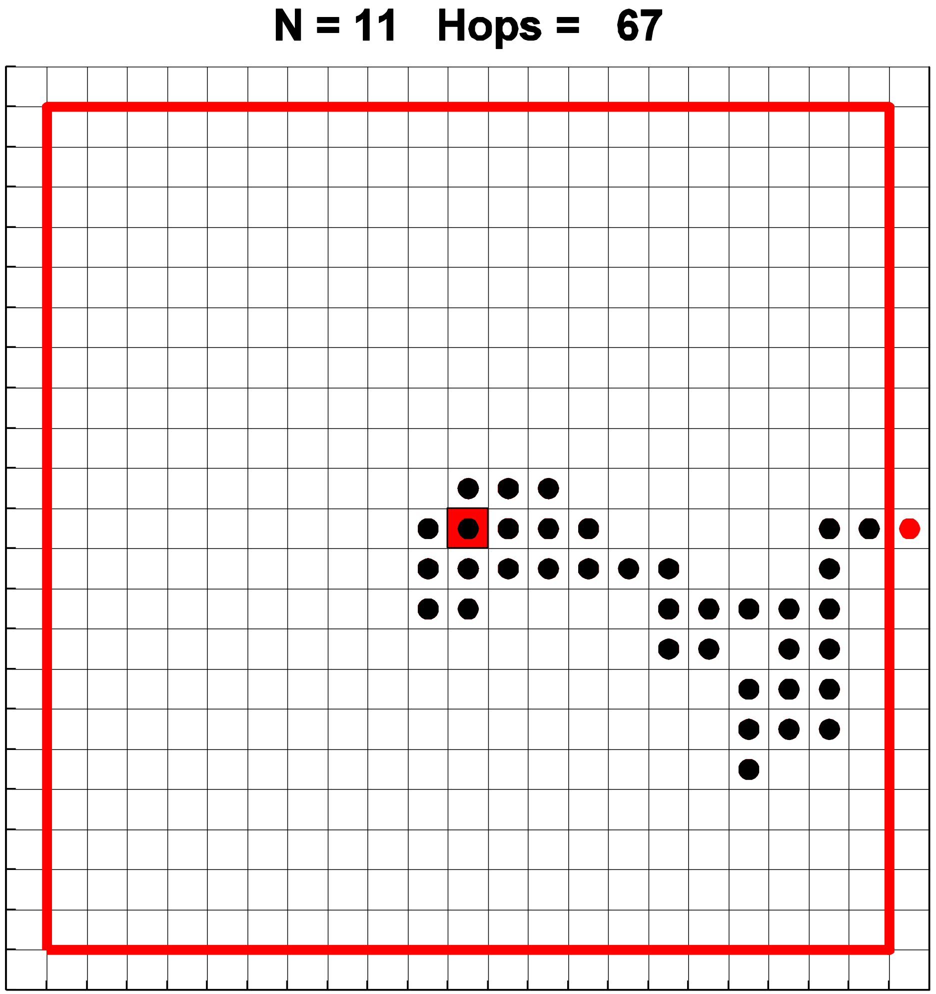
Two rand() calls summed ⇒ distribution not obvious ⇒ sim!

2-dimensional random walk

Start in the middle tile, (0,0).

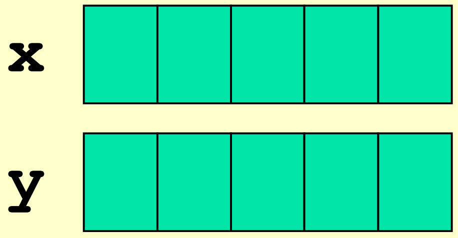
For each step, randomly choose between N,E,S,W and then walk one tile. Each tile is 1×1.

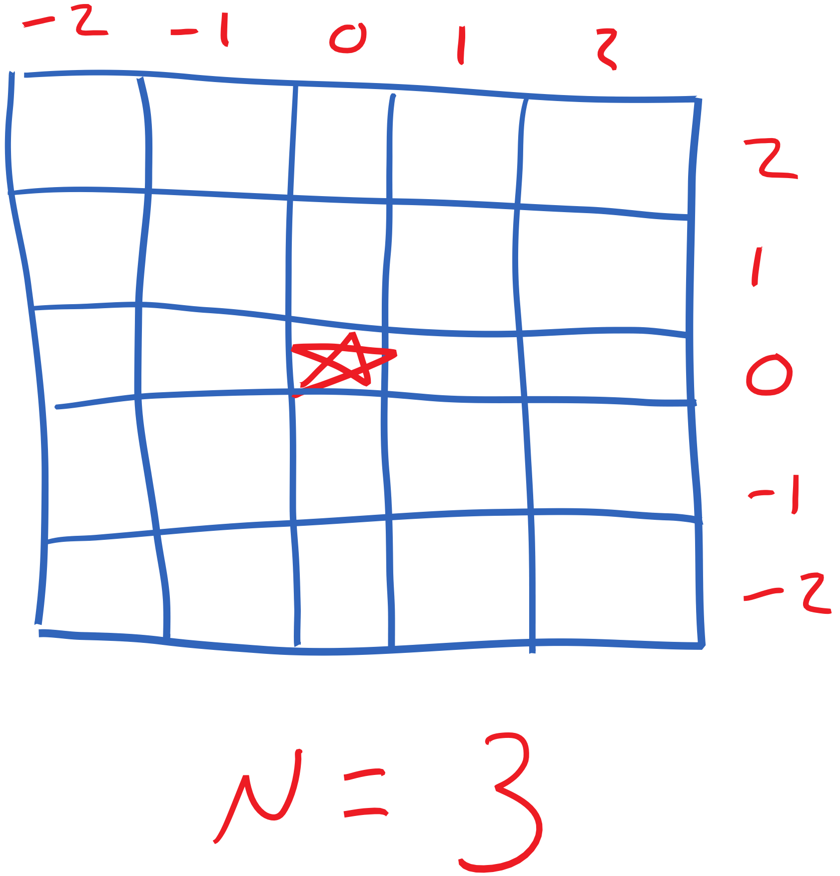
Walk until you reach the boundary.



```
function [x, y] = RandomWalk2D(N)
% 2D random walk in 2N-1 by 2N-1 grid.
% Walk randomly from (0,0) to an edge.
% Vectors x,y represent the path.
```

By the end of the function …

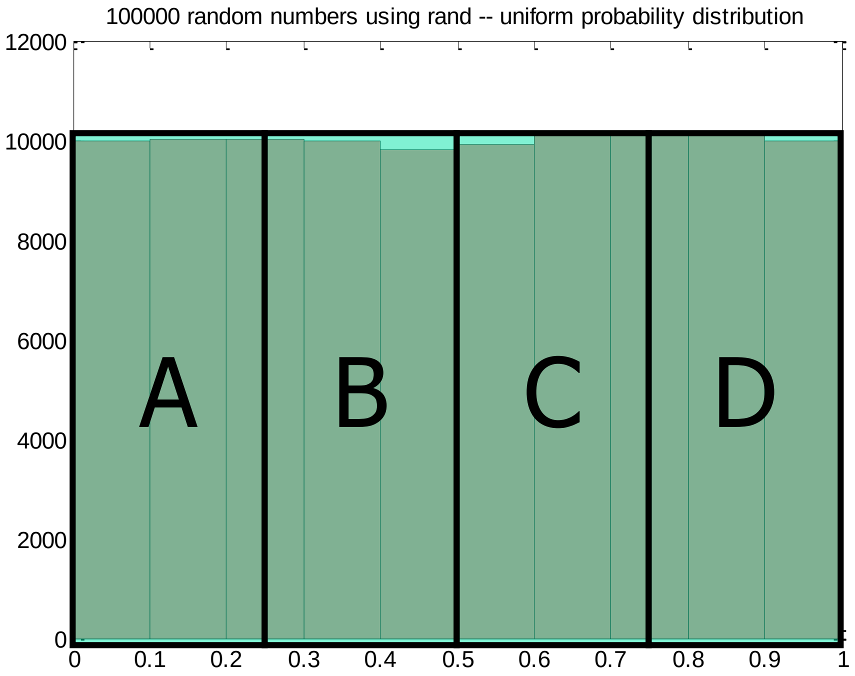




```
function [x, y] = RandomWalk2D(N)

k=0;  xc=0;  yc=0;

while current position not past an edge
    % Choose random dir, update xc,yc




    % Record new location in x, y

end
```

```
function [x, y] = RandomWalk2D(N)

k=0;  xc=0;  yc=0;

while  abs(xc)<N && abs(yc)<N
    % Choose random dir, update xc,yc




    % Record new location in x, y
    k=k+1;  x(k)=xc;  y(k)=yc;
end
```

Making a random choice



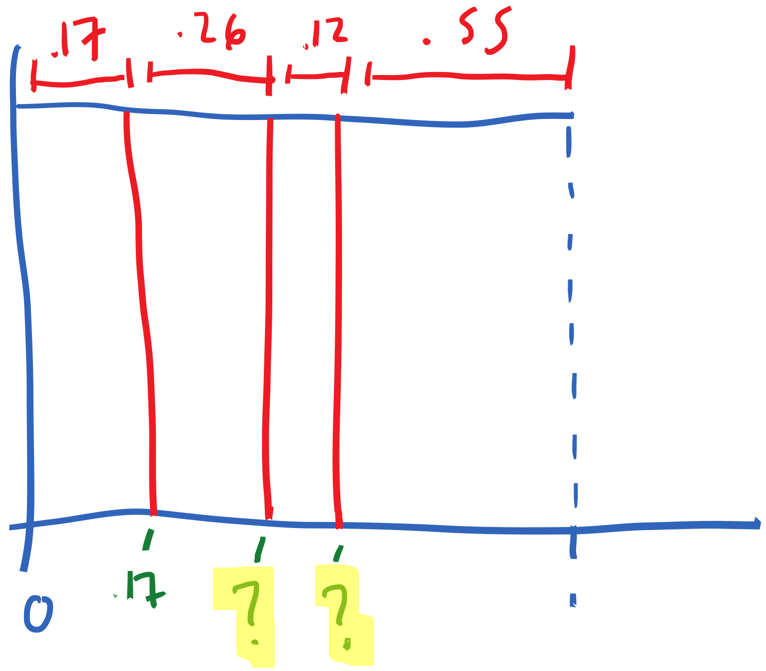
- Likelihood of `rand()` being between two numbers is proportional to their difference – *width*

```
% Standing at (xc,yc)
% Randomly select a step
    r= rand();
    if r < 0.25
        yc= yc + 1;   % north
    elseif r < 0.5
        xc= xc + 1;   % east
    elseif r < 0.75
        yc= yc - 1;   % south
    else
        xc= xc - 1;   % west
    end
```

See RandomWalk2D.m

Custom likelihoods

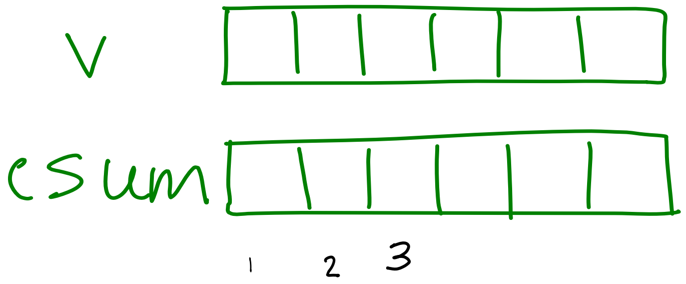
- Suppose you want outcomes with the following likelihoods:
  17%, 26%, 12%, 55%
  What should the thresholds be? Do these even add up to 100%?
- Trick: keep a running sum of the widths



Exercise

- Write a program fragment that calculates the cumulative sums of a given vector **`v`**.
- The cumulative sums should be stored in a vector of the same length as **`v`**.

1, 3, 5, 0	**`v`**
1, 4, 9, 9	cumulative sums of **`v`**



$$csum(k) = csum(k-1) + v(k)$$

$$csum(3) = v(1) + v(2) + v(3)$$

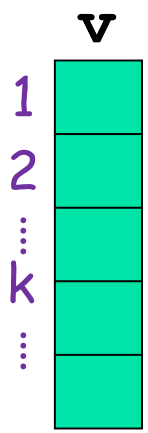
$$csum(4) = \underbrace{v(1) + v(2) + v(3)}_{csum(3)} + v(4)$$

```
csum(1) = v(1);
for k = 2 : length(v)
    csum(k) = csum(k-1) + v(k);
end
```

Demo: Random walk with biased probabilities

Loop patterns for processing a vector

```
% Given a vector v

for k=1:length(v)

   % Work with v(k)
   % E.g., disp(v(k))

end
```




```
% Given a vector v
k = 1;
while k<=length(v)
    % and possibly other
    % continuation conditions

   % Work with v(k)
   % E.g., disp(v(k))

   k = k+1;

end
```