- Previous Lecture (and Discussion):
  - Branching (`if`, `elseif`, `else`, `end`)
  - Relational operators (`<`, `>=`, `==`, `~=`, …, etc.)

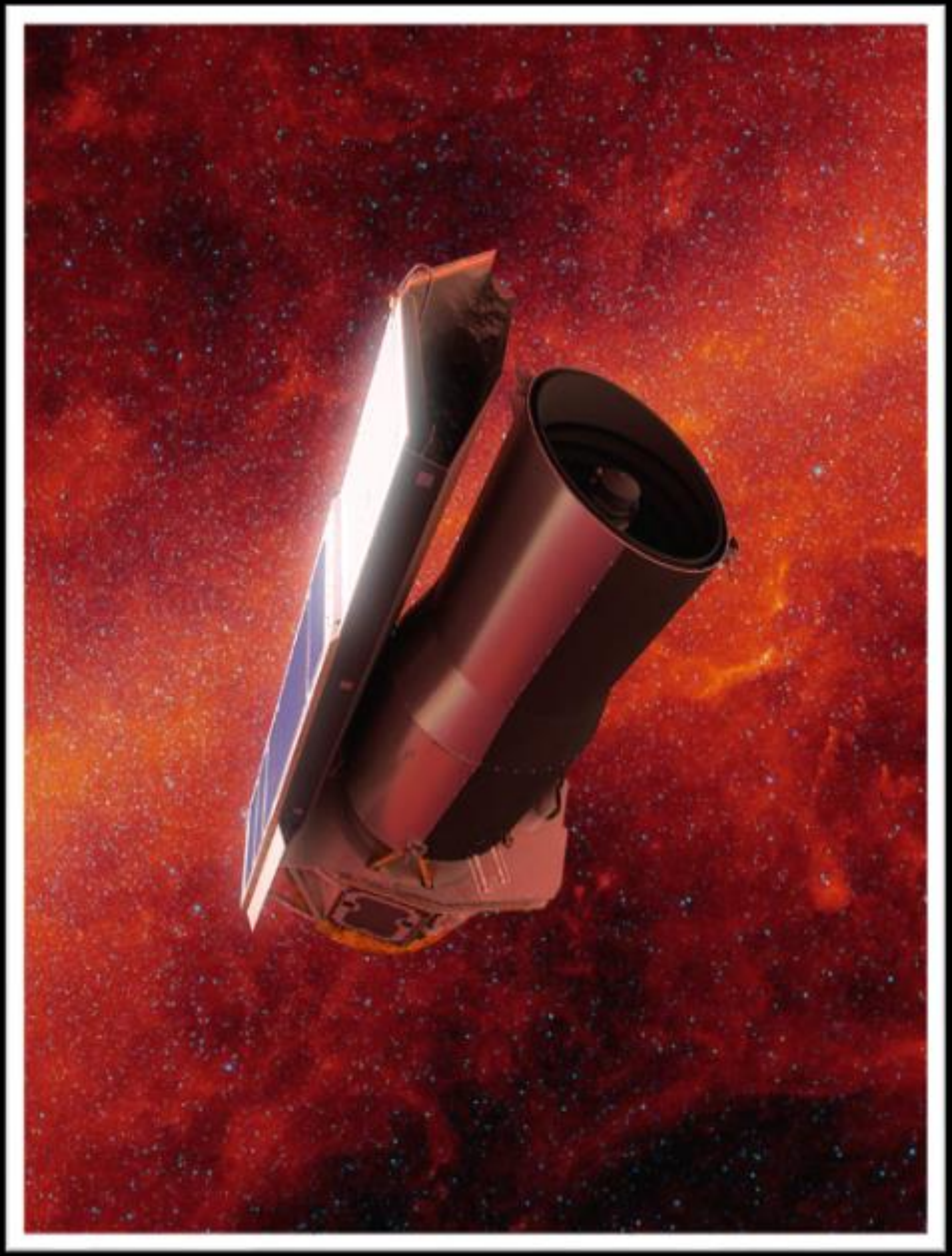
- Today's Lecture:
  - Logical operators (`&&`, `||`, `~`) and "short-circuiting"
  - More branching—*nesting*
  - Top-down design

- Announcements:
  - Project 1 (P1) due Tuesday 2/4 at 11pm
  - On project due dates (e.g., 2/4), course staff will not check off exercises during office/consulting hours so that we can devote our effort to helping students with the project due. Thanks for your understanding.
  - Register your clicker on Canvas – questions will count for credit next time
  - Lunch with instructors! Fri, 11:50, sign up on website

Farewell, Spitzer

Spitzer Space Telescope (SIRTF)

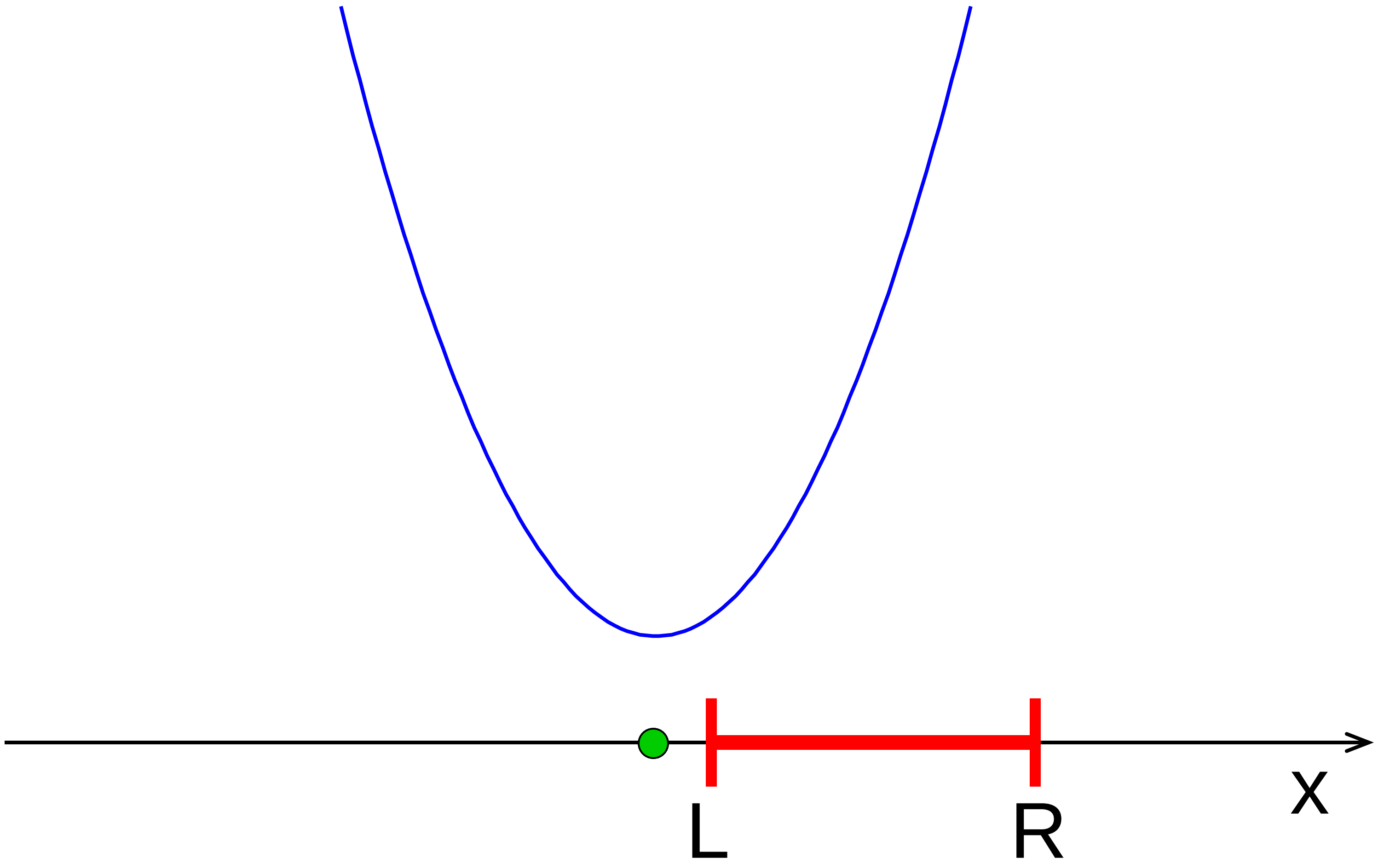
2003–2020



Minimum is at L, R, or x_c

$$q(x) = x^2 + bx + c$$

$$x_c = -b/2$$



Problem 3

Write a code fragment that prints "yes" if xc is in the interval and "no" if it is not.

So what is the requirement?

```
% Determine whether xc is in
% [L,R]
xc = -b/2;

if ________________

   disp('Yes')
else
   disp('No')
end
```

So what is the requirement?

```
% Determine whether xc is in
% [L,R]
xc = -b/2;

if L<=xc && xc<=R

   disp('Yes')
else
   disp('No')
end
```

The `if` construct

```
if  boolean expression1
    statements to execute if expression1 is true
elseif  boolean expression2
    statements to execute if expression1 is false
    but expression2 is true
:
else
    statements to execute if all previous conditions
    are false
end
```

Can have any number of elseif branches
but at most one else branch

The value of a **boolean expression** is either <u>true</u> or <u>false</u>.

```
(L<=xc)  &&  (xc<=R)
```

Above (compound) boolean expression is made up of two (simple) boolean expressions. Each has a value that is either *true* or *false*.

Connect boolean expressions by boolean operators and (&&), or (||)

Also available is the not operator (~)

Logical operators

&& logical and: Are both conditions true?

E.g., we ask "is $L \le x_c$ and $x_c \le R$?"

In our code: `L<=xc && xc<=R`

Logical operators

`&&` logical <u>and</u>: Are both conditions true?

E.g., we ask "is $L \leq x_c$ and $x_c \leq R$?"

In our code: `L<=xc && xc<=R`

`||` logical <u>or</u>: Is at least one condition true?

E.g., we can ask if x_c is outside of $[L,R]$,

i.e., "is $x_c < L$ or $R < x_c$?"

In code: `xc<L || R<xc`

Logical operators

&& logical <u>and</u>: Are both conditions true?

E.g., we ask "is $L \leq x_c$ and $x_c \leq R$?"

In our code: `L<=xc && xc<=R`

|| logical <u>or</u>: Is at least one condition true?

E.g., we can ask if x_c is outside of $[L,R]$,

i.e., "is $x_c < L$ or $R < x_c$?"

In code: `xc<L || R<xc`

~ logical <u>not</u>: Negation

E.g., we can ask if x_c is not outside $[L,R]$.

In code: `~(xc<L || R<xc)`

"Truth table"

X, Y represent boolean expressions.
E.g., d>3.14

X	Y	X && Y "and"	X || Y "or"	~Y "not"
F	F			
F	T			
T	F			
T	T			

"Truth table"

X, Y represent boolean expressions.
E.g., d>3.14

X	Y	X && Y "and"	X || Y "or"	~Y "not"
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"Truth table"

X, Y represent boolean expressions.
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X	Y	X && Y "and"	X || Y "or"	~Y "not"
F	F			
F	T	F	T	F
T	F			
T	T			

Checkpoint

- How many entries in the table are True?

A: 4

B: 5

C: 8

D: other

“Truth table”

X, Y represent boolean expressions.
E.g., d>3.14

X	Y	X && Y “and”	X || Y “or”	~Y “not”
F	F	F	F	T
F	T	F	T	F
T	F	F	T	T
T	T	T	T	F

“Truth table”

Matlab uses 0 to represent false,
1 to represent true

X	Y	X && Y “and”	X || Y “or”	~Y “not”
0	0	0	0	1
0	1	0	1	0
1	0	0	1	1
1	1	1	1	0

Logical operators "short-circuit"

$\underbrace{a > b}_{\text{true}}$ && c > d

Go on

$\underbrace{a > b}_{\text{false}}$ && c > d

Stop

Entire expression is false since
the first part is false

A **&&** expression short-circuits to false if the left operand evaluates to *false*.

A || expression short-circuits to ______________________ if

____

____

Logical operators "short-circuit"

$$\underbrace{a > b}_{\text{false}} \ || \ c > d$$

Go on

$$\underbrace{a > b}_{\text{true}} \ || \ c > d$$

Stop

Entire expression is true since the first part is true

A **&&** expression short-circuits to false if the left operand evaluates to *false*.

A **||** expression short-circuits to <u>true</u> if <u>the left operand evaluates to *true*</u>.

Why short-circuit?

- Right-hand Boolean expression may be *expensive* or potentially *invalid*
- Much clearer than alternatives

```
if (x < 0.5) || (tan(x) < 1)
   % ...
end


if (x ~= 0) && (y/x > 1e-8)
   % ...
end
```

Logical operators are required when connecting multiple Boolean expressions

Why is it wrong to use the expression

$$\cancel{\texttt{L <= xc <= R}}$$

for checking if x_c is in $[L,R]$?

L<=xc && xc<=R

Example: Suppose **`L`** is 5, **`R`** is 8, and **`xc`** is 10. We know that 10 is not in [5,8], but the expression

$$\underbrace{\underbrace{\texttt{L <= xc}}_{1}\texttt{ <= R}}_{1}$$ gives…

Stepping back…

Variables a, b, and c are integers between 1 and 100. Does this fragment correctly identify when lines of length a, b, and c could form a right triangle?

```
if a^2 + b^2 == c^2
    disp('Right tri')
else
    disp('No right tri')
end
```

A: correct

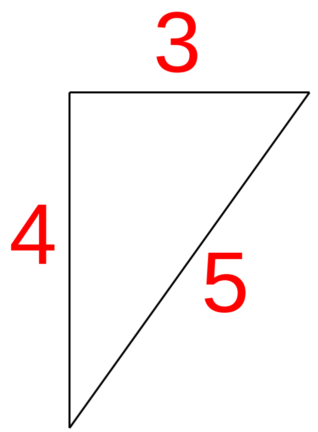
B: false positives

C: false negatives

D: both B & C

```
a = 5;
b = 3;
c = 4;
if (a^2 + b^2 == c^2)


   disp('Right tri')
else
   disp('No right tri')
end
```

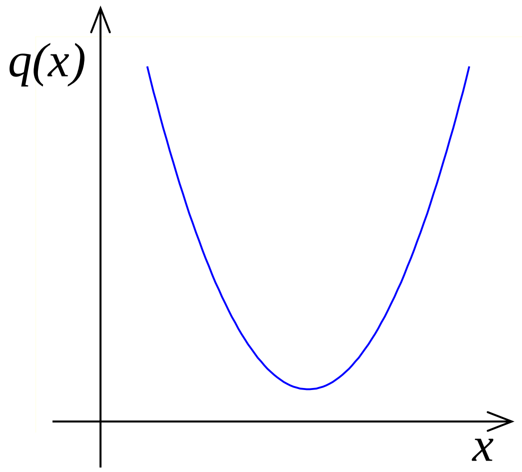


This fragment prints "No" even though we have a right triangle!

```
a = 5;
b = 3;
c = 4;
if (a^2 + b^2 == c^2) || ...
   (a^2 + c^2 == b^2) || ...
   (b^2 + c^2 == a^2)
   disp('Right tri')
else
   disp('No right tri')
end
```

Consider the quadratic function

$$q(x) = x^2 + bx + c$$

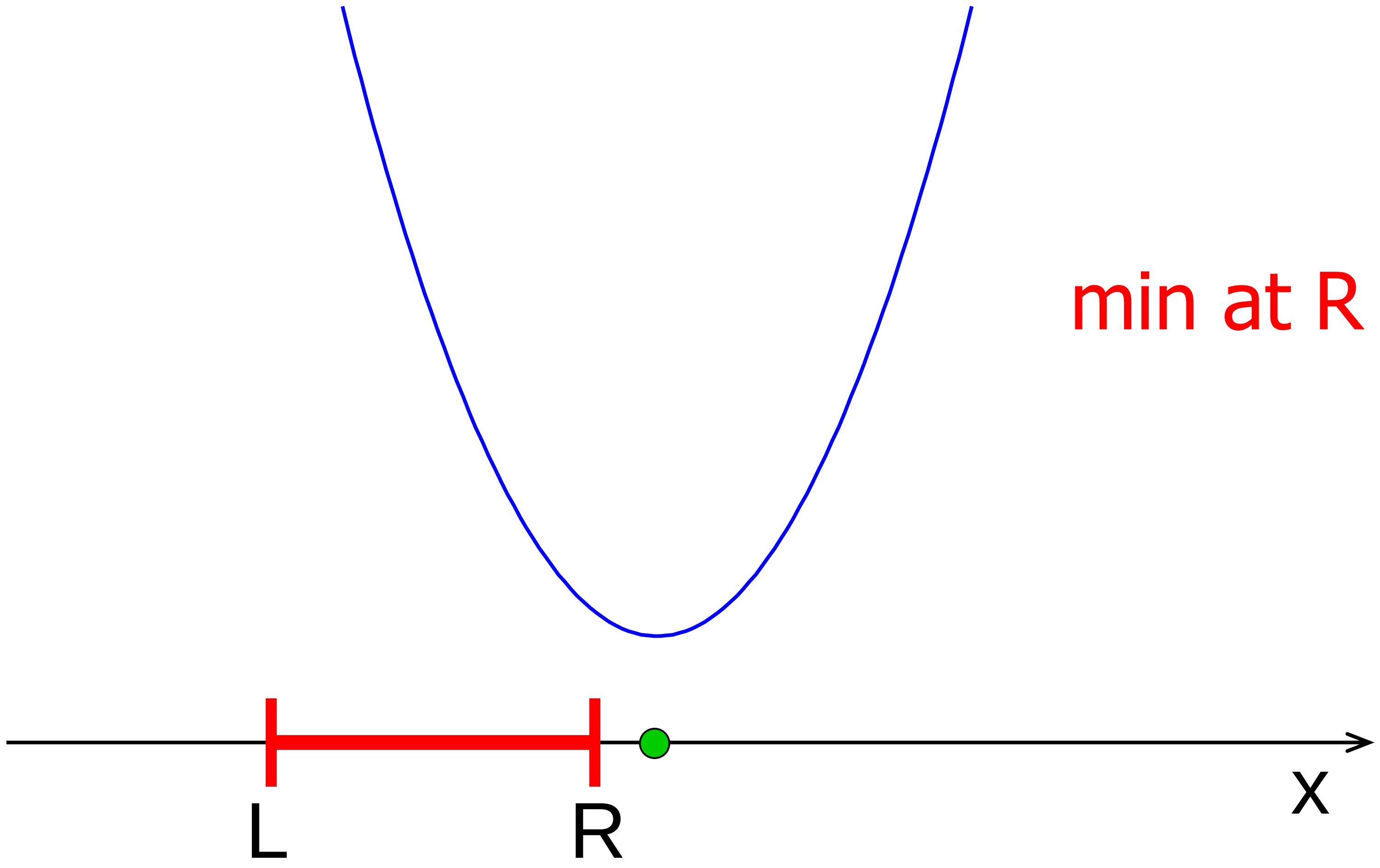


on the interval $[L\ ,\ R]$:

- Is the function strictly increasing in $[L\ ,\ R]$?
- Which is smaller, $q(L)$ or $q(R)$?
- What is the minimum value of $q(x)$ in $[L\ ,\ R]$?

$$q(x) = x^2 + bx + c$$

$$x_c = -b/2$$

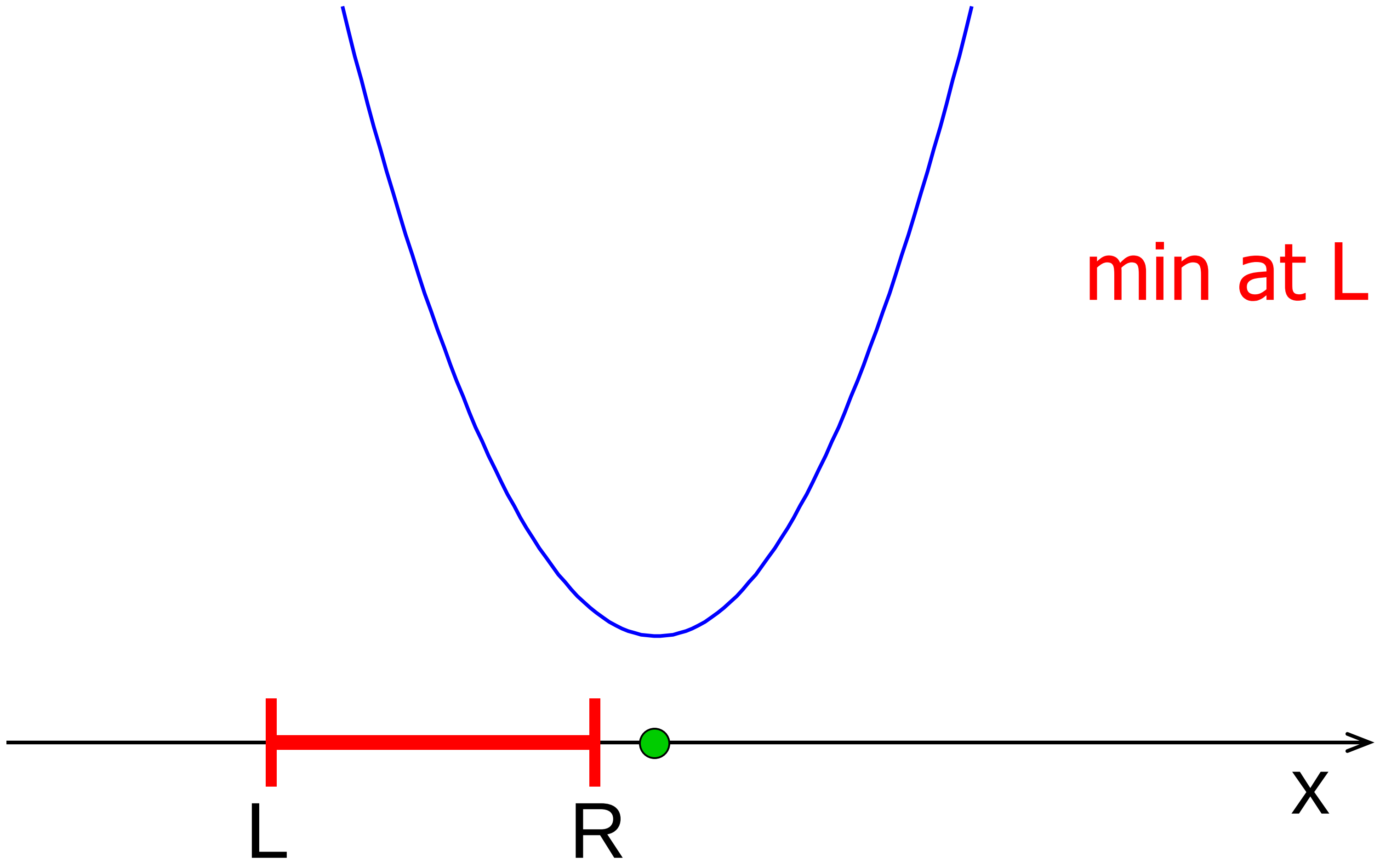


min at R

$$q(x) = x^2 + bx + c$$

$$x_c = -b/2$$

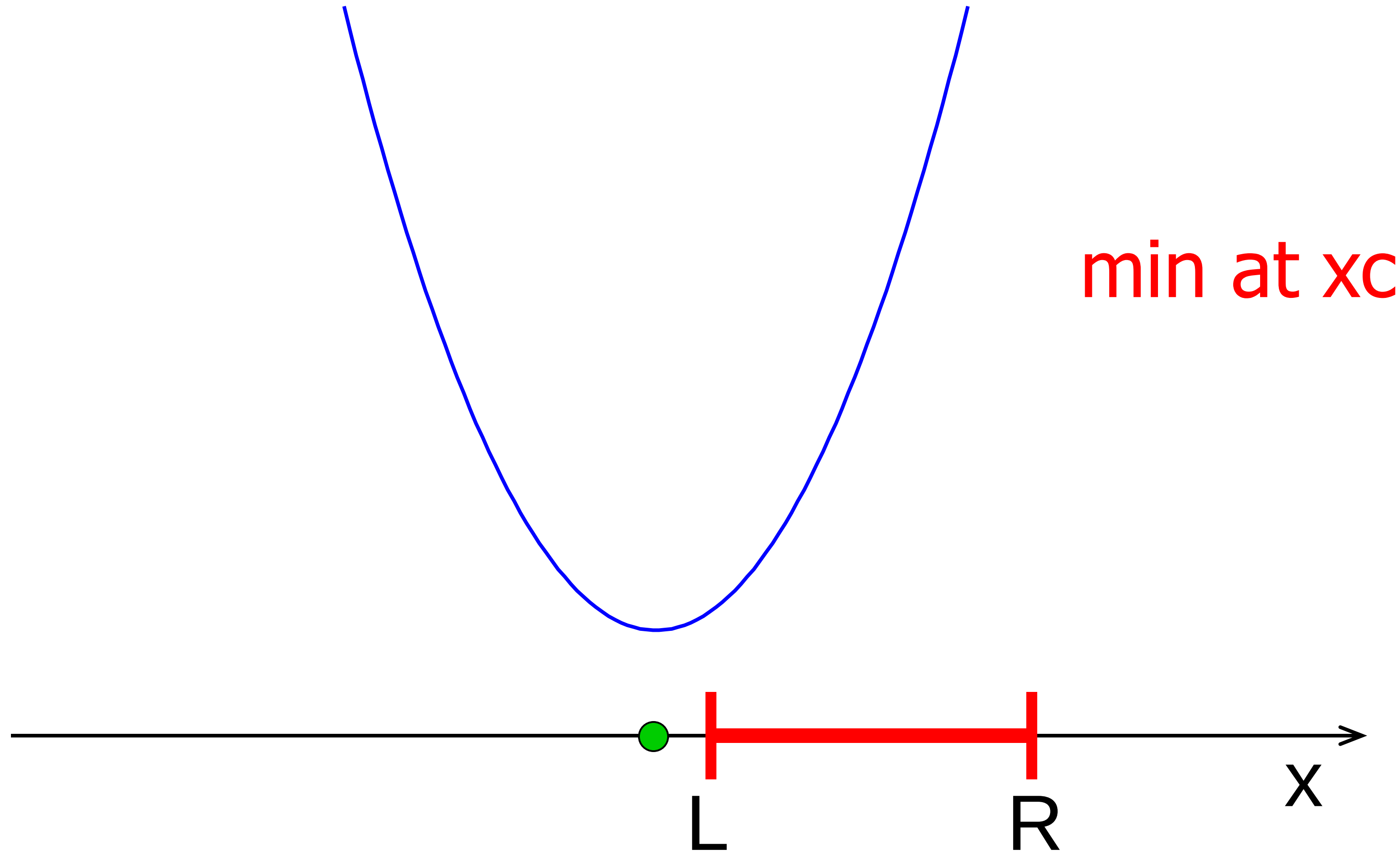
min at L



$$q(x) = x^2 + bx + c$$

$x_c = -b/2$

min at xc



Conclusion

If x_c is between L and R

Then min is at x_c

Otherwise

Min value is at one of the endpoints

Start with pseudocode

If *xc* is between *L* and *R*

    Min is at *xc*

Otherwise

    Min is at one of the endpoints

We have decomposed the problem into three pieces! Can choose to work with any piece next: the if-else construct/condition, min at xc, or min at an endpoint

Set up structure first: if-else, condition

```
if   L<=xc  &&  xc<=R
```

Then min is at *xc*

```
else
```

Min is at one of the endpoints

```
end
```

Now refine our solution-in-progress. I'll choose to work on the if-branch next

Refinement: filled in detail for task "min at xc"

```
if  L<=xc && xc<=R
   % min is at xc
   qMin= xc^2 + b*xc + c;

else
```

Min is at one of the endpoints

```
end
```

Continue with refining the solution… else-branch next

Refinement: detail for task “min at an endpoint”

```
if  L<=xc && xc<=R
    % min is at xc
   qMin= xc^2 + b*xc + c;
else
    % min is at one of the endpoints
    if  % xc left of bracket
       % min is at L
    else  % xc right of bracket
       % min is at R
    end
end
```

Continue with the refinement, i.e., replace comments with code

Refinement: detail for task “min at an endpoint”

```
if  L<=xc && xc<=R
    % min is at xc
    qMin= xc^2 + b*xc + c;
else
    % min is at one of the endpoints
    if  xc < L
        qMin= L^2 + b*L + c;
    else
        qMin= R^2 + b*R + c;
    end
end
```

Final solution (given b,c,L,R,xc)

```
if  L<=xc && xc<=R
   % min is at xc
   qMin= xc^2 + b*xc + c;
else
   % min is at one of the endpoints
   if  xc < L
      qMin= L^2 + b*L + c;
   else
      qMin= R^2 + b*R + c;
   end
end
```

An if-statement can appear within a branch—just like any other kind of statement!

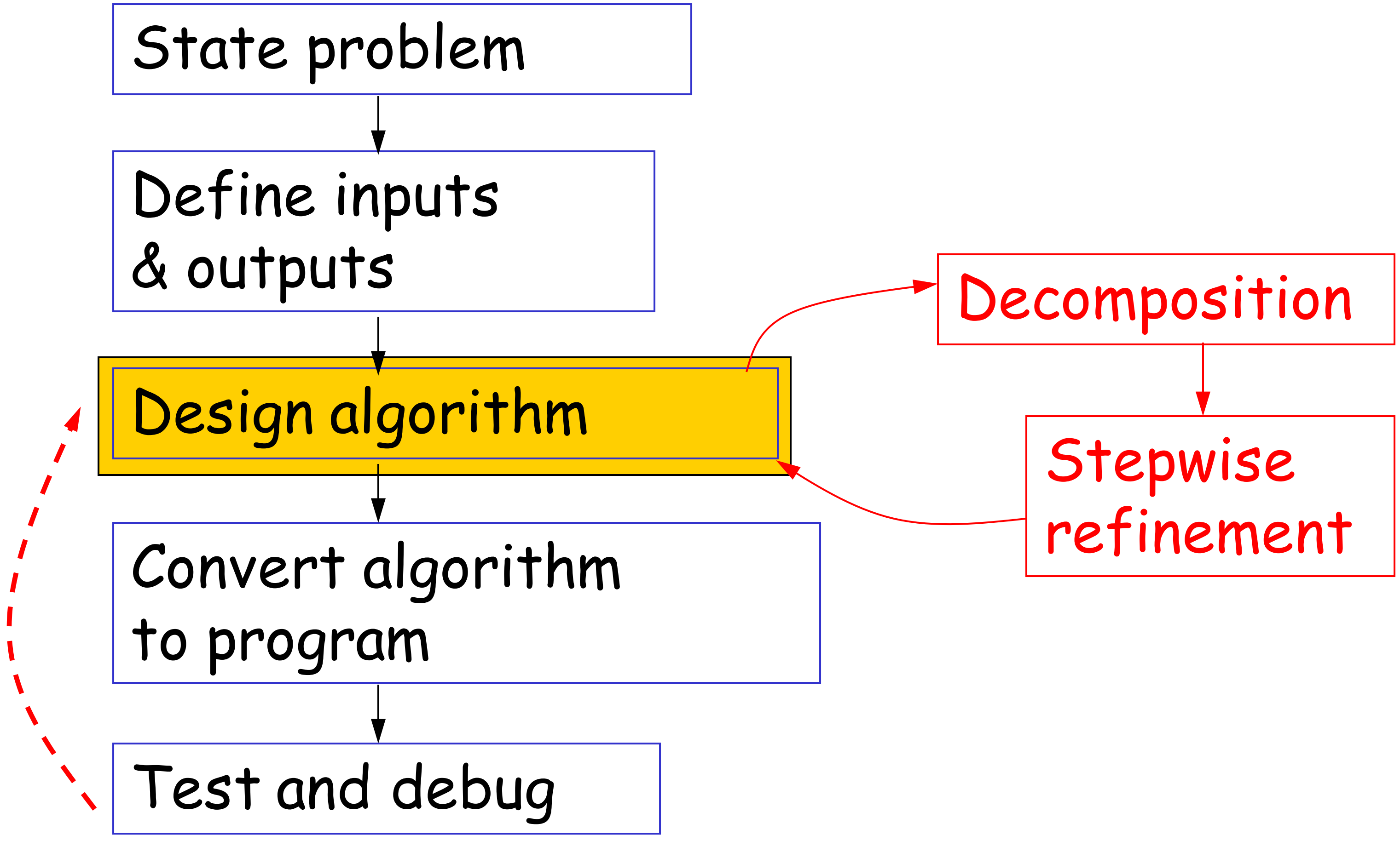
See `quadMin.m`
`quadMinGraph.m`

Notice that there are 3 alternatives→can use elseif!

```
if  L<=xc && xc<=R
  % min is at xc
  qMin= xc^2 + b*xc + c;
else
  % min at one endpt
  if  xc < L
    qMin= L^2 + b*L + c;
  else
    qMin= R^2 + b*R + c;
  end
end
```

```
if  L<=xc && xc<=R
  % min is at xc
  qMin= xc^2 + b*xc + c;
elseif xc < L
  qMin= L^2 + b*L + c;
else
  qMin= R^2 + b*R + c;
end
```

Top-Down Design



An algorithm is an idea. To use an algorithm you must choose a programming language and implement the algorithm.

If xc is between L and R

Then min value is at xc

Otherwise

Min value is at one of the endpoints

```
if  L<=xc && xc<=R
   % min is at xc

else
   % min is at one of the endpoints




end
```

```
if  L<=xc && xc<=R
    % min is at xc

else
    % min is at one of the endpoints




end
```

```
if  L<=xc && xc<=R
   % min is at xc
   qMin= xc^2 + b*xc + c;
else
   % min is at one of the endpoints




end
```

```
if  L<=xc && xc<=R
   % min is at xc
   qMin= xc^2 + b*xc + c;
else
   % min is at one of the endpoints




end
```

```
if  L<=xc && xc<=R
   % min is at xc
   qMin= xc^2 + b*xc + c;
else
   % min is at one of the endpoints
   if  xc < L

   else

   end
end
```

```
if  L<=xc && xc<=R
   % min is at xc
   qMin= xc^2 + b*xc + c;
else
   % min is at one of the endpoints
   if  xc < L
      qMin= L^2 + b*L + c;
   else
      qMin= R^2 + b*R + c;
   end
end
```

Testing and debugging

- An integral part of the design loop
- The programmer's job, not someone else's
  - Don't ask TAs "is this right?"; *Run your own tests*, then ask for guidance on failures
- Doesn't need to be formal, but does need to be thought through
- Testing tips
  - Know what your immediate goal is
  - Look for simple cases, compare with hand-calcs
  - Think about corner cases – try to break things while still respecting input constraints

Checkpoint: Should we use this code to decide your grade?

```
score= input('Enter score: ');
if score>55
     disp('D')
elseif score>65
     disp('C')
elseif score>80
     disp('B')
elseif score>93
     disp('A')
else
     disp('Try again')
end
```

A: yes

B: no – high scores might get low grade

C: no – low scores might get high grade

D: no – some scores might get no grade

Question

A stick of unit length is split into two pieces. The breakpoint is randomly selected. On average, how long is the shorter piece?

Physical experiment?

Thought experiment? → analysis

Computational experiment! → simulation

Need to repeat many trials!