

- Previous Lecture:

- Vectorized code
- 2-d array—matrix

- Today's Lecture:

- More examples on matrices
- Optional reading: contour plot (7.2, 7.3 in *Insight*)

- Announcement:

- Fall Break next Mon & Tues: no lec, dis, office/consulting hrs. Attendance at 10/12 (W) discussion is optional, but do the exercise even if you don't attend. Attend any of the 10/12 dis sections if you wish. Location is Hollister 464
- Optional review sessions: W7:30-9p, W5-6:30p. Location: Olin 255

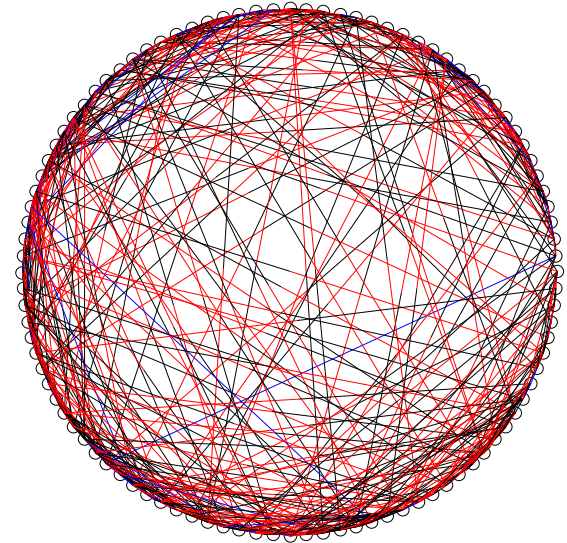
Storing and using data in tables

A company has 3 factories that make 5 products with these costs:

C

10	36	22	15	62
12	35	20	12	66
13	37	21	16	59

What is the best way to fill a given purchase order?



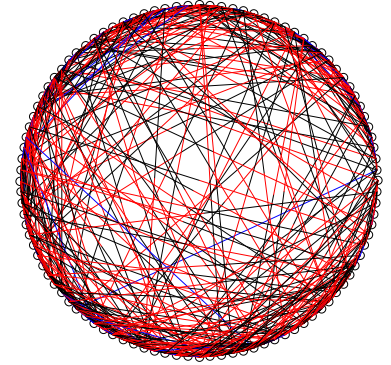
Connections
between webpages

0	0	1	0	1	0	0
1	0	0	1	1	1	0
0	1	0	1	1	1	1
1	0	1	1	0	1	0
0	0	1	1	0	1	1
0	0	1	0	1	0	1
0	1	1	0	1	1	0

Pattern for traversing a matrix M

```
[nr, nc] = size(M)
for r= 1:nr
    % At row r
    for c= 1:nc
        % At column c (in row r)
        %
        % Do something with M(r,c) ...
    end
end
end
```

Matrix example: Random Web



- N web pages can be represented by an N -by- N Link Array A .
- $A(i,j)$ is 1 if there is a link on webpage j to webpage i
- Generate a random link array and display the connectivity:
 - There is no link from a page to itself
 - If $i \neq j$ then $A(i,j) = 1$ with probability $\frac{1}{1+|i-j|}$
➡ There is more likely to be a link if i is close to j

An event happens with probability p , $0 < p < 1$

```
% Flip a fair coin
r= rand;
if r <= .5
    disp('heads')
else
    disp('tails')
end
```



```
% Unfair coin: shows heads
% twice as often as tails
r= rand;
if r <= 2/3
    disp('heads')
else
    disp('tails')
end
```



```
% Event X happens with probability p
r= rand;
if r <= p
    % Code for event X
end
```

```
function A = RandomLinks(n)
% A is n-by-n matrix of 1s and 0s
% representing n webpages

A = zeros(n,n);
for i= 1:n
    for j= 1:n
        r= rand;
        if i~=j && r <= 1/(1 + abs(i-j));
            A(i,j)= 1;
        end
    end
end
end
```

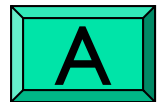
Random web
N = 20

```
0 1 1 1 0 0 0 0 0 1 0 0 1 0 0 0 0 0 0 0
1 0 0 0 1 0 0 0 1 1 1 0 0 0 0 0 0 1 0 0
0 1 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
0 0 1 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 1 0 0 0 0 0 0 1 1 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 1 0 0 0
0 1 1 1 1 1 0 0 0 1 0 1 1 0 0 0 0 0 0 0
0 0 0 0 0 0 1 0 0 0 0 1 0 0 0 0 0 0 1 1
0 1 0 0 0 0 0 0 0 1 0 0 1 0 0 0 1 0 0 0
0 0 0 0 0 0 0 1 1 0 1 0 0 0 0 0 0 0 0 1
0 0 0 0 0 0 1 0 0 0 0 0 1 1 0 0 0 0 0 0
0 0 0 0 0 0 1 0 0 1 0 0 0 0 0 0 0 0 0 1
0 0 0 1 0 0 0 0 1 1 0 1 0 1 1 0 0 0 0 0
0 0 0 0 0 0 1 0 0 0 0 0 0 0 1 1 0 0 0 0
0 0 0 0 0 1 0 1 0 0 0 0 0 1 0 0 1 0 0 0 1
0 0 0 0 0 0 1 0 0 0 1 0 0 0 0 0 1 0 1 0
0 1 0 0 0 0 0 0 1 0 0 0 0 1 0 1 0 1 1 0
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 1 0 0 1
0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 1 0
```

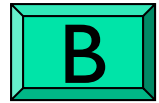
% Given an nr-by-nc matrix M.

% What is A?

```
for r= 1: nr
    for c= 1: nc
        A(c,r)= M(r,c);
    end
end
```



A is M with the columns in reverse order



A is M with the rows in reverse order

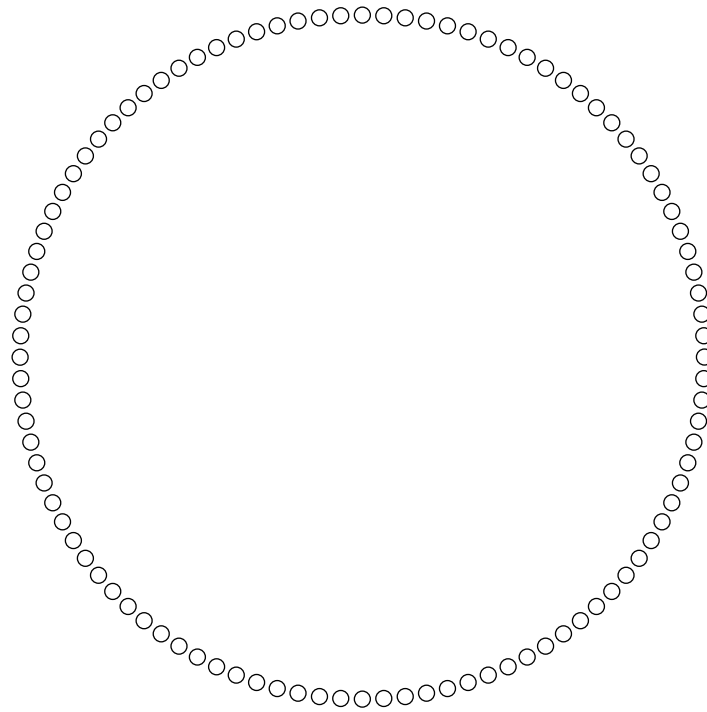


A is the transpose of M



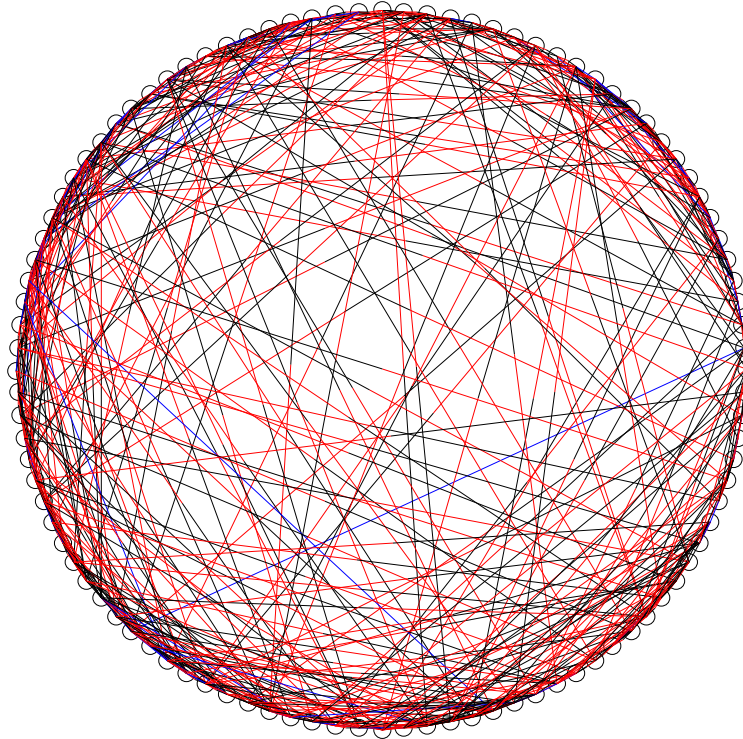
A and M are the same

Represent the web pages graphically...



100 Web pages arranged in a circle.
Next display the links....

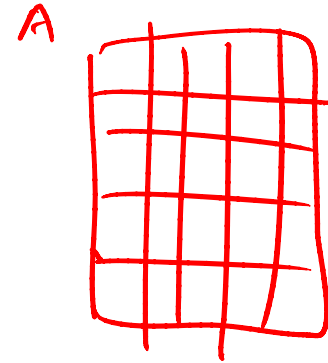
Represent the web pages graphically...



Bidirectional links are blue. Unidirectional link is black as it leaves page j, red when it arrives at page i.

```
for i = 1:n  
    for j = 1:n
```

```
    end  
end
```



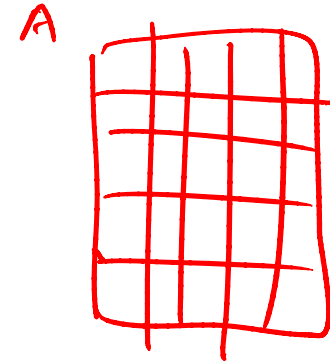
```
for i = 1:n
```

```
    for j = 1:n
```

```
        if A(i,j) == 1 && A(j,i) == 1
            % Blue
```

```
        elseif A(i,j) == 1
            % Black-Red
            j → mid    mid → i
```

```
        end
    end
end
```



Somewhat inefficient: each blue line gets drawn twice.
See **ShowRandomLinks.m**

A(1,5)

A(5,1)

A(12,10)

A(10,12)

*Transpose—like
switching row and
column indices*



0	1	1	1	0	0	0	0	1	0	0	1	0	0	0	0	0	0	0	0
1	0	0	0	1	0	0	0	1	1	1	0	0	0	0	0	0	0	1	0
0	1	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	1	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	1	0	0	0	0	0	0	1	1	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	1	0	0	0	0
0	1	1	1	1	1	0	0	0	1	0	1	1	0	0	0	0	0	0	0
0	0	0	0	0	0	1	0	0	0	0	1	0	0	0	0	0	0	1	1
0	1	0	0	0	0	0	0	0	1	0	0	1	0	0	0	1	0	0	0
0	0	0	0	0	0	0	1	1	0	1	0	0	0	0	0	0	0	0	1
0	0	0	0	0	0	1	0	0	0	0	0	1	1	0	0	0	0	0	0
0	0	0	0	0	0	1	0	0	1	0	0	0	0	0	0	0	0	0	1
0	0	0	1	0	0	0	0	1	1	0	1	0	1	1	0	0	0	0	0
0	0	0	0	0	0	1	0	0	0	0	0	0	0	1	1	0	0	0	0
0	0	0	0	0	1	0	1	0	0	0	0	1	0	0	1	0	0	0	1
0	0	0	0	0	0	1	0	0	0	1	0	0	0	0	0	0	1	0	1
0	1	0	0	0	0	0	1	0	0	0	0	1	0	1	0	1	1	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	0	0
0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	1

A Cost/Inventory Problem

- A company has 3 factories that make 5 different products
- The cost of making a product varies from factory to factory
- The inventory/capacity varies from factory to factory

Problems

A customer submits a purchase order that is to be filled by a single factory.

1. How much would it cost a factory to fill the order?
2. Does a factory have enough inventory/capacity to fill the order?
3. Among the factories that can fill the order, who can do it most cheaply?

Available data

$C(i, j)$ is what it costs factory i to make product j

$Inv(i, j)$ is the inventory in factory i of product j

$PO(j)$ is the number of product j 's that the client wants

C

10	36	22	15	62
12	35	20	12	66
13	37	21	16	59

Inv

38	5	99	34	42
82	19	83	12	42
51	29	21	56	87

PO

1	0	12	29	5
---	---	----	----	---

C

10	36	22	15	62
12	35	20	12	66
13	37	21	16	59

P0

1	0	12	29	5
---	---	----	----	---

Cost for
factory I:

$$1*10 + 0*36 + 12*22 + 29*15 + 5*62$$

C

10	36	22	15	62
12	35	20	12	66
13	37	21	16	59

P0

1	0	12	29	5
---	---	----	----	---

Cost for
factory 1:

```
s = 0; %Sum of cost
for j=1:5
    s = s + C(1,j)*P0(j)
end
```

C

10	36	22	15	62
12	35	20	12	66
13	37	21	16	59

P0

1	0	12	29	5
---	---	----	----	---

Cost for
factory 2:

```
s = 0; %Sum of cost
for j=1:5
    s = s + C(2,j)*P0(j)
end
```

C

10	36	22	15	62
12	35	20	12	66
13	37	21	16	59

P0

1	0	12	29	5
---	---	----	----	---

Cost for
factory **i**:

```
s = 0; %Sum of cost
for j=1:5
    s = s + C(i,j)*P0(j)
end
```

Encapsulate...

```
function TheBill = iCost(i,C,P0)
% The cost when factory i fills the
% purchase order

nProd = length(P0);
TheBill = 0;
for j=1:nProd
    TheBill = TheBill + C(i,j)*P0(j);
end
```

Finding the Cheapest

```
iBest = 0; minBill = inf;  
for i=1:nFact  
    iBill = iCost(i,C,P0);  
    if iBill < minBill  
        % Found an Improvement  
        iBest = i; minBill = iBill;  
    end  
end
```

inf – a special value that can be regarded as positive infinity

x = 10/0 assigns **inf** to **x**

y = 1+x assigns **inf** to **y**

z = 1/x assigns **0** to **z**

w < inf is always true if **w** is numeric

Inventory/Capacity Considerations

What if a factory lacks the inventory/capacity to fill the purchase order?

Such a factory should be excluded from the find-the-cheapest computation.

Who Can Fill the Order?

Inv

38	5	99	34	42
82	19	83	12	42
51	29	21	56	87

Yes

No

Yes

P0

1	0	12	29	5
---	---	----	----	---

Wanted: A True/False Function



DO is "true" if **factory i** can fill the order.

DO is "false" if **factory i** cannot fill the order.

Example: Check inventory of factory 2

Inv	38	5	99	34	42
	82	19	83	12	42
	51	29	21	56	87
P0	1	0	12	29	5

Method 1: check the
inventory for every product

Initialization

Inv

38	5	99	34	42
82	19	83	12	42
51	29	21	56	87

D0

1

P0

1	0	12	29	5
---	---	----	----	---

Still True...

Inv	38	5	99	34	42
	82	19	83	12	42
	51	29	21	56	87
					D0
					1
P0	1	0	12	29	5

D0 = D0 && (Inv(2,1) >= P0(1))

Still True...

Inv	38	5	99	34	42
	82	19	83	12	42
	51	29	21	56	87
					D0
					1
P0	1	0	12	29	5

D0 = D0 && (Inv(2,2) >= P0(2))

Still True...

Inv	38	5	99	34	42
	82	19	83	12	42
	51	29	21	56	87
P0	1	0	12	29	5

D0 1

$D0 = D0 \ \&\& \ (\text{Inv}(2, 3) \geq P0(3))$

No Longer True...

Inv

38	5	99	34	42
82	19	83	12	42
51	29	21	56	87

D0 0

P0

1	0	12	29	5
---	---	----	----	---

D0 = D0 && (Inv(2,4) >= P0(4))

Stay False...

Inv	38	5	99	34	42
	82	19	83	12	42
	51	29	21	56	87
					D0
					0
P0	1	0	12	29	5

D0 = D0 && (Inv(2,5) >= P0(5))

Encapsulate...

```
function D0 = iCanDo(i, Inv, P0)
% D0 is true if factory i can fill
% the purchase order. Otherwise, false

nProd = length(P0);
D0 = 1;
for j = 1:nProd
    D0 = D0 && ( Inv(i, j) >= P0(j) );
end
```

Encapsulate...

```
function DO = iCanDo(i, Inv, P0)
% DO is true if factory i can fill
% the purchase order. Otherwise, false
nProd = length(P0);
j = 1;
while j <= nProd && Inv(i, j) >= P0(j)
    j = j+1;
end
DO = _____;
```

Encapsulate...

```
function DO = iCanDo(i, Inv, P0)
% DO is true if factory i can fill
% the purchase order. Otherwise, false
nProd = length(P0);
j = 1;
while j <= nProd && Inv(i, j) >= P0(j)
    j = j+1;
end
DO = (j > nProd);
```