

- Previous Lecture:
  - Vectors
  - Color computation
  - Linear interpolation
  - Introduction to vectorized computation
- Today's Lecture:
  - "Clean up" details on vectors
  - Vectorized operations
  - 2-d array—matrix
- Announcements:
  - Discussion this week in classrooms as listed in Student Center
  - Prelim I on Oct 13 (Thursday) at 7:30pm

Initialize arrays if dimensions are known  
... instead of "building" the array one component at a time

```
% Initialize y
x=linspace(a,b,n);
y=zeros(1,n);
for k=1:n
    y(k)=myF(x(k));
end
```

```
% Build y on the fly
x=linspace(a,b,n);
for k=1:n
    y(k)=myF(x(k));
end
```

↑  
Much faster for large n!

Lecture 13

9

### Drawing a polygon (multiple line segments)

```
% Draw a rectangle with the lower-left
% corner at (a,b), width w, height h.
x= [          ]; % x data
y= [          ]; % y data
plot(x, y)
```

Fill in the missing vector values!

Lecture 12

10

### Coloring a polygon (fill)

```
% Draw a rectangle with the lower-left
% corner at (a,b), width w, height h,
% and fill it with a color named by c.
x= [a a+w a+w a a]; % x data
y= [b b b+h b+h b]; % y data
fill(x, y, c)
```

Built-in function `fill` does the "wrap-around" automatically.

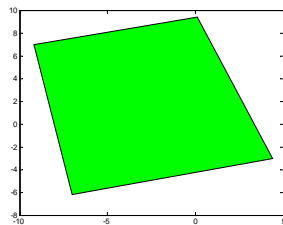
Lecture 12

13

### Coloring a polygon (fill)

```
x= [0.1 -9.2 -7 4.4];
y= [9.4 7 -6.2 -3];
fill(x,y,'g')
```

Can be a vector  
(RGB values)



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14

### Vectorized code

—a Matlab-specific feature

See Sec 4.1 for list of vectorized arithmetic operations

- Code that performs element-by-element arithmetic/relational/logical operations on array operands in one step
- Scalar operation:  $x + y$   
where  $x, y$  are scalar variables
- Vectorized code:  $x + y$   
where  $x$  and/or  $y$  are vectors. If  $x$  and  $y$  are both vectors, they must be of the same shape and length

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15

### Vectorized addition

$$\begin{array}{rcl}
 \mathbf{x} & \begin{bmatrix} 2 & 1 & .5 & 8 \end{bmatrix} \\
 + & \mathbf{y} & \begin{bmatrix} 1 & 2 & 0 & 1 \end{bmatrix} \\
 \hline
 = & \mathbf{z} & \begin{bmatrix} 3 & 3 & .5 & 9 \end{bmatrix}
 \end{array}$$

Matlab code: `z = x + y`

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16

### Vectorized multiplication

$$\begin{array}{rcl}
 & \mathbf{a} & \begin{bmatrix} 2 & 1 & .5 & 8 \end{bmatrix} \\
 \times & \mathbf{b} & \begin{bmatrix} 1 & 2 & 0 & 1 \end{bmatrix} \\
 \hline
 = & \mathbf{c} & \begin{bmatrix} 2 & 2 & 0 & 8 \end{bmatrix}
 \end{array}$$

Matlab code: `c = a .* b`

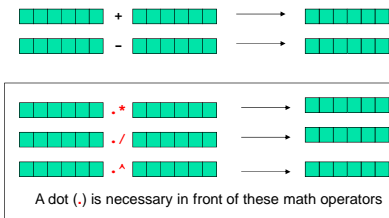


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18

### Vectorized element-by-element arithmetic operations on arrays

See full list of ops in §4.1



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19

### Shift

$$\begin{array}{rcl}
 \mathbf{x} & 3 \\
 + & \mathbf{y} & \begin{bmatrix} 2 & 1 & .5 & 8 \end{bmatrix} \\
 \hline
 = & \mathbf{z} & \begin{bmatrix} 5 & 4 & 3.5 & 11 \end{bmatrix}
 \end{array}$$

Matlab code: `z = x + y`

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20

### Reciprocate

$$\begin{array}{rcl}
 \mathbf{x} & 1 \\
 / & \mathbf{y} & \begin{bmatrix} 2 & 1 & .5 & 8 \end{bmatrix} \\
 \hline
 = & \mathbf{z} & \begin{bmatrix} .5 & 1 & 2 & .125 \end{bmatrix}
 \end{array}$$

Matlab code: `z = x ./ y`



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21

### Vectorized element-by-element arithmetic operations between an array and a scalar

See full list of ops in §4.1



A dot (.) is necessary in front of these math operators

Not necessary but OK to use dot: `array.*scalar`, `array.*array`, `array./scalar`

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22

Can we plot this?

See plotComparison.m

$$f(x) = \frac{\sin(5x)\exp(-x/2)}{1+x^2} \quad \text{for } -2 \leq x \leq 3$$

Yes!

```
x = linspace(-2,3,200);
y = sin(5*x).*exp(-x/2)./(1 + x.^2);
plot(x,y)
```

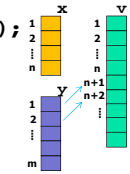
↑ ↑ ↑  
Element-by-element arithmetic  
operations on arrays

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23

Concatenating 2 vectors—copy 2 vectors into a new one

```
% given column vectors x and y
v= zeros(length(x)+length(y),1);
for k=1:length(x)
    v(k)= x(k);
end
for k=1:length(y)
    v(length(x)+k)= y(k);
end
```



This is **non-vectorized code**—operations are performed on one component (scalar) at a time

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28

Concatenating 2 vectors—copy 2 vectors into a new one

```
% given column vectors x and y
v= zeros(length(x)+length(y),1);
for k=1:length(x)
    v(k)= x(k);
end
for k=1:length(y)
    v(length(x)+k)= y(k);
end
```

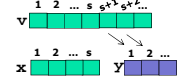
Below is **vectorized code**—ops are performed on multiple components (a vector) at the same time:  
 $v = [x; y];$

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29

Split a vector in 2—copy values into 2 vectors

```
% given row vector v
s= ceil(rand*length(v)); % split after v(s)
x= zeros(1,s);
y= zeros(1,length(v)-s);
for k=1:s
    x(k)= v(k);
end
for k=1:length(y)
    y(k)= v(s+k);
end
```



This is **non-vectorized code**—operations are performed on one component (scalar) at a time

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33

Split a vector in 2—copy values into 2 vectors

```
% given row vector v
s= ceil(rand*length(v)); % split after v(s)
x= zeros(1,s);
y= zeros(1,length(v)-s);
for k=1:s
    x(k)= v(k);
end
for k=1:length(y)
    y(k)= v(s+k);
end
```

Below is **vectorized code**: multiple components (subvectors) are affected/accessed at the same time:

$x = v(1:s);$   
 $y = v(s+1:length(v));$

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34

End of  
Prelim 1 material

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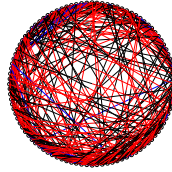
35

Storing and using data in tables

A company has 3 factories that make 5 products with these costs:

C	10	36	22	15	62
	12	35	20	12	66
	13	37	21	16	59

What is the best way to fill a given purchase order?



Connections between webpages

0	0	1	0	1	0	0
1	0	0	1	1	1	0
0	1	0	1	1	1	1
1	0	1	1	0	1	0
0	0	1	1	0	1	1
0	0	1	0	1	0	1
0	1	1	0	1	1	0

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36

2-d array: **matrix**

- An array is a **named** collection of **like** data organized into rows and columns
- A 2-d array is a table, called a **matrix**
- Two **indices** identify the position of a value in a matrix, e.g.,

`mat(r,c)`

refers to component in row *r*, column *c* of matrix *mat*

- Array index starts at **1**
- Rectangular**: all rows have the same #of columns

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37

## Creating a matrix

- Built-in functions: **ones**, **zeros**, **rand**
  - E.g., `zeros(2,3)` gives a 2-by-3 matrix of 0s
  - E.g., `zeros(2)` gives a 2-by-2 matrix of 0s
- “Build” a matrix using square brackets, `[ ]`, but the dimension must match up:
  - `[x y]` puts *y* to the right of *x* → 

4	0	3
5	1	9
  - `[x; y]` puts *y* below *x* → 

4	0	3
1	1	1
  - `[4 0 3; 5 1 9]` creates the matrix
  - `[4 0 3; ones(1,3)]` gives
  - `[4 0 3; ones(3,1)]` doesn't work

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39

Working with a matrix:  
**size** and individual components

2	-1	.5	0	-3
3	8	6	7	7
5	-3	8.5	9	10
52	81	.5	7	2

Given a matrix *M*

```
[nr, nc] = size(M) % nr is #of rows,
                  % nc is #of columns
nr = size(M, 1) % # of rows
nc = size(M, 2) % # of columns

M(2,4) = 1;
disp(M(3,1))
M(1,nc) = 4;
```

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40

## Example: minimum value in a matrix

```
function val = minInMatrix(M)
% val is the smallest value in matrix M
```



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45

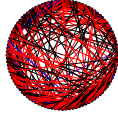
Pattern for traversing a matrix *M*

```
[nr, nc] = size(M)
for r = 1:nr
    % At row r
    for c = 1:nc
        % At column c (in row r)
        %
        % Do something with M(r,c) ...
    end
end
```

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47

## Matrix example: Random Web



- N web pages can be represented by an N-by-N Link Array  $A$ .
  - $A(i,j)$  is 1 if there is a link on webpage  $j$  to webpage  $i$
  - Generate a random link array and display the connectivity:
    - There is no link from a page to itself
    - If  $i \neq j$  then  $A(i,j) = 1$  with probability  $\frac{1}{1+|i-j|}$
- ⇒ There is more likely to be a link if  $i$  is close to  $j$

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48

```
function A = RandomLinks(n)
% A is n-by-n matrix of 1s and 0s
% representing n webpages

A = zeros(n,n);
for i=1:n
    for j=1:n
        r = rand(1);
        if i~=j && r<= 1/(1 + abs(i-j));
            A(i,j) = 1;
        end
    end
end
```

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49