

■ Previous Lecture:

- Vectors
- Color computation
- Linear interpolation
- Introduction to vectorized computation

■ Today's Lecture:

- “Clean up” details on vectors
- Vectorized operations
- 2-d array—matrix

■ Announcements:

- Discussion this week in classrooms as listed in Student Center
- Optional review sessions on Oct 12 (Wednesday): 5-6:30pm and 7:30-9pm. Both in Olin Hall room 255.
- Prelim I on Oct 13 (Thursday) at 7:30pm

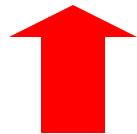
Initialize arrays if dimensions are known

... instead of “building” the array one component at a time

```
% Initialize y
x=linspace(a,b,n);
y=zeros(1,n);
for k=1:n
    y(k)=myF(x(k));
end
```

```
% Build y on the fly
x=linspace(a,b,n);

for k=1:n
    y(k)=myF(x(k));
end
```



Much faster for large n!

Drawing a polygon (multiple line segments)

```
% Draw a rectangle with the lower-left  
% corner at (a,b), width w, height h.  
x= [          ]; % x data  
y= [          ]; % y data  
plot(x, y)
```

Fill in the missing vector values!

Drawing a polygon (multiple line segments)

```
% Draw a rectangle with the lower-left  
% corner at (a,b), width w, height h.  
x= [a    a+w    a+w    a    a ]; % x data  
y= [b    b    b+h    b+h    b ]; % y data  
plot(x, y)
```

Coloring a polygon (fill)

```
% Draw a rectangle with the lower-left  
% corner at (a,b), width w, height h,  
% and fill it with a color named by c.  
x= [a  a+w  a+w  a  a];  % x data  
y= [b  b    b+h  b+h  b];  % y data  
fill(x, y, c)
```



A built-in function

Coloring a polygon (fill)

```
% Draw a rectangle with the lower-left  
% corner at (a,b), width w, height h,  
% and fill it with a color named by c.  
x= [a  a+w  a+w  a  a];  % x data  
y= [b  b    b+h  b+h  b];  % y data  
fill(x, y, c)
```

Built-in function **fill** does the
"wrap-around" automatically.

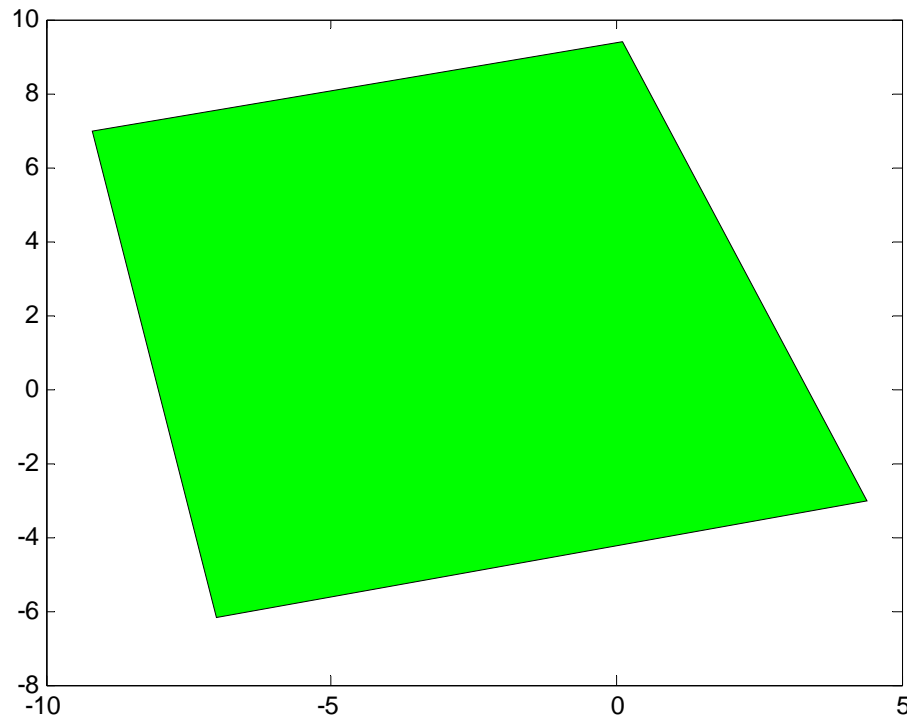
Coloring a polygon (fill)

```
x= [0.1 -9.2 -7 4.4];
```

```
y= [9.4 7 -6.2 -3];
```

```
fill(x,y,'g')
```

Can be a vector
(RGB values)



Vectorized code

—a Matlab-specific feature

See Sec 4.1 for list of vectorized arithmetic operations

- Code that performs element-by-element arithmetic/relational/logical operations on array operands in one step
- Scalar operation: $x + y$
where x, y are scalar variables
- **Vectorized code:** $x + y$
where x and/or y are vectors. If x and y are both vectors, they must be of the **same shape and length**

Vectorized addition

$$\begin{array}{rcl} & \mathbf{x} & \begin{array}{|c|c|c|c|} \hline 2 & 1 & .5 & 8 \\ \hline \end{array} \\ + & \mathbf{y} & \begin{array}{|c|c|c|c|} \hline 1 & 2 & 0 & 1 \\ \hline \end{array} \\ \hline = & \mathbf{z} & \begin{array}{|c|c|c|c|} \hline 3 & 3 & .5 & 9 \\ \hline \end{array} \end{array}$$

Matlab code: $\mathbf{z} = \mathbf{x} + \mathbf{y}$

Vectorized subtraction

$$\begin{array}{r} \mathbf{x} \quad \begin{array}{|c|c|c|c|} \hline 2 & 1 & .5 & 8 \\ \hline \end{array} \\ - \quad \mathbf{y} \quad \begin{array}{|c|c|c|c|} \hline 1 & 2 & 0 & 1 \\ \hline \end{array} \\ \hline = \quad \mathbf{z} \quad \begin{array}{|c|c|c|c|} \hline 1 & -1 & .5 & 7 \\ \hline \end{array} \end{array}$$

Matlab code: $\mathbf{z} = \mathbf{x} - \mathbf{y}$

Vectorized multiplication

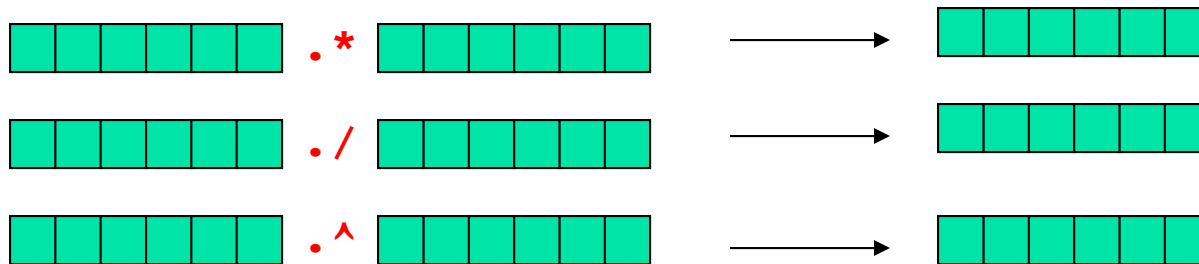
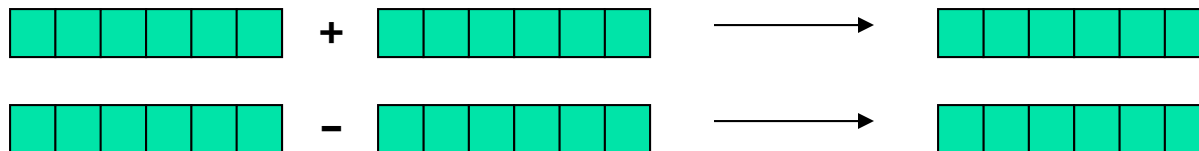
$$\begin{array}{rcl} & \mathbf{a} & \begin{bmatrix} 2 & 1 & .5 & 8 \end{bmatrix} \\ \times & \mathbf{b} & \begin{bmatrix} 1 & 2 & 0 & 1 \end{bmatrix} \\ \hline = & \mathbf{c} & \begin{bmatrix} 2 & 2 & 0 & 8 \end{bmatrix} \end{array}$$

Matlab code: `c = a .* b`



Vectorized

element-by-element arithmetic operations on arrays



A dot (.) is necessary in front of these math operators

Shift

$$\begin{array}{rcl} & \mathbf{x} & \boxed{3} \\ + & \mathbf{y} & \boxed{2 \quad 1 \quad .5 \quad 8} \\ \hline = & \mathbf{z} & \boxed{5 \quad 4 \quad 3.5 \quad 11} \end{array}$$

Matlab code: **z = x + y**

Reciprocate

$$\begin{array}{rcl} & \mathbf{x} & \boxed{1} \\ / & \mathbf{y} & \boxed{2 \quad 1 \quad .5 \quad 8} \\ \hline = & \mathbf{z} & \boxed{.5 \quad 1 \quad 2 \quad .125} \end{array}$$

Matlab code: $\mathbf{z} = \mathbf{x} \cdot / \mathbf{y}$



Vectorized

element-by-element arithmetic operations between an array and a scalar

$$\begin{bmatrix} \square & \square & \square & \square & \square & \square & \square \end{bmatrix} + \square$$

$$\begin{bmatrix} \square & \square & \square & \square & \square & \square & \square \end{bmatrix} - \square$$

$$\begin{bmatrix} \square & \square & \square & \square & \square & \square & \square \end{bmatrix} * \square$$

$$\begin{bmatrix} \square & \square & \square & \square & \square & \square & \square \end{bmatrix} / \square$$

$$\square + \begin{bmatrix} \square & \square & \square & \square & \square & \square & \square \end{bmatrix}$$

$$\square - \begin{bmatrix} \square & \square & \square & \square & \square & \square & \square \end{bmatrix}$$

$$\square * \begin{bmatrix} \square & \square & \square & \square & \square & \square & \square \end{bmatrix}$$

$$\begin{bmatrix} \square & \square & \square & \square & \square & \square & \square \end{bmatrix} \text{.}^\wedge \square$$

$$\square \text{.} / \begin{bmatrix} \square & \square & \square & \square & \square & \square & \square \end{bmatrix}$$

$$\square \text{.}^\wedge \begin{bmatrix} \square & \square & \square & \square & \square & \square & \square \end{bmatrix}$$

A dot (.) is necessary in front of these math operators

Not necessary but OK to use dot:

$$\begin{bmatrix} \square & \square \end{bmatrix} \text{.} * \square, \quad \square \text{.} * \begin{bmatrix} \square & \square \end{bmatrix}, \quad \begin{bmatrix} \square & \square \end{bmatrix} \text{.} / \square$$

Can we plot this?

$$f(x) = \frac{\sin(5x) \exp(-x/2)}{1+x^2} \quad \text{for} \quad -2 \leq x \leq 3$$

Yes!

```
x = linspace(-2,3,200);  
y = sin(5*x).*exp(-x/2)./(1 + x.^2);  
plot(x,y)
```



Element-by-element arithmetic
operations on arrays

Concatenating 2 vectors—copy 2 vectors into a new one

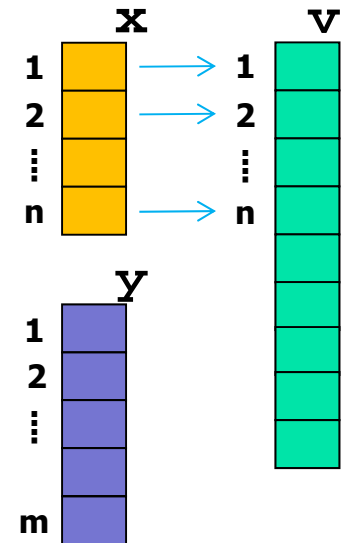
% given column vectors **x** and **y**

```
v= zeros(length(x)+length(y),1);
```

```
for k=1:length(x)
```

```
    v(k)= x(k);
```

```
end
```



Concatenating 2 vectors—copy 2 vectors into a new one

% given column vectors **x** and **y**

```
v= zeros(length(x)+length(y),1);
```

```
for k=1:length(x)
```

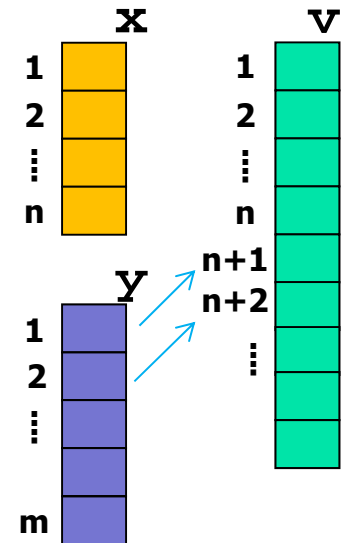
```
    v(k)= x(k);
```

```
end
```

```
for k=1:length(y)
```

```
    v(length(x)+k)= y(k);
```

```
end
```



This is **non-vectorized code**—operations are performed on one component (scalar) at a time

Concatenating 2 vectors—copy 2 vectors into a new one

```
% given column vectors x and y
v= zeros(length(x)+length(y),1);
for k=1:length(x)
    v(k)= x(k);
end
for k=1:length(y)
    v(length(x)+k)= y(k);
end
```

Below is **vectorized code**—ops are performed on multiple components (a vector) at the same time:

$v = [x; y];$

Split a vector in 2—copy values into 2 vectors

% given row vector v

s= ceil(rand*length(v)); **% split after v(s)**

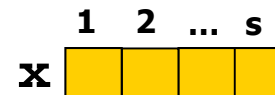
x= zeros(1,s);

y= zeros(1,length(v)-s);

for k=1:s

x(k)= v(k);

end



Split a vector in 2—copy values into 2 vectors

`% given row vector v`

`s= ceil(rand*length(v)); % split after v(s)`

`x= zeros(1,s);`

`y= zeros(1,length(v)-s);`

`for k=1:s`

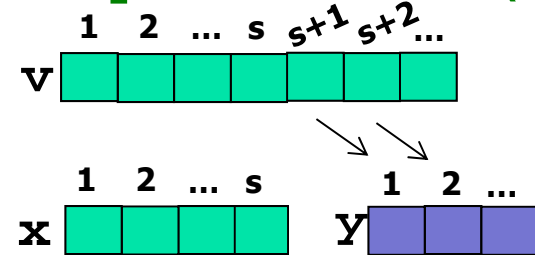
`x(k)= v(k);`

`end`

`for k=1:length(y)`

`y(k)= v(s+k);`

`end`



This is non-vectorized code—operations are performed on one component (scalar) at a time

Split a vector in 2—copy values into 2 vectors

```
% given row vector v
```

```
s= ceil(rand*length(v)); % split after v(s)
```

```
x= zeros(1,s);
```

```
y= zeros(1,length(v)-s);
```

```
for k=1:s
```

```
    x(k)= v(k);
```

```
end
```

```
for k=1:length(y)
```

```
    y(k)= v(s+k);
```

```
end
```

Below is **vectorized code**:
multiple components
(subvectors) are
affected/accessed at the same
time:

```
x= v(1:s);
```

```
y= v(s+1:length(v));
```

End of
Prelim 1 material

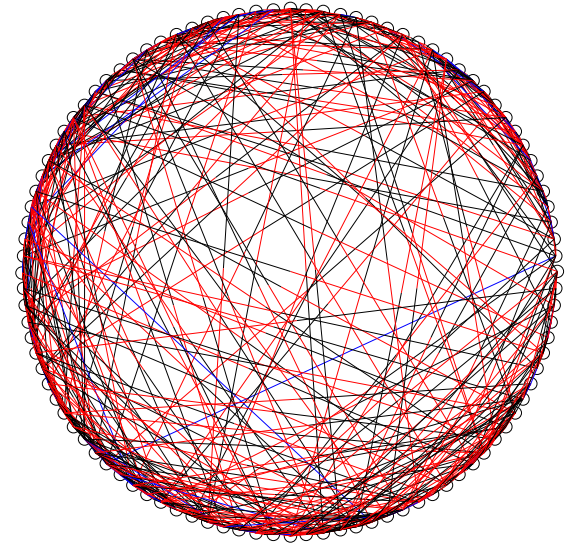
Storing and using data in tables

A company has 3 factories that make 5 products with these costs:

C

10	36	22	15	62
12	35	20	12	66
13	37	21	16	59

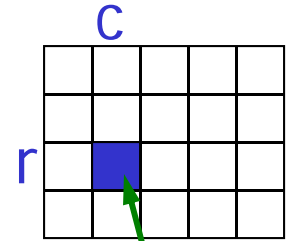
What is the best way to fill a given purchase order?



Connections
between webpages

0	0	1	0	1	0	0
1	0	0	1	1	1	0
0	1	0	1	1	1	1
1	0	1	1	0	1	0
0	0	1	1	0	1	1
0	0	1	0	1	0	1
0	1	1	0	1	1	0

2-d array: **matrix**



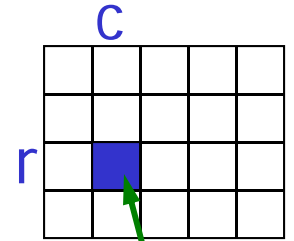
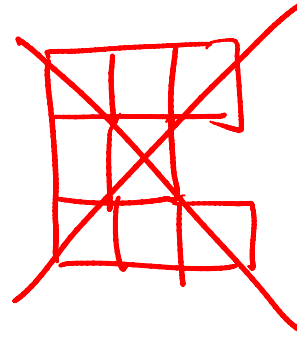
- An array is a **named** collection of **like** data organized into rows and columns
- A 2-d array is a table, called a **matrix**
- Two **indices** identify the position of a value in a matrix, e.g.,

mat(r, c)

refers to component in row **r**, column **c** of matrix **mat**

- Array index starts at **1**
- **Rectangular**: all rows have the same #of columns

2-d array: **matrix**



- An array is a **named** collection of **like** data organized into rows and columns
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`mat(r, c)`

refers to component in row **r**, column **c** of matrix **mat**

- Array index starts at **1**
- **Rectangular**: all rows have the same #of columns

Creating a matrix

- Built-in functions: `ones`, `zeros`, `rand`
 - E.g., `zeros(2,3)` gives a 2-by-3 matrix of 0s
 - E.g., `zeros(2)` gives a 2-by-2 matrix of 0s
- “Build” a matrix using square brackets, `[]`, but the dimension must match up:
 - `[x y]` puts `y` to the right of `x`
 - `[x; y]` puts `y` below `x`
 - `[4 0 3; 5 1 9]` creates the matrix
 - `[4 0 3; ones(1,3)]` gives
 - `[4 0 3; ones(3,1)]` doesn't work



4	0	3
5	1	9

4	0	3
1	1	1

Working with a matrix:
size and individual components

Given a matrix M

2	-1	.5	0	-3
3	8	6	7	7
5	-3	8.5	9	10
52	81	.5	7	2

```
[nr, nc]= size(M)    % nr is #of rows,  
                    % nc is #of columns
```

```
nr= size(M, 1)    % # of rows
```

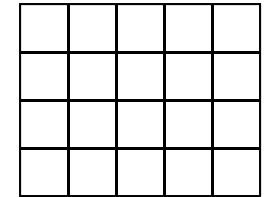
```
nc= size(M, 2)    % # of columns
```

```
M(2,4)= 1;
```

```
disp(M(3,1))
```

```
M(1,nc)= 4;
```

Example: minimum value in a matrix



```
function val = minInMatrix(M)
```

% val is the smallest value in matrix M

Example: minimum value in a matrix

```
function val = minInMatrix(M)
```

% val is the smallest value in matrix M

```
[nr, nc] = size(M);
```

```
val = M(1,1);
```

```
for r = 1:nr
```

```
    % At row r
```

```
    for c = 1:nc
```

```
        % At col c (at row r)
```

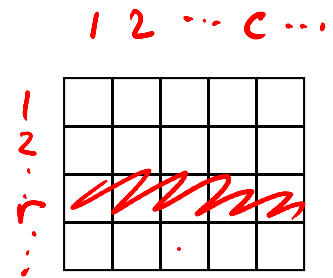
```
        if M(r,c) < val
```

```
            val = M(r,c);
```

```
        end
```

```
    end
```

```
end
```



Pattern for traversing a matrix M

```
[nr, nc] = size(M)
for r= 1:nr
    % At row r
    for c= 1:nc
        % At column c (in row r)
        %
        % Do something with M(r,c) ...
    end
end
end
```