- Previous Lecture:
  - 2-d array—matrix
- Today's Lecture:
  - More examples on matrices
  - Optional reading: contour plot (7.2, 7.3 in *Insight*)
- Announcement:
  - Fall Break next Mon & Tues: no lec, dis, office/consulting hrs. Attendance at 10/15 (W) dis is optional, but the exercise is required. Attend any of the 10/15 dis sections if you wish. Location is ~~Hollister 401~~ Upson B7
  - *Optional* review sessions: W 5-6:30p, W 7:30-9p. Location: Hollister B14

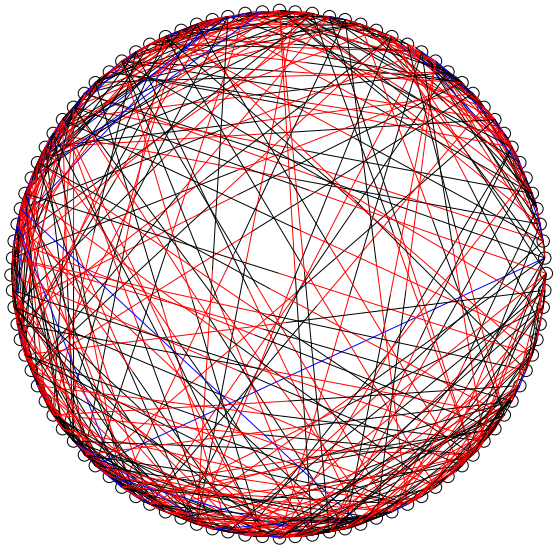
# Storing and using data in *tables*

A company has 3 factories that make 5 products with these costs:

C

| | | | | |
|---|---|---|---|---|
| 10 | 36 | 22 | 15 | 62 |
| 12 | 35 | 20 | 12 | 66 |
| 13 | 37 | 21 | 16 | 59 |

What is the best way to fill a given purchase order?



Connections between webpages

```
0 0 1 0 1 0 0
1 0 0 1 1 1 0
0 1 0 1 1 1 1
1 0 1 1 0 1 0
0 0 1 1 0 1 1
0 0 1 0 1 0 1
0 1 1 0 1 1 0
```

# Pattern for traversing a matrix M

```
[nr, nc] = size(M)
for  r= 1:nr
      % At row r
      for  c= 1:nc
            % At column c (in row r)
            %
            % Do something with M(r,c) …
      end
end
```

```
% Given an nr-by-nc matrix M.
% What is A?
for  r= 1: nr
     for c= 1: nc
          A(c,r)= M(r,c);
     end
end
```

A A is M with the columns in reverse order

B A is M with the rows in reverse order

C A is the transpose of M

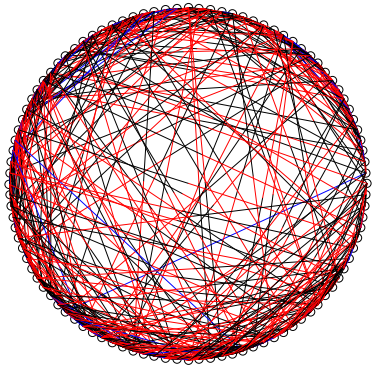
D A and M are the same

```
% Given an nr-by-nc matrix M.
% What is A?
for  r= 1: nr
     for c= 1: nc
          A(c,r)= M(r,c);
     end
end
```

M

| | | | |
|---|---|---|---|
| 0 | 3 | 2 | 5 |
| 4 | 13 | 20 | 6 |
| 11 | 26 | 9 | 1 |

# Matrix example: Random Web



- N web pages can be represented by an N-by-N Link Array A.
- A(i,j) is 1 if there is a link on webpage j to webpage i
- Generate a random link array and display the connectivity:
  - There is no link from a page to itself
  - If i≠j then A(i,j) = 1 with probability $\frac{1}{1+|i-j|}$

⇨ There is more likely to be a link if i is close to j

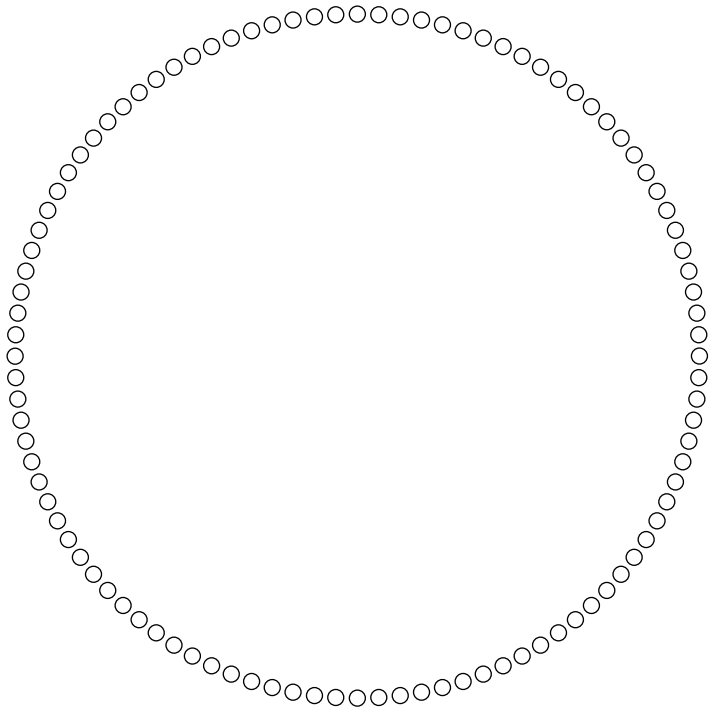
```
function A = RandomLinks(n)
% A is n-by-n matrix of 1s and 0s
% representing n webpages

A = zeros(n,n);
for i=1:n
  for j=1:n
    r = rand;
    if i~=j && r <= 1/(1 + abs(i-j));
        A(i,j) = 1;
    end
  end
end
```

*Random web*

**N = 20**

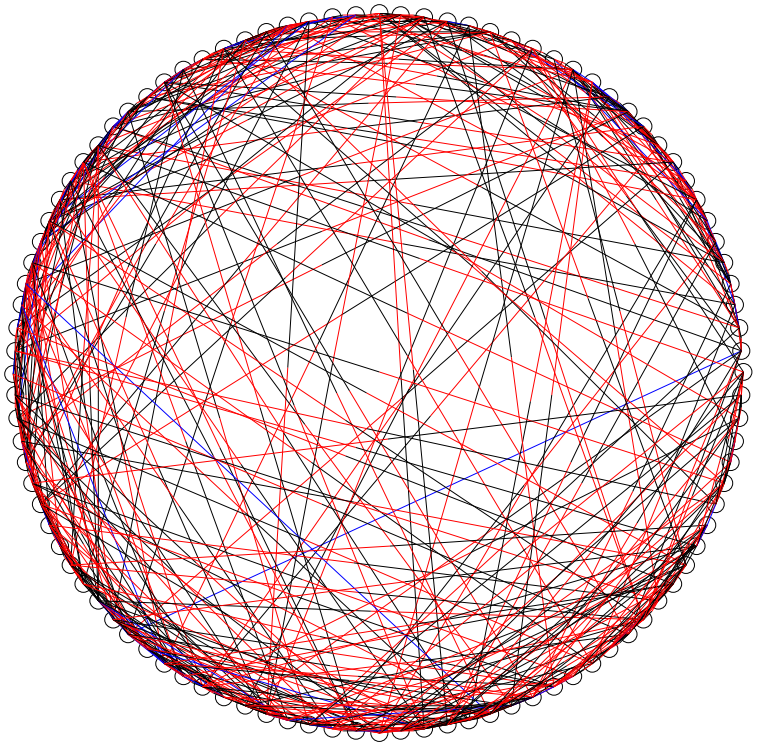
```
0 1 1 1 0 0 0 0 0 1 0 0 1 0 0 0 0 0 0 0
1 0 0 0 1 0 0 0 1 1 1 0 0 0 0 0 0 1 0 0
0 1 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
0 0 1 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 1 0 0 0 0 0 0 1 1 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 1 0 0 0 0
0 1 1 1 1 1 0 0 0 1 0 1 1 0 0 0 0 0 0 0
0 0 0 0 0 0 1 0 0 0 0 1 0 0 0 0 0 0 1 1
0 1 0 0 0 0 0 0 0 1 0 0 1 0 0 0 1 0 0 0
0 0 0 0 0 0 0 1 1 0 1 0 0 0 0 0 0 0 0 1
0 0 0 0 0 0 1 0 0 0 0 0 1 1 0 0 0 0 0 0
0 0 0 0 0 0 1 0 0 1 0 0 0 0 0 0 0 0 0 1
0 0 0 1 0 0 0 0 1 1 0 1 0 1 1 0 0 0 0 0
0 0 0 0 0 0 1 0 0 0 0 0 0 0 1 1 0 0 0 0
0 0 0 0 0 1 0 1 0 0 0 0 1 0 0 1 0 0 0 1
0 0 0 0 0 0 1 0 0 0 1 0 0 0 0 0 1 0 1 0
0 1 0 0 0 0 0 0 1 0 0 0 0 1 0 1 0 1 1 0
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 1 0 0 1
0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 1 0 0
```

## Represent the web pages graphically…



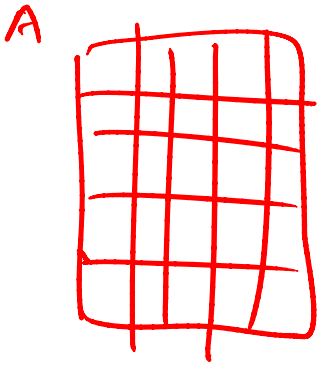
100 Web pages arranged in a circle.
Next display the links….

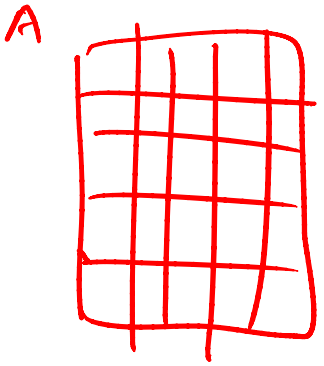
Represent the web pages graphically…



Bidirectional links are blue. Unidirectional link is black as it leaves page j, red when it arrives at page i.

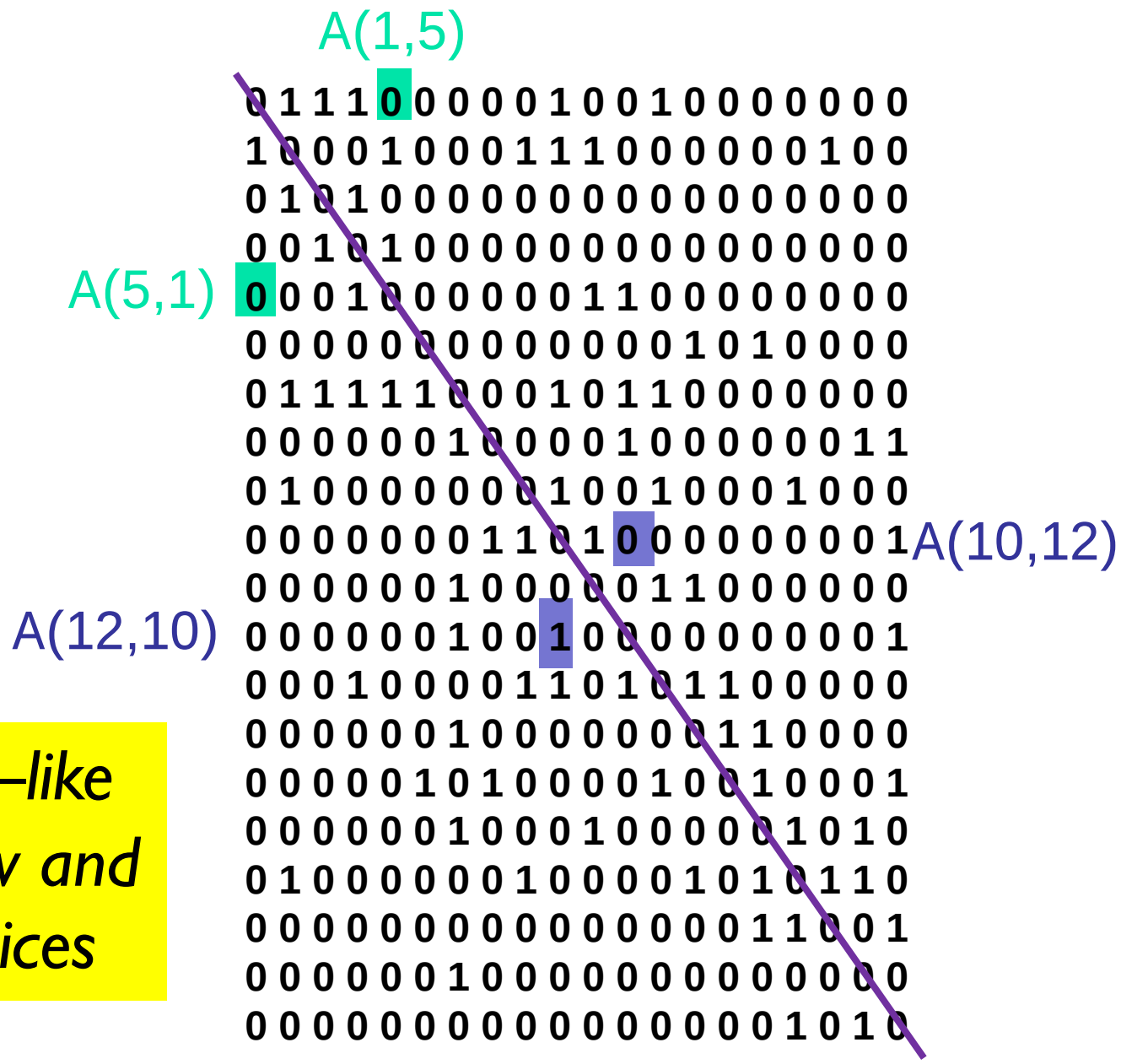
```
for i = 1:n
    for j = 1:n
end
end
```






```
for i = 1:n
    for j = 1:n
        if A(i,j) == 1 && A(j,i) == 1
            % Blue

        elseif A(i,j) == 1
            % Black - Red
            % j -> mid   mid -> i

        end
    end
end
```

Somewhat inefficient: each blue line gets drawn twice.
See **`ShowRandomLinks.m`**




*Transpose—like switching row and column indices*

# A Cost/Inventory Problem

- A company has 3 factories that make 5 different products
- The cost of making a product varies from factory to factory
- The inventory/capacity varies from factory to factory

# Problems

A customer submits a purchase order that is to be filled by a single factory.

1. How much would it cost a factory to fill the order?
2. Does a factory have enough inventory/capacity to fill the order?
3. Among the factories that can fill the order, who can do it most cheaply?

# Cost Array

C

| | | | | |
|---|---|---|---|---|
| 10 | 36 | 22 | 15 | 62 |
| 12 | 35 | 20 | 12 | 66 |
| 13 | 37 | 21 | 16 | 59 |

The value of `C(i,j)` is what it costs factory i to make product j.

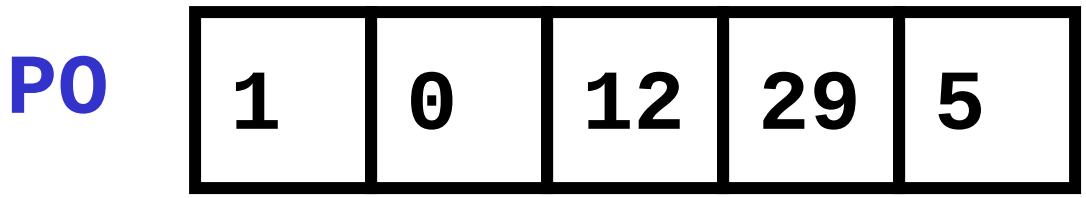
# Inventory (or Capacity) Array

**Inv**

| | | | | |
|---|---|---|---|---|
| 38 | 5 | 99 | 34 | 42 |
| 82 | 19 | 83 | 12 | 42 |
| 51 | 29 | 21 | 56 | 87 |

The value of `Inv(i,j)` is the inventory in factory i of product j.

# Purchase Order




The value of `PO(j)` is the number of product j's that the customer wants

C

| | | | | |
|---|---|---|---|---|
| 10 | 36 | 22 | 15 | 62 |
| 12 | 35 | 20 | 12 | 66 |
| 13 | 37 | 21 | 16 | 59 |

P0

| | | | | |
|---|---|---|---|---|
| 1 | 0 | 12 | 29 | 5 |

Cost for factory 1:

1*10 + 0*36 + 12*22 + 29* 15 + 5*62

C

| | | | | |
|---|---|---|---|---|
| 10 | 36 | 22 | 15 | 62 |
| 12 | 35 | 20 | 12 | 66 |
| 13 | 37 | 21 | 16 | 59 |

PO

| | | | | |
|---|---|---|---|---|
| 1 | 0 | 12 | 29 | 5 |

Cost for factory 1:

```
s = 0;  %Sum of cost
for j=1:5
   s = s + C(1,j)*PO(j)
end
```

C

| | | | | |
|---|---|---|---|---|
| 10 | 36 | 22 | 15 | 62 |
| 12 | 35 | 20 | 12 | 66 |
| 13 | 37 | 21 | 16 | 59 |

PO

| | | | | |
|---|---|---|---|---|
| 1 | 0 | 12 | 29 | 5 |

Cost for factory 2:

```
s = 0;  %Sum of cost
for j=1:5
   s = s + C(2,j)*PO(j)
end
```

C

| | | | | |
|---|---|---|---|---|
| 10 | 36 | 22 | 15 | 62 |
| 12 | 35 | 20 | 12 | 66 |
| 13 | 37 | 21 | 16 | 59 |

PO

| | | | | |
|---|---|---|---|---|
| 1 | 0 | 12 | 29 | 5 |

Cost for factory i:

```
s = 0;  %Sum of cost
for j=1:5
   s = s + C(i,j)*PO(j)
end
```

# Encapsulate…

```
function  TheBill = iCost(i,C,PO)
% The cost when factory i fills the
% purchase order

nProd = length(PO);
TheBill = 0;
for j=1:nProd
   TheBill = TheBill + C(i,j)*PO(j);
end
```

# Finding the Cheapest

```
iBest = 0;  minBill = inf;
for i=1:nFact
    iBill = iCost(i,C,PO);
    if iBill < minBill
        % Found an Improvement
        iBest = i; minBill = iBill;
    end
end
```

**`inf`** – a special value that can be regarded as positive infinity

| | |
|---|---|
| `x = 10/0` | assigns `inf` to `x` |
| `y = 1+x` | assigns `inf` to `y` |
| `z = 1/x` | assigns `0` to `z` |
| `w < inf` | is always true if `w` is numeric |

# Inventory/Capacity Considerations

What if a factory lacks the inventory/capacity to fill the purchase order?

Such a factory should be excluded from the find-the-cheapest computation.

# Who Can Fill the Order?

**Inv**

| | | | | | |
|---|---|---|---|---|---|
| 38 | 5 | 99 | 34 | 42 | Yes |
| 82 | 19 | 83 | 12 | 42 | No |
| 51 | 29 | 21 | 56 | 87 | Yes |

**PO**

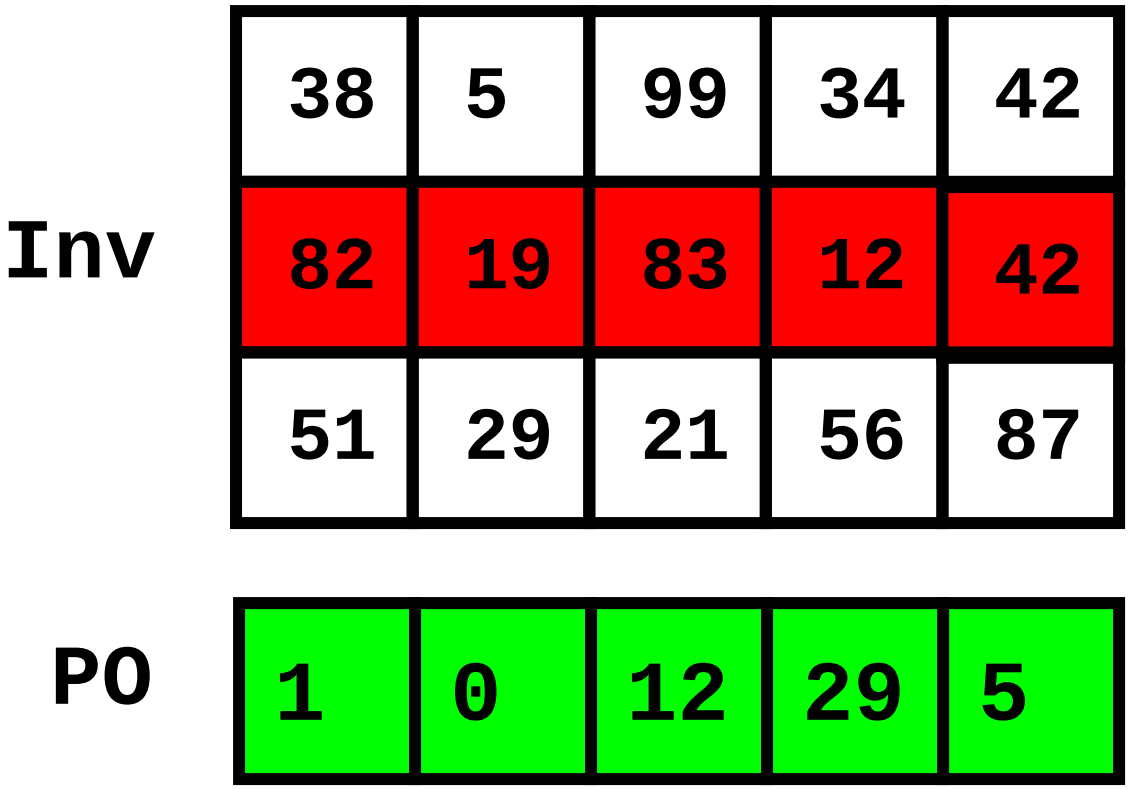
| | | | | |
|---|---|---|---|---|
| 1 | 0 | 12 | 29 | 5 |

# Wanted: A True/False Function


**DO** is "true" if factory i can fill the order.

**DO** is "false" if factory i cannot fill the order.

# Example: Check inventory of factory 2




Method 1: check the inventory for every product

# Initialization

**Inv**

| | | | | |
|---|---|---|---|---|
| 38 | 5 | 99 | 34 | 42 |
| 82 | 19 | 83 | 12 | 42 |
| 51 | 29 | 21 | 56 | 87 |

**DO** 1

**PO**

| | | | | |
|---|---|---|---|---|
| 1 | 0 | 12 | 29 | 5 |

# Still True…

Inv

| | | | | |
|---|---|---|---|---|
| 38 | 5 | 99 | 34 | 42 |
| 82 | 19 | 83 | 12 | 42 |
| 51 | 29 | 21 | 56 | 87 |

DO 1

PO

| | | | | |
|---|---|---|---|---|
| 1 | 0 | 12 | 29 | 5 |

```
DO = DO && ( Inv(2,1) >= PO(1) )
```

# Still True…

Inv

| | | | | |
|---|---|---|---|---|
| 38 | 5 | 99 | 34 | 42 |
| 82 | 19 | 83 | 12 | 42 |
| 51 | 29 | 21 | 56 | 87 |

DO 1

PO

| | | | | |
|---|---|---|---|---|
| 1 | 0 | 12 | 29 | 5 |

```
DO = DO && ( Inv(2,2) >= PO(2) )
```

# Still True…

Inv

| | | | | |
|---|---|---|---|---|
| 38 | 5 | 99 | 34 | 42 |
| 82 | 19 | 83 | 12 | 42 |
| 51 | 29 | 21 | 56 | 87 |

DO 1

PO

| | | | | |
|---|---|---|---|---|
| 1 | 0 | 12 | 29 | 5 |

```
DO = DO && ( Inv(2,3) >= PO(3) )
```

# No Longer True…

Inv

| | | | | |
|---|---|---|---|---|
| 38 | 5 | 99 | 34 | 42 |
| 82 | 19 | 83 | 12 | 42 |
| 51 | 29 | 21 | 56 | 87 |

DO 0

PO

| | | | | |
|---|---|---|---|---|
| 1 | 0 | 12 | 29 | 5 |

```
DO = DO && ( Inv(2,4) >= PO(4) )
```

# Stay False…

Inv

| | | | | |
|---|---|---|---|---|
| 38 | 5 | 99 | 34 | 42 |
| 82 | 19 | 83 | 12 | 42 |
| 51 | 29 | 21 | 56 | 87 |

DO 0

PO

| | | | | |
|---|---|---|---|---|
| 1 | 0 | 12 | 29 | 5 |

```
DO = DO && ( Inv(2,5) >= PO(5) )
```

# Encapsulate…

```
function  DO = iCanDo(i,Inv,PO)
% DO is true if factory i can fill
% the purchase order. Otherwise, false

nProd = length(PO);
DO = 1;
for j = 1:nProd
     DO = DO && ( Inv(i,j) >= PO(j) );
end
```

# Encapsulate…

```
function  DO = iCanDo(i,Inv,PO)
% DO is true if factory i can fill
% the purchase order. Otherwise, false
nProd = length(PO);
j = 1;
while j<=nProd && Inv(i,j)>=PO(j)
   j = j+1;
end
DO = ________;
```

# Encapsulate…

```
function  DO = iCanDo(i,Inv,PO)
% DO is true if factory i can fill
% the purchase order. Otherwise, false
nProd = length(PO);
j = 1;
while j<=nProd && Inv(i,j)>=PO(j)
    j = j+1;
end
DO = ____________;
```

**DO** should be true when…

A `j < nProd`

B `j == nProd`

C `j > nProd`

# Encapsulate…

```
function  DO = iCanDo(i,Inv,PO)
% DO is true if factory i can fill
% the purchase order. Otherwise, false
nProd = length(PO);
j = 1;
while j<=nProd && Inv(i,j)>=PO(j)
   j = j+1;
end
DO = (j>nProd);
```

# Back To Finding the Cheapest

```
iBest = 0; minBill = inf;
for i=1:nFact
    iBill = iCost(i,C,PO);
    if iBill < minBill
        % Found an Improvement
        iBest = i; minBill = iBill;
    end
end
```

Don't bother with this unless there is sufficient inventory.

# Back To Finding the Cheapest

```
iBest = 0; minBill = inf;
for i=1:nFact
    if iCanDo(i,Inv,PO)
        iBill = iCost(i,C,PO);
        if iBill < minBill
        % Found an Improvement
            iBest = i; minBill = iBill;
        end
    end
end
```

See **Cheapest.m**
for alternative implementation

# Finding the Cheapest