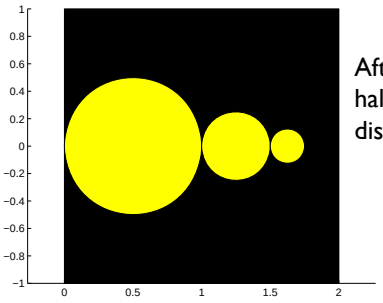
- Previous Lecture:
  - Examples on vectors and simulation
- Today's Lecture:
  - Finite vs. Infinite; Discrete vs. Continuous
  - Vectors and vectorized code
  - Color computation with *linear interpolation*
  - `plot` and `fill`
- Announcements:
  - Dinner tonight: 5pm, meet outside Appel
  - Project 3 due Monday 10/5 at 11pm
  - Tutoring available through Engineering or by course staff
  - Prelim 1 on Oct 15th at 7:30pm. Email TA Wayne Uy (wtu4) now if you have an exam conflict (specify conflicting course and instructor contact info)

Screen Granularity



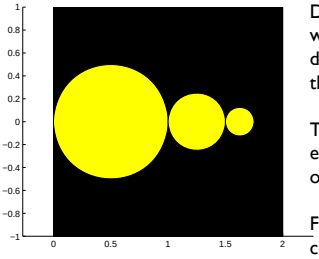
After how many halvings will the disks disappear?

Xeno's Paradox

- A wall is two feet away
- Take steps that repeatedly halve the remaining distance
- You never reach the wall because the distance traveled after n steps =

$$1 + \tfrac{1}{2} + \tfrac{1}{4} + \ldots + 1/2^n = 2 - 1/2^n$$

Example: "Xeno" disks

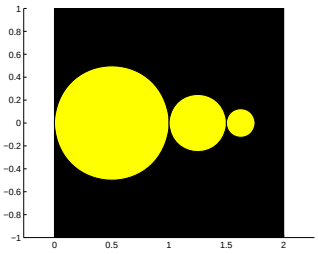


Draw a sequence of 20 disks where the (k+1)th disk has a diameter that is half that of the kth disk.

The disks are tangent to each other and have centers on the x-axis.

First disk has diameter 1 and center (1/2, 0).

Example: "Xeno" disks



What do you need to keep track of?

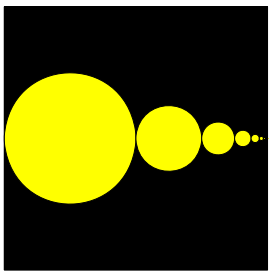
- Diameter (d)
- Position
  Left tangent point (x)

Disk	x	d
1	0	1
2	0+1	1/2
3	0+1+1/2	1/4


```
% Xeno Disks

DrawRect(0,-1,2,2,'k')
% Draw 20 Xeno disks


for k= 1:20




end
```

```
% Xeno Disks

DrawRect(0,-1,2,2,'k')
% Draw 20 Xeno disks
d= 1;
x= 0;  % Left tangent point
for k= 1:20
   % Draw kth disk

   % Update x, d for next disk



end
```

Here's the output… Shouldn't there be 20 disks?

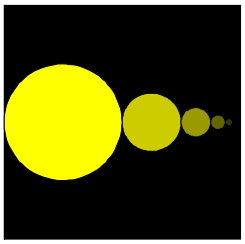


The "screen" is an array of dots called pixels.

Disks smaller than the dots don't show up.

The 20th disk has radius<.000001

Fading Xeno disks



- First disk is yellow
- Last disk is black (invisible)
- *Interpolate* the color in between

Color is a 3-vector, sometimes called the RGB values

- Any color is a mix of red, green, and blue
- Example:

```
colr= [0.4   0.6   0]
```

- Each component is a real value in [0,1]
- [0 0 0] is black
- [1 1 1] is white


```
% Draw n fading Xeno disks
d= 1;
x= 0;  % Left tangent point
yellow= [1 1 0];
black=  [0 0 0];
for k= 1:n
   % Compute color of kth disk


   % Draw kth disk
   DrawDisk(x+d/2, 0, d/2, _______)
   x= x+d;
   d= d/2;
end
```


Example: 3 disks fading from yellow to black

```
r= 1;  % radius of  disk
yellow= [1 1 0];
black = [0 0 0];
```

.5	*	1	1	0	→	.5	.5	0
.5	*	0	0	0	→	0	0	0

Vectorized multiplication

```
% Left disk yellow, at x=1
DrawDisk(1,0,r,yellow)
% Right disk black, at x=5
DrawDisk(5,0,r,black)

% Middle disk with average color, at x=3
colr= 0.5*yellow + 0.5*black;
DrawDisk(3,0,r,colr)
```

Example: 3 disks fading from yellow to black

```
r= 1;  % radius of  disk
yellow= [1 1 0];
black = [0 0 0];

% Left disk yellow, at x=1
DrawDisk(1,0,r,yellow)
% Right disk black, at x=5
DrawDisk(5,0,r,black)

% Middle disk with average color, at x=3
colr= 0.5*yellow + 0.5*black;
DrawDisk(3,0,r,colr)
```

Vectorized addition

	.5	.5	0
+	0	0	0
=	.5	.5	0

Vectorized code allows an operation on multiple values at the same time

```
yellow= [1 1 0];
black = [0 0 0];

% Average color via vectorized op
colr= 0.5*yellow + 0.5*black;
% Average color via scalar op
for k = 1:length(black)
   colr(k)= 0.5*yellow(k) + 0.5*black(k);
end
```

Vectorized addition

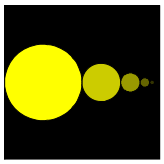
	.5	.5	0
+	0	0	0
=	.5	.5	0

Operation performed on vectors

Operation performed on scalars

Use ***linear interpolation*** to obtain the colors. Each disk has a color that is a linear combination of yellow and black. Let f be a fraction in (0,1) …

```
f= ???
colr= f*black + (1-f)*yellow;
```



Linear interpolation

x	g(x)
:	:
9	110
10	118
10.25	?
10.50	?
10.75	?
11	126
12	134
:	:

$g(10.5) = \frac{1}{2}\, g(11) + \frac{1}{2}\, g(10)$

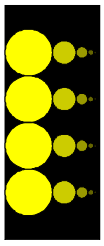
g(10) = 0/4·g(11) + 4/4·g(10)
g(10.25) = 1/4·g(11) + 3/4·g(10)
g(10.50) = 2/4·g(11) + 2/4·g(10)
g(10.75) = 3/4·g(11) + 1/4·g(10)
g(11) = 4/4·g(11) + 0/4·g(10)

f · g(11) + (1-f) · g(10)


```
% Draw n fading Xeno disks
d= 1;
x= 0;  % Left tangent point
yellow= [1 1 0];
black=  [0 0 0];
for k= 1:n
   % Compute color of kth disk
   f= ???
   colr= f*black + (1-f)*yellow;
   % Draw kth disk
   DrawDisk(x+d/2, 0, d/2, colr)
   x= x+d;
   d= d/2;
end
```

A. k/n
B. k/(n-1)
C. (k-1)/n
D. (k-1)/(n-1)
E. (k-1)/(n+1)

Rows of Xeno disks



```
for y = __ : __ : __
    Code to draw one
    row of Xeno disks
    at some y-coordinate
end
```

Be careful with "initializations"

Does this script print anything?

```
k = 0;
while 1 + 1/2^k > 1
   k = k+1;
end
disp(k)
```

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Computer Arithmetic—floating point arithmetic

Suppose you have a calculator with a window like this:

+	2	4	1	-	3

representing 2.41×10^{-3}

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Floating point addition

+	2	4	1	-	3
+	1	0	0	-	3

Result:

+	3	4	1	-	3

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Floating point addition

+	2	4	1	-	3
+	1	0	0	-	4

Result:

+	2	5	1	-	3

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Floating point addition

+	2	4	1	-	3
+	1	0	0	-	6

Result:

+	2	4	1	-	3

Not enough room to represent .002411

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The loop DOES terminate given the limitations of floating point arithmetic!

```
k = 0;
while 1 + 1/2^k > 1
   k = k+1;
end
disp(k)
```

1 + 1/2^53 is calculated to be just 1, so "53" is printed.

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Computer arithmetic is *inexact*

- There is error in computer arithmetic—floating point arithmetic—due to limitation in "hardware." Computer memory is finite.
- What is $1 + 10^{-16}$?
  - 1.0000000000000001 in real arithmetic
  - 1 in floating point arithmetic (IEEE)
- Read Sec 4.3

Vectorized code
—a Matlab-specific feature

See Sec 4.1 for list of vectorized arithmetic operations

- Code that performs element-by-element arithmetic/relational/logical operations on array operands in one step
- Scalar operation: x + y
  where x, y are scalar variables
- Vectorized code: x + y
  where x and/or y are vectors. If x and y are both vectors, they must be of the same shape and length

Vectorized addition

	x	2	1	.5	8
+	y	1	2	0	1
=	z	3	3	.5	9

Matlab code: `z= x + y`

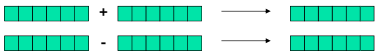
Vectorized multiplication

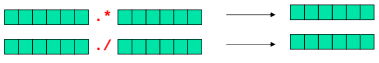
	a	2	1	.5	8
×	b	1	2	0	1
=	c	2	2	0	8

Matlab code: `c= a .* b`

Vectorized
element-by-element arithmetic operations on arrays

See full list of ops in §4.1





A dot (.) is necessary in front of these math operators

Shift

	x	3			
+	y	2	1	.5	8
=	z	5	4	3.5	11

Matlab code: `z= x + y`

Reciprocate

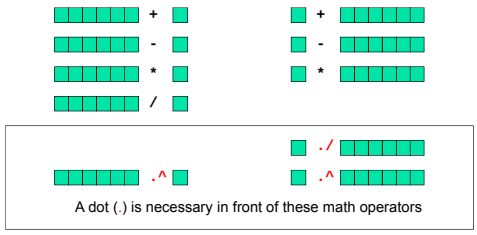
	x	1			
/	y	2	1	.5	8
=	z	.5	1	2	.125

Matlab code: `z= x ./ y`

Vectorized

See full list of ops in §4.1

element-by-element arithmetic operations between an array and a scalar



The dot in `.*` , `.*` , `./` not necessary but OK

Plot using vectorized computation

See `plotComparison.m`

$$f(x) = \frac{\sin(5x)\exp(-x/2)}{1+x^2}$$ for -2 <= x <= 3

```
x = linspace(-2,3,200);
y = sin(5*x).*exp(-x/2)./(1 + x.^2);
plot(x,y)
```

vectorized arithmetic operations on arrays

Element-by-element arithmetic operations on arrays…

Also called "vectorized code"

```
x = linspace(-2,3,200);
y = sin(5*x).*exp(-x/2)./(1 + x.^2);
```

`x` and `y` are vectors

What's the difference?

The operators are (mostly) the same; the operands may be scalars or vectors.

When an operand is a vector, you have "vectorized code."

Contrast with scalar operations that we've used previously…

```
a = 2.1;
b = sin(5*a);
```

`a` and `b` are scalars

Drawing a polygon (multiple line segments)

```
% Draw a rectangle with the lower-left
% corner at (a,b), width w, height h.
x= [                    ];  % x data
y= [                    ];  % y data
plot(x, y)
```

Fill in the missing vector values!


```
x= [0.1 -9.2  -7  4.4];
y= [9.4  7   -6.2  -3];
fill(x,y,'g')
```

Can be a vector (RGB values)