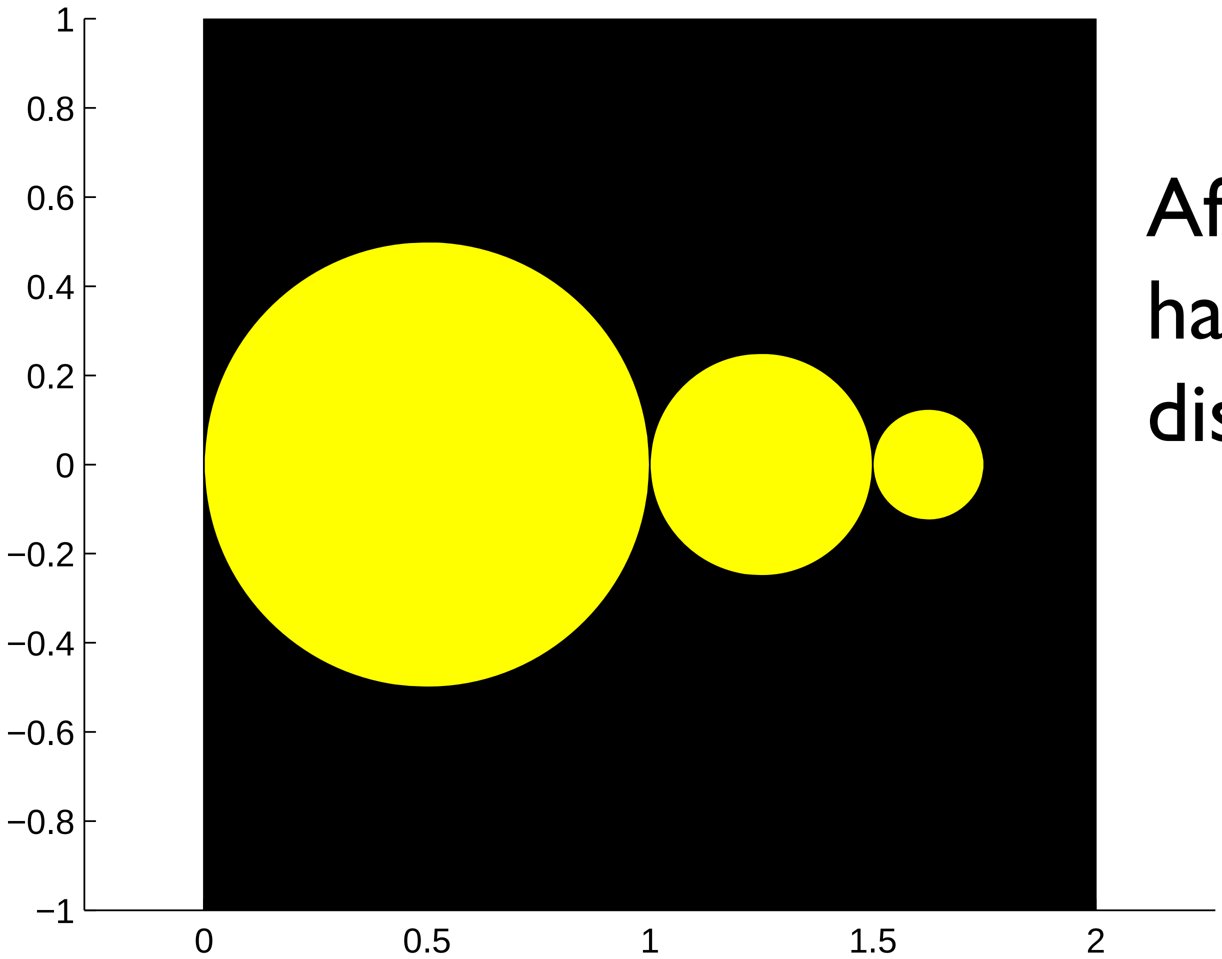
- Previous Lecture:
  - Examples on vectors and simulation

- Today's Lecture:
  - Finite vs. Infinite; Discrete vs. Continuous
  - Vectors and vectorized code
  - Color computation with *linear interpolation*
  - `plot` and `fill`

- Announcements:
  - Dinner tonight: 5pm, meet outside Appel
  - Project 3 due Monday 10/5 at 11pm
  - Tutoring available through Engineering or by course staff
  - Prelim 1 on Oct 15th at 7:30pm. Email TA Wayne Uy (wtu4) now if you have an exam conflict (specify conflicting course and instructor contact info)

Screen Granularity



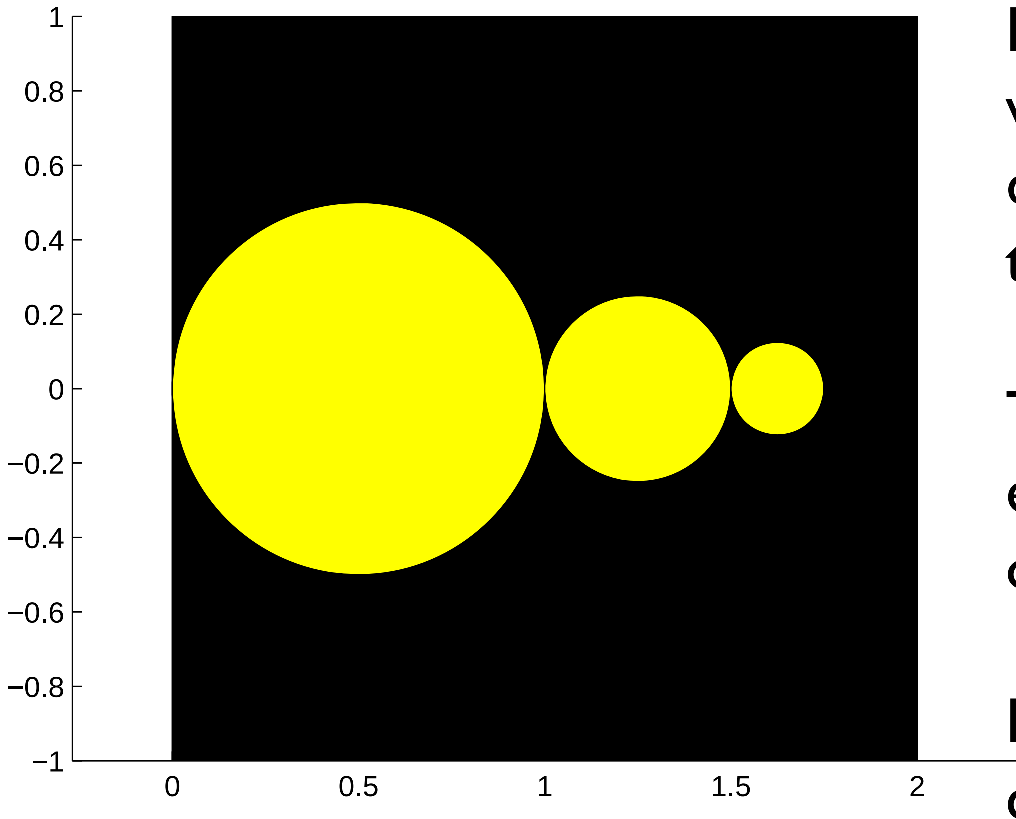
After how many halvings will the disks disappear?

Xeno's Paradox

- A wall is two feet away
- Take steps that repeatedly halve the remaining distance
- You never reach the wall because the distance traveled after n steps =

$$1 + \tfrac{1}{2} + \tfrac{1}{4} + \ldots + 1/2^n = 2 - 1/2^n$$

Example: "Xeno" disks

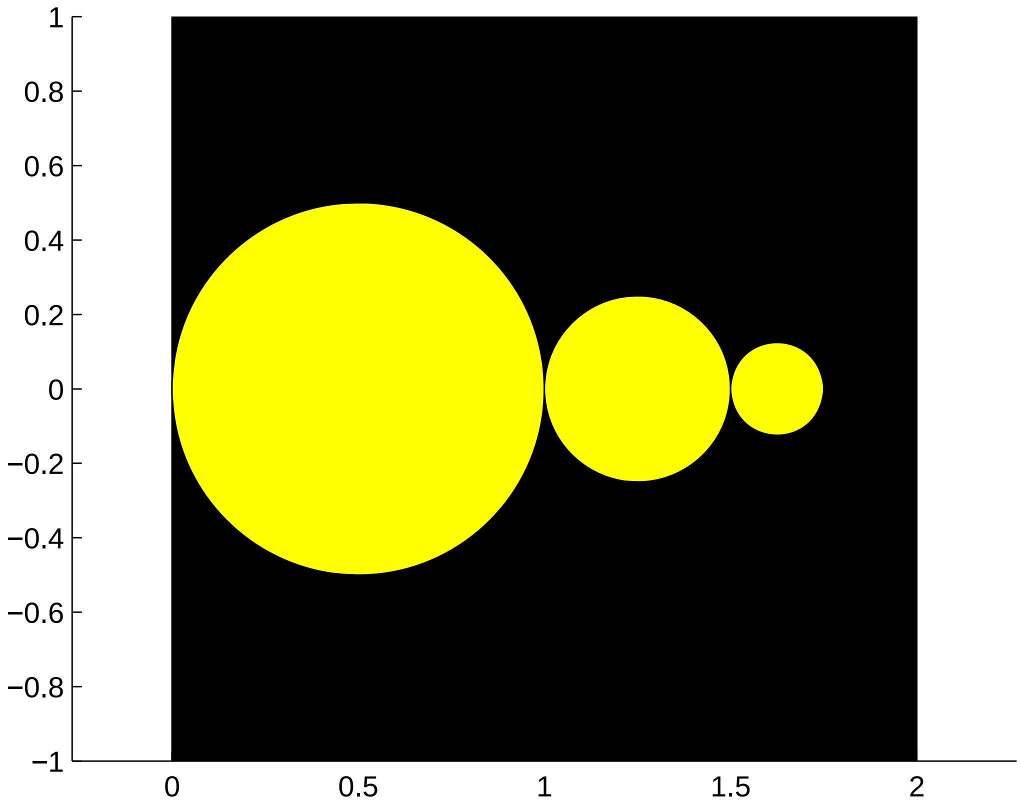


Draw a sequence of 20 disks where the (k+1)th disk has a diameter that is half that of the kth disk.

The disks are tangent to each other and have centers on the x-axis.

First disk has diameter 1 and center (1/2, 0).

Example: "Xeno" disks



What do you need to keep track of?

- Diameter (d)
- Position
  Left tangent point (x)

Disk	x	d
1	**0**	**1**
2	**0+1**	**1/2**
3	**0+1+1/2**	**1/4**

```
% Xeno Disks

DrawRect(0,-1,2,2,'k')
% Draw 20 Xeno disks
```

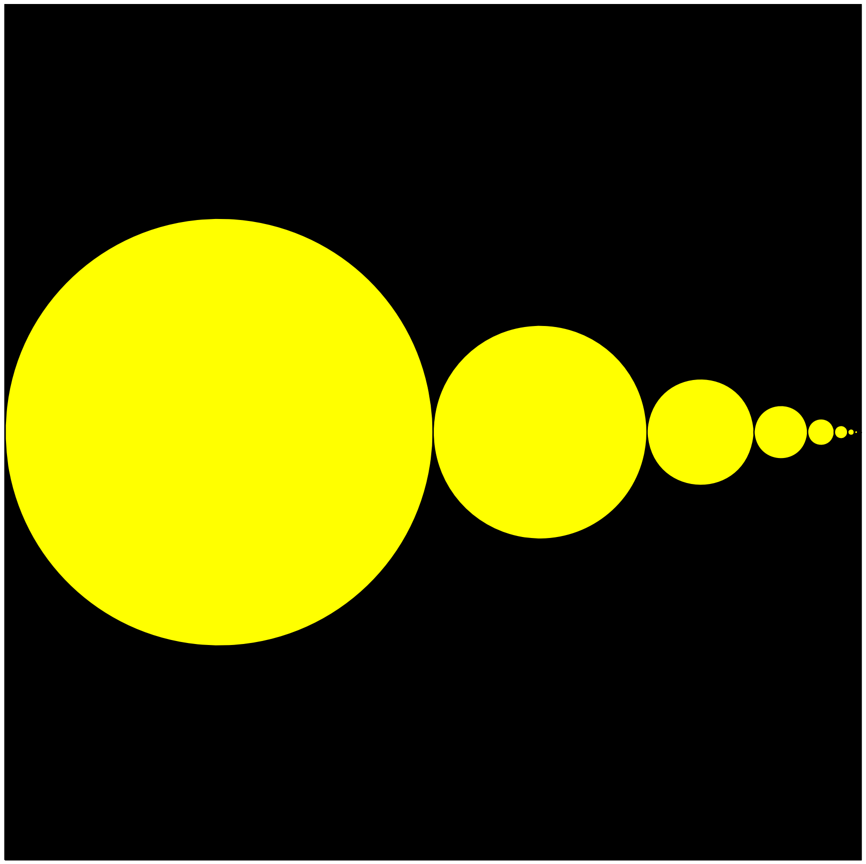
```
% Xeno Disks

DrawRect(0,-1,2,2,'k')
% Draw 20 Xeno disks


for k= 1:20






end
```

```
% Xeno Disks

DrawRect(0,-1,2,2,'k')
% Draw 20 Xeno disks
d= 1;
x= 0;  % Left tangent point
for k= 1:20









end
```

```
% Xeno Disks

DrawRect(0,-1,2,2,'k')
% Draw 20 Xeno disks
d= 1;
x= 0;  % Left tangent point
for k= 1:20
   % Draw kth disk


   % Update x, d for next disk




end
```

```
% Xeno Disks

DrawRect(0,-1,2,2,'k')
% Draw 20 Xeno disks
d= 1;
x= 0;  % Left tangent point
for k= 1:20
   % Draw kth disk
     DrawDisk(x+d/2, 0, d/2, ‘y’)
   % Update x, d for next disk
     x= x+d;
     d= d/2;

end
```

Here's the output… Shouldn't there be 20 disks?

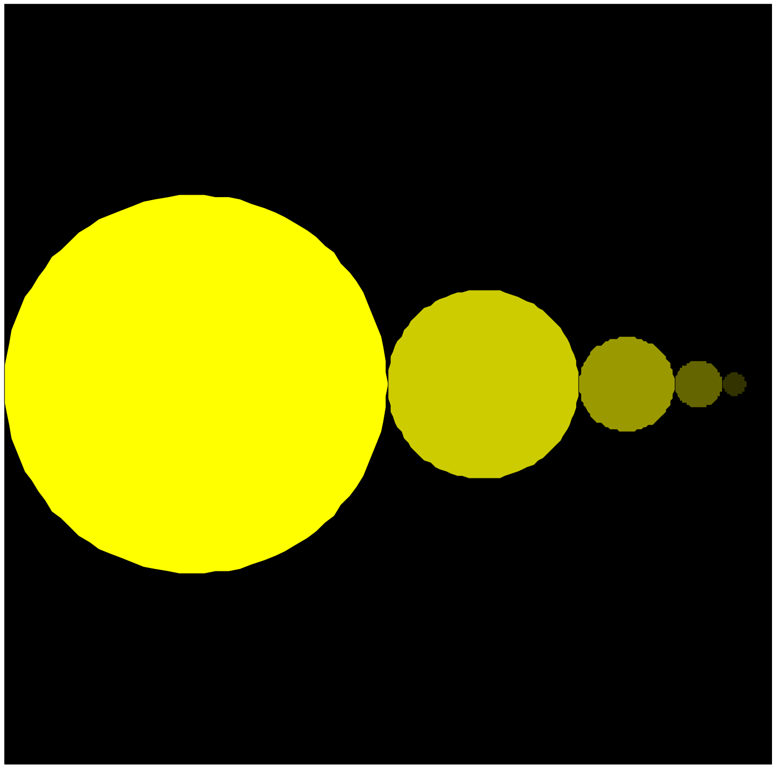


The "screen" is an array of dots called pixels.

Disks smaller than the dots don't show up.

The 20th disk has radius<.000001

Fading Xeno disks

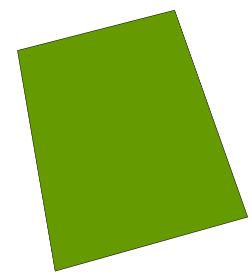


- First disk is yellow
- Last disk is black (invisible)
- *Interpolate* the color in between

Color is a 3-vector, sometimes called the RGB values

- Any color is a mix of red, green, and blue
- Example:

```
colr= [0.4  0.6  0]
```



- Each component is a real value in [0,1]
- [0 0 0] is black
- [1 1 1] is white

```
% Draw n Xeno disks
d= 1;
x= 0;  % Left tangent point


for k= 1:n



   % Draw kth disk
   DrawDisk(x+d/2, 0, d/2, ‘y’)
   x= x+d;
   d= d/2;
end
```

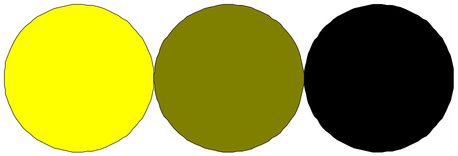
```
% Draw n Xeno disks
d= 1;
x= 0;  % Left tangent point


for k= 1:n



    % Draw kth disk
    DrawDisk(x+d/2, 0, d/2, [1 1 0])
    x= x+d;
    d= d/2;
end
```

A vector of length 3

```
% Draw n fading Xeno disks
d= 1;
x= 0;  % Left tangent point
yellow= [1 1 0];
black=  [0 0 0];
for k= 1:n
   % Compute color of kth disk



   % Draw kth disk
   DrawDisk(x+d/2, 0, d/2, _______)
   x= x+d;
   d= d/2;
end
```

Example: 3 disks fading from yellow to black

```
r= 1;  % radius of  disk
yellow= [1 1 0];
black = [0 0 0];

% Left disk yellow, at x=1
DrawDisk(1,0,r,yellow)
% Right disk black, at x=5
DrawDisk(5,0,r,black)

% Middle disk with average color, at x=3
colr= 0.5*yellow + 0.5*black;
DrawDisk(3,0,r,colr)
```



Example: 3 disks fading from yellow to black

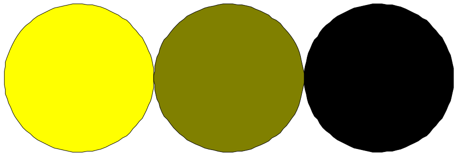
```
r= 1;  % radius of  disk
yellow= [1 1 0];
black = [0 0 0];
```

$.5 * [1\ 1\ 0] \rightarrow [.5\ .5\ 0]$

$.5 * [0\ 0\ 0] \rightarrow [0\ 0\ 0]$

Vectorized multiplication

```
% Left disk yellow, at x=1
DrawDisk(1,0,r,yellow)
% Right disk black, at x=5
DrawDisk(5,0,r,black)

% Middle disk with average color, at x=3
colr= 0.5*yellow + 0.5*black;
DrawDisk(3,0,r,colr)
```

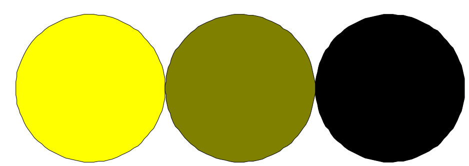


Example: 3 disks fading from yellow to black

```
r= 1;  % radius of  disk
yellow= [1 1 0];
black = [0 0 0];

% Left disk yellow, at x=1
DrawDisk(1,0,r,yellow)
% Right disk black, at x=5
DrawDisk(5,0,r,black)

% Middle disk with average color, at x=3
colr= 0.5*yellow + 0.5*black;
DrawDisk(3,0,r,colr)
```

Vectorized addition

	.5	.5	0
+	0	0	0
=	.5	.5	0



Vectorized code allows an operation on multiple values at the same time

Vectorized addition

	.5	.5	0
+	0	0	0
=	.5	.5	0

```
yellow= [1 1 0];
black = [0 0 0];

% Average color via vectorized op
colr= 0.5*yellow + 0.5*black;
```

Operation performed on vectors

```
% Average color via scalar op
for k = 1:length(black)
   colr(k)= 0.5*yellow(k) + 0.5*black(k);
end
```

Operation performed on scalars

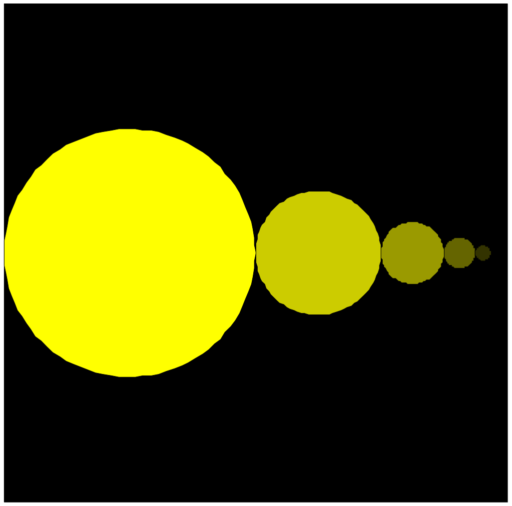
```
% Draw n fading Xeno disks
d= 1;
x= 0;  % Left tangent point
yellow= [1 1 0];
black=  [0 0 0];
for k= 1:n
   % Compute color of kth disk



   % Draw kth disk
   DrawDisk(x+d/2, 0, d/2, _______)
   x= x+d;
   d= d/2;
end
```

```
% Draw n fading Xeno disks
d= 1;
x= 0;  % Left tangent point
yellow= [1 1 0];
black=  [0 0 0];
for k= 1:n
   % Compute color of kth disk

   colr= ___*black + ___*yellow;
   % Draw kth disk
   DrawDisk(x+d/2, 0, d/2, colr)
   x= x+d;
   d= d/2;
end
```

```
% Draw n fading Xeno disks
d= 1;
x= 0;  % Left tangent point
yellow= [1 1 0];
black=  [0 0 0];
for k= 1:n
   % Compute color of kth disk
   f= ???
   colr= f*black + (1-f)*yellow;
   % Draw kth disk
   DrawDisk(x+d/2, 0, d/2, colr)
   x= x+d;
   d= d/2;
end
```

Use ***linear interpolation*** to obtain the colors. Each disk has a color that is a linear combination of yellow and black. Let f be a fraction in (0,1) …

```
f= ???
colr= f*black + (1-f)*yellow;
```



Linear interpolation

x	$g(x)$
:	:
9	110
10	118
11	126
12	134
:	:

Linear interpolation

x	g(x)
:	:
9	110
10	118
10.25	?
10.50	?
10.75	?
11	126
12	134
:	:

$$g(10.5) = [\, g(11) + g(10) \,] / 2$$

Linear interpolation

x	$g(x)$
:	:
9	110
10	118
10.25	?
10.50	?
10.75	?
11	126
12	134
:	:

$$g(10.5) = \tfrac{1}{2}\, g(11) + \tfrac{1}{2}\, g(10)$$

$$g(10.25) = 1/4 \cdot g(11) + 3/4 \cdot g(10)$$
$$g(10.50) = 2/4 \cdot g(11) + 2/4 \cdot g(10)$$
$$g(10.75) = 3/4 \cdot g(11) + 1/4 \cdot g(10)$$

Linear interpolation

x	g(x)
:	:
9	110
10	118
10.25	?
10.50	?
10.75	?
11	126
12	134
:	:

$$g(10.5) = \tfrac{1}{2}\, g(11) + \tfrac{1}{2}\, g(10)$$

$$\begin{aligned}
g(10) &= 0/4 \cdot g(11) + 4/4 \cdot g(10) \\
g(10.25) &= 1/4 \cdot g(11) + 3/4 \cdot g(10) \\
g(10.50) &= 2/4 \cdot g(11) + 2/4 \cdot g(10) \\
g(10.75) &= 3/4 \cdot g(11) + 1/4 \cdot g(10) \\
g(11) &= 4/4 \cdot g(11) + 0/4 \cdot g(10)
\end{aligned}$$

$$\boxed{f \cdot g(11) + (1-f) \cdot g(10)}$$

```
% Draw n fading Xeno disks
d= 1;
x= 0;  % Left tangent point
yellow= [1 1 0];
black=  [0 0 0];
for k= 1:n
    % Compute color of kth disk
    f= ???
    colr= f*black + (1-f)*yellow;
    % Draw kth disk
    DrawDisk(x+d/2, 0, d/2, colr)
    x= x+d;
    d= d/2;
end
```

A. k/n
B. k/(n-1)
C. (k-1)/n
D. (k-1)/(n-1)
E. (k-1)/(n+1)

Linear interpolation

x	g(x)
:	:
9	110
10	118
10.25	?
10.50	?
10.75	?
11	126
12	134
:	:

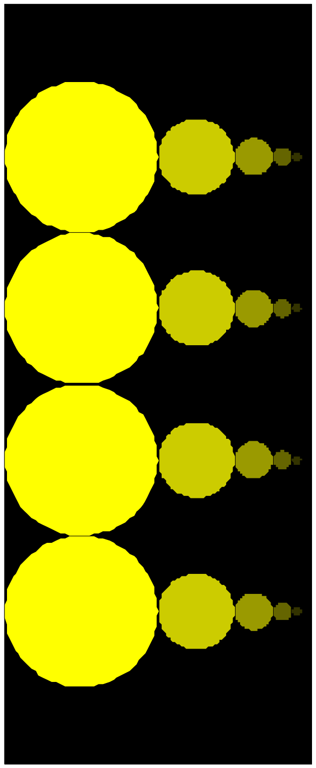
$$g(10) = 0/4 \cdot g(11) + 4/4 \cdot g(10)$$
$$g(10.25) = 1/4 \cdot g(11) + 3/4 \cdot g(10)$$
$$g(10.50) = 2/4 \cdot g(11) + 2/4 \cdot g(10)$$
$$g(10.75) = 3/4 \cdot g(11) + 1/4 \cdot g(10)$$
$$g(11) = 4/4 \cdot g(11) + 0/4 \cdot g(10)$$

$$f \cdot g(11) + (1-f) \cdot g(10)$$

```
% Draw n fading Xeno disks
d= 1;
x= 0;  % Left tangent point
yellow= [1 1 0];
black=  [0 0 0];
for k= 1:n
   % Compute color of kth disk
   f= ???
   colr= f*black + (1-f)*yellow;
   % Draw kth disk
   DrawDisk(x+d/2, 0, d/2, colr)
   x= x+d;
   d= d/2;
end
```

A k/n
B k/(n-1)
C (k-1)/n
D (k-1)/(n-1)
E (k-1)/(n+1)

Rows of Xeno disks



```
for y = __ : __ : __

    Code to draw one
    row of Xeno disks
    at some y-coordinate

end
```

Be careful with "initializations"

```
yellow=[1 1 0];  black=[0 0 0];

d= 1;

x= 0;

for k= 1:n
   % Compute color of kth disk
   f= (k-1)/(n-1);
   colr= f*black + (1-f)*yellow;
   % Draw kth disk
   DrawDisk(x+d/2, 0, d/2, colr)
   x=x+d;  d=d/2;
end
```

Where to put the loop header **for y=__:__:__**

A ⇨

```
yellow=[1 1 0];  black=[0 0 0];
```

B ⇨

```
d= 1;
```

C ⇨

```
x= 0;
```

D ⇨

```
for k= 1:n
   % Compute color of kth disk
   f= (k-1)/(n-1);
   colr= f*black + (1-f)*yellow;
   % Draw kth disk
   DrawDisk(x+d/2, 0, d/2, colr)
   x=x+d;  d=d/2;
end
```

y

end

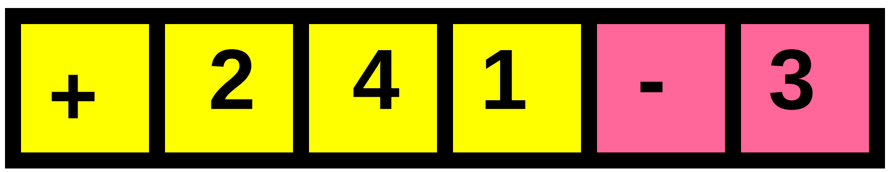
```
    yellow=[1 1 0];  black=[0 0 0];
for y= __:__:__
    d= 1;
                initializations necessary for each row
    x= 0;

    for k= 1:n
       % Compute color of kth disk
       f= (k-1)/(n-1);
       colr= f*black + (1-f)*yellow;
       % Draw kth disk
       DrawDisk(x+d/2, 0, d/2, colr)
       x=x+d;  d=d/2;
    end                                  y
end
```

Does this script print anything?

```
k = 0;
while 1 + 1/2^k > 1
    k = k+1;
end
disp(k)
```

Computer Arithmetic—floating point arithmetic

Suppose you have a calculator with
a window like this:



representing 2.41×10^{-3}

Floating point addition

+	2	4	1	-	3

+	1	0	0	-	3

Result:

+	3	4	1	-	3

Floating point addition

+	2	4	1	-	3

+	1	0	0	-	4

Result:

+	2	5	1	-	3

Floating point addition

+	2	4	1	-	3

+	1	0	0	-	5

Result:

+	2	4	2	-	3

Floating point addition

+	2	4	1	-	3

+	1	0	0	-	6

Result:

+	2	4	1	-	3

Floating point addition

+	2	4	1	-	3

+	1	0	0	-	6

Result:

+	2	4	1	-	3

Not enough room to represent .002411

The loop DOES terminate given the limitations of floating point arithmetic!

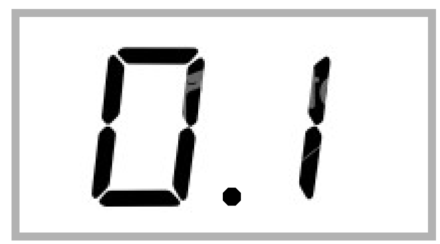
```
k = 0;
while 1 + 1/2^k > 1
    k = k+1;
end
disp(k)
```

1 + 1/2^53 is calculated to be just 1, so "53" is printed.

Patriot missile failure

www.namsa.nato.int/gallery/systems

In 1991, a Patriot Missile failed, resulting in 28 deaths and about 100 injured. The cause?



Inexact representation of time/number

- System clock represented time in tenths of a second: a clock tick every 1/10 of a second

- Time = number of clock ticks x 0.1

"exact" value

.000110011001100110011001100110011…

.00011001100110011001100011 value in Patriot system

Error of .000000095 every clock tick

Resulting error

… after 100 hours

.000000095 x (100x60x60)

0.34 second

At a velocity of 1700 m/s, missed target by more than 500 meters!

Computer arithmetic is *inexact*

- There is error in computer arithmetic—floating point arithmetic—due to limitation in "hardware." Computer memory is finite.

- What is $1 + 10^{-16}$?
  - 1.0000000000000001 in real arithmetic
  - 1 in floating point arithmetic (IEEE)

- Read Sec 4.3