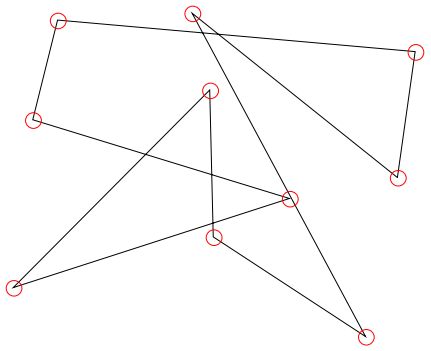
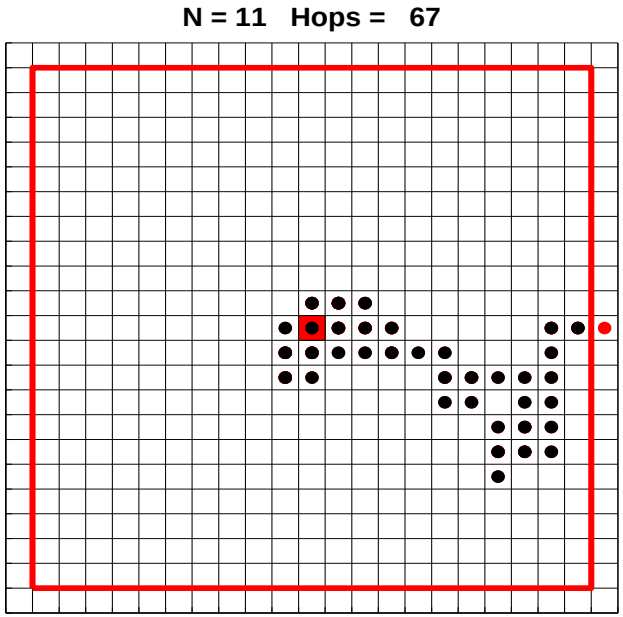
- Previous Lecture:
  - 1-d array—vector
  - Probability and random numbers
- Today's Lecture:
  - More examples on vectors and simulation
- Announcement:
  - Discussion this week in Upson B7 lab
  - Project 3 due on Mon 10/5

Simulation

- Imitates real system
- Requires judicious use of random numbers
- Requires many trials
- → opportunity to practice working with vectors!



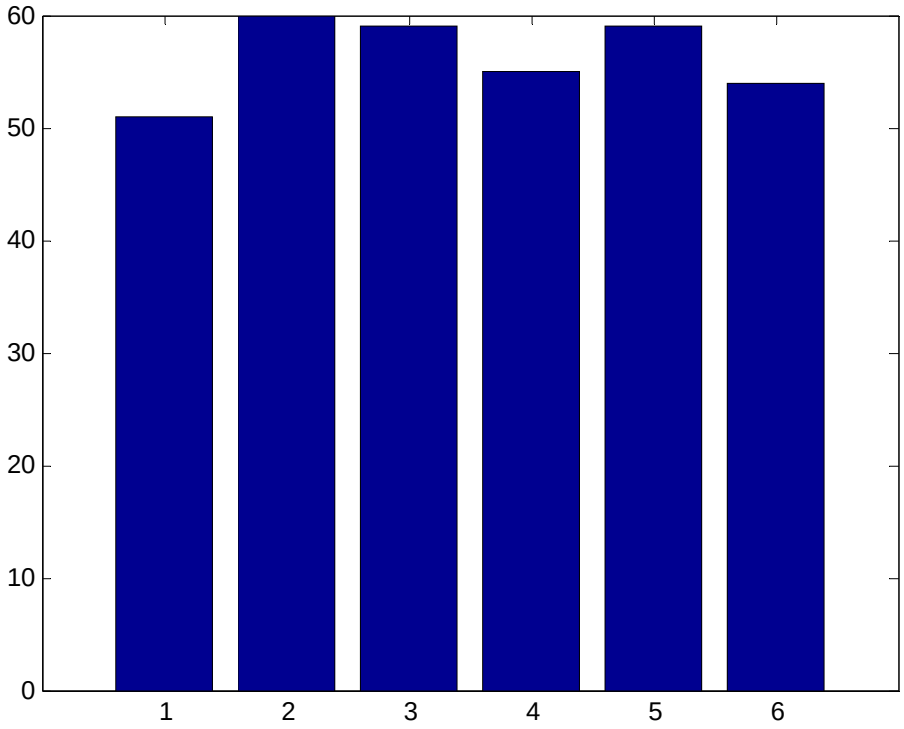


Simulation result

Data in bins

```
bar(1:6, count)
```

Bin numbers



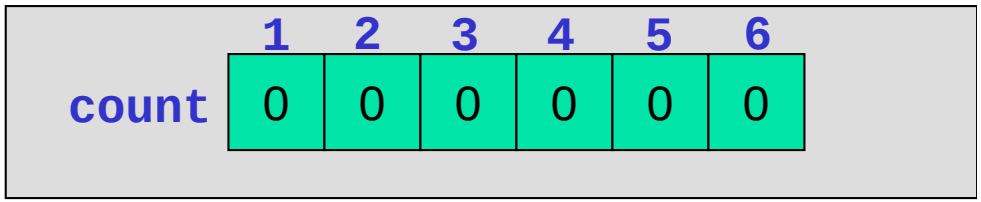
51	60	59	55	59	54
1	2	3	4	5	6

Keep tally on repeated rolls of a fair die

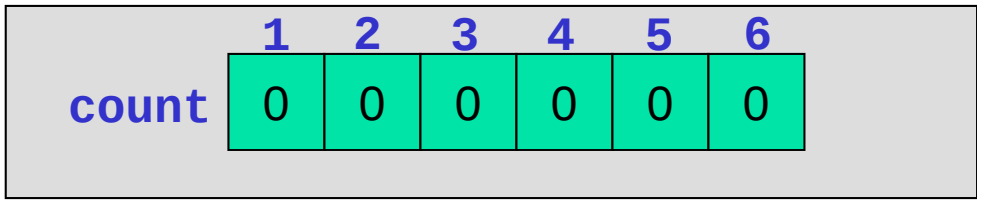
Repeat the following:

```
% roll the die

% increment correct "bin"
```

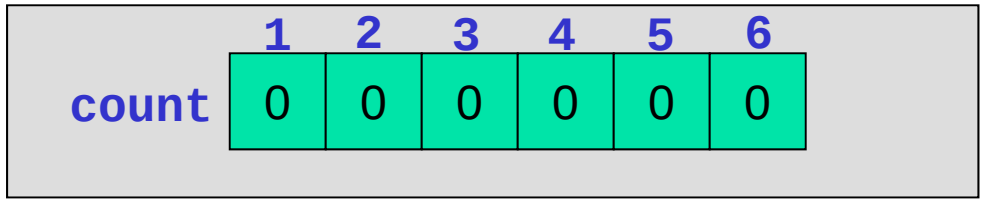
```
function count = rollDie(rolls)

FACES= 6;                          % #faces on die
count= zeros(1,FACES);

% Count outcomes of rolling a FAIR die
for k= 1:rolls
    % Roll the die

    % Increment the appropriate bin

end

% Show histogram of outcome
```



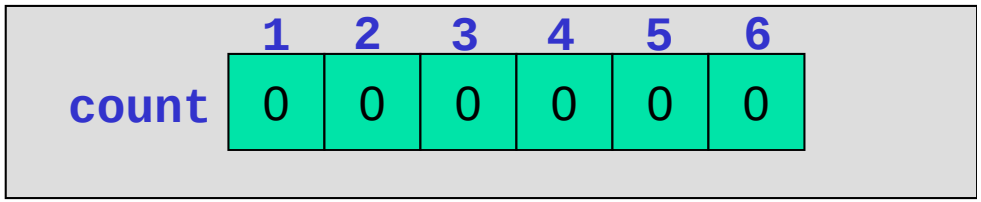
```
function count = rollDie(rolls)

FACES= 6;                          % #faces on die
count= zeros(1,FACES);

% Count outcomes of rolling a FAIR die
for k= 1:rolls
    % Roll the die
    face= ceil(rand*FACES);
    % Increment the appropriate bin

end

% Show histogram of outcome
```



```
% Count outcomes of rolling a FAIR die
count= zeros(1,6);
for k= 1:100
    face= ceil(rand*6);
    if face==1
        count(1)= count(1) + 1;
    elseif face==2
        count(2)= count(2) + 1;
    ⋮
    elseif face==5
        count(5)= count(5) + 1;
    else
        count(6)= count(6) + 1;
    end
end
```



```
function count = rollDie(rolls)

FACES= 6;                          % #faces on die
count= zeros(1,FACES);

% Count outcomes of rolling a FAIR die
for k= 1:rolls
    % Roll the die
    face= ceil(rand*FACES);
    % Increment the appropriate bin
    count(face)= count(face) + 1;
end

% Show histogram of outcome
```



% Simulate the rolling of 2 fair dice
totalOutcome= ???

A `ceil(rand*12)`

B `ceil(rand*11)+1`

C `floor(rand*11)+2`

D *2 of the above*

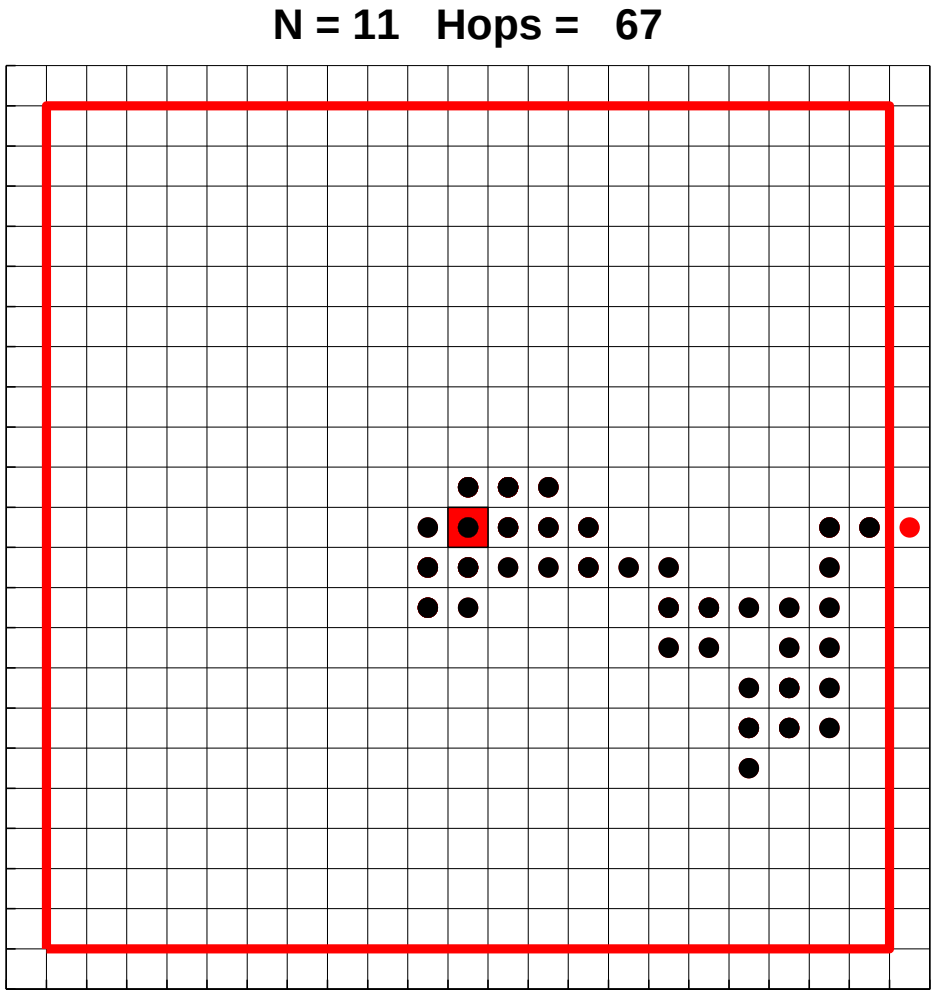
E *None of the above*

2-dimensional random walk

Start in the middle tile, (0,0).

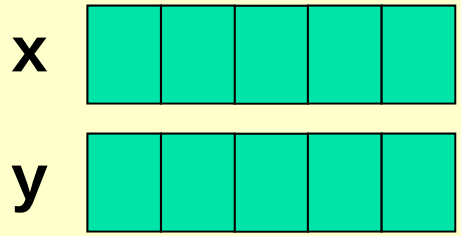
For each step, randomly choose between N,E,S,W and then walk one tile. Each tile is 1×1.

Walk until you reach the boundary.



```
function [x, y] = RandomWalk2D(N)
% 2D random walk in 2N-1 by 2N-1 grid.
% Walk randomly from (0,0) to an edge.
% Vectors x,y represent the path.
```

By the end of the function …



```
function [x, y] = RandomWalk2D(N)

k=0;  xc=0;  yc=0;

while   not at an edge
   % Choose random dir, update xc,yc



   % Record new location in x, y

end
```

```
function [x, y] = RandomWalk2D(N)

k=0;  xc=0;  yc=0;

while  abs(xc)<N && abs(yc)<N
    % Choose random dir, update xc,yc



    % Record new location in x, y

end
```

```
function [x, y] = RandomWalk2D(N)

k=0;  xc=0;  yc=0;

while  abs(xc)<N && abs(yc)<N
   % Choose random dir, update xc,yc



   % Record new location in x, y
   k=k+1;  x(k)=xc;  y(k)=yc;
end
```

```
% Standing at (xc,yc)
% Randomly select a step
    r= rand(1);
    if r < .25
        yc= yc + 1;  % north
    elseif r < .5
        xc= xc + 1;  % east
    elseif r < .75
        yc= yc -1;   % south
    else
        xc= xc -1;   % west
    end
```

See **RandomWalk2D.m**

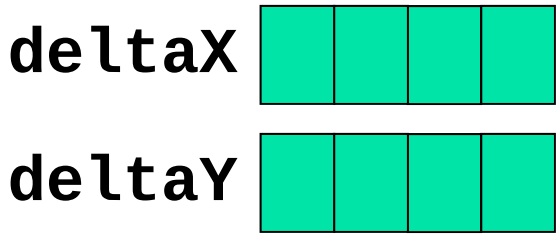
Another representation for the random step

- Observe that each update has the form

$$xc = xc + \Delta x$$

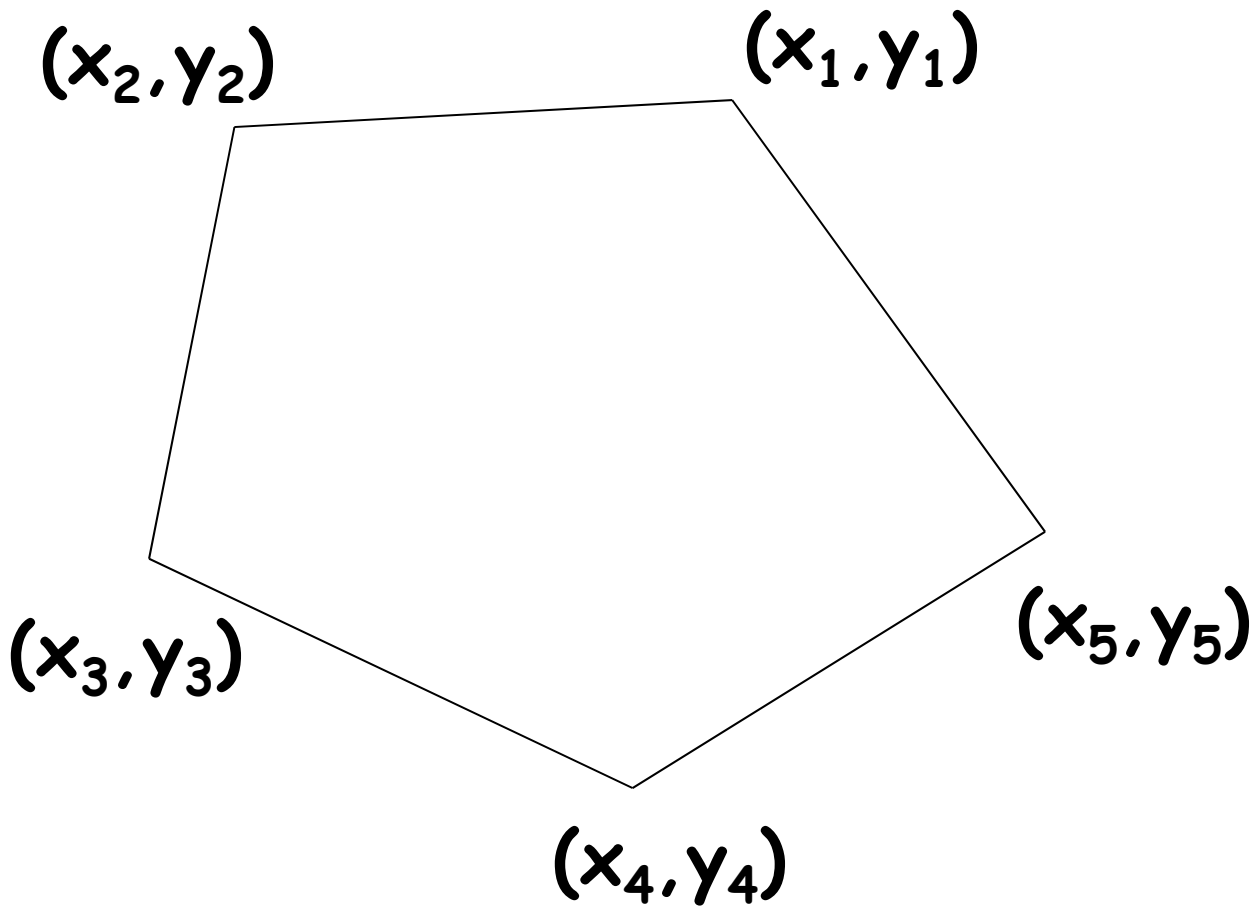
$$yc = yc + \Delta y$$

  no matter which direction is taken.
- So let's get rid of the if statement!
- Need to create two "change vectors" deltaX and deltaY

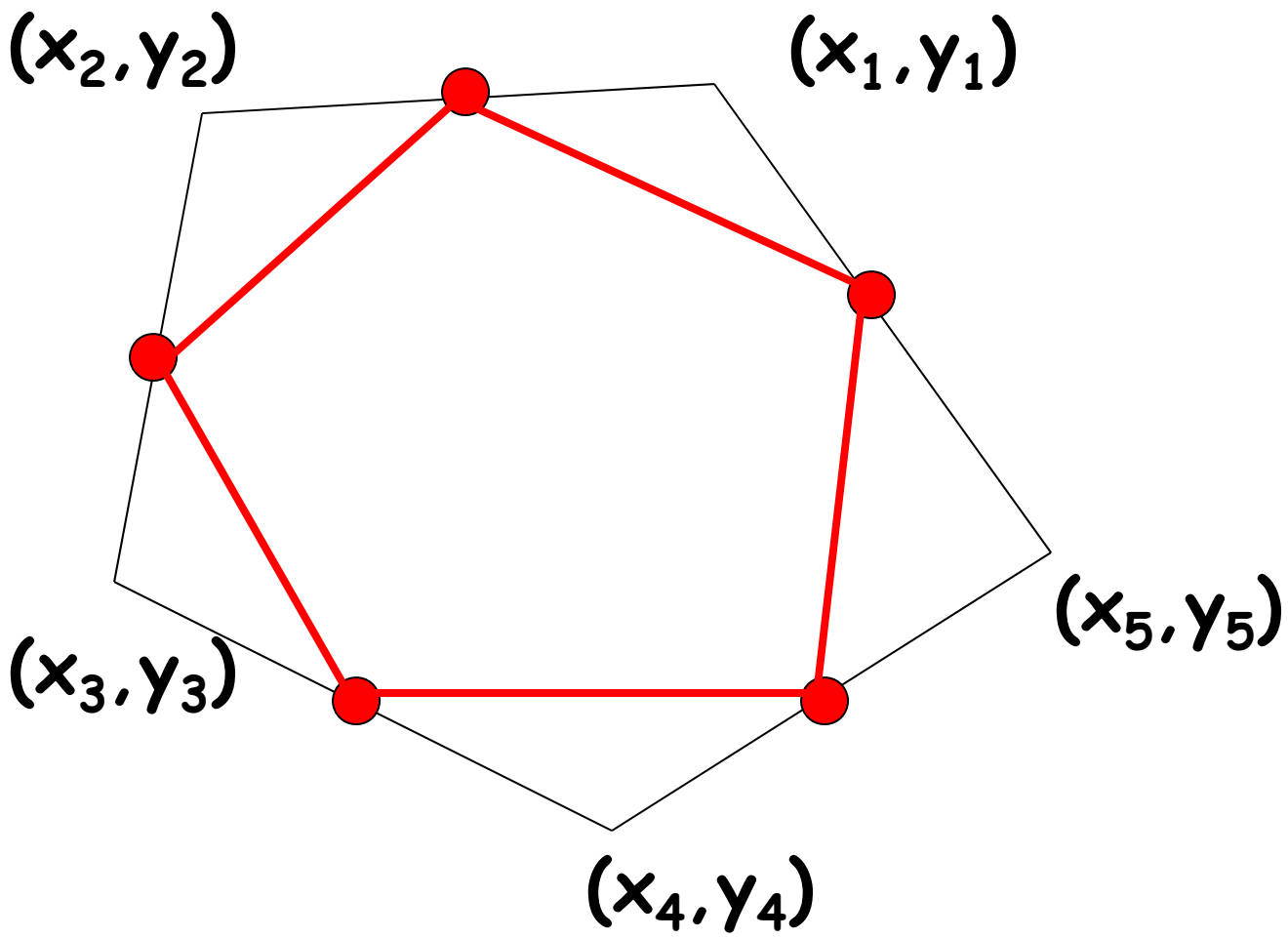


See **RandomWalk2D_v2.m**

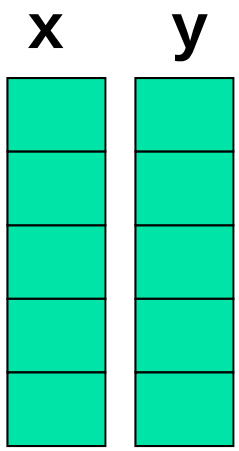
Example: polygon smoothing



Example: polygon smoothing

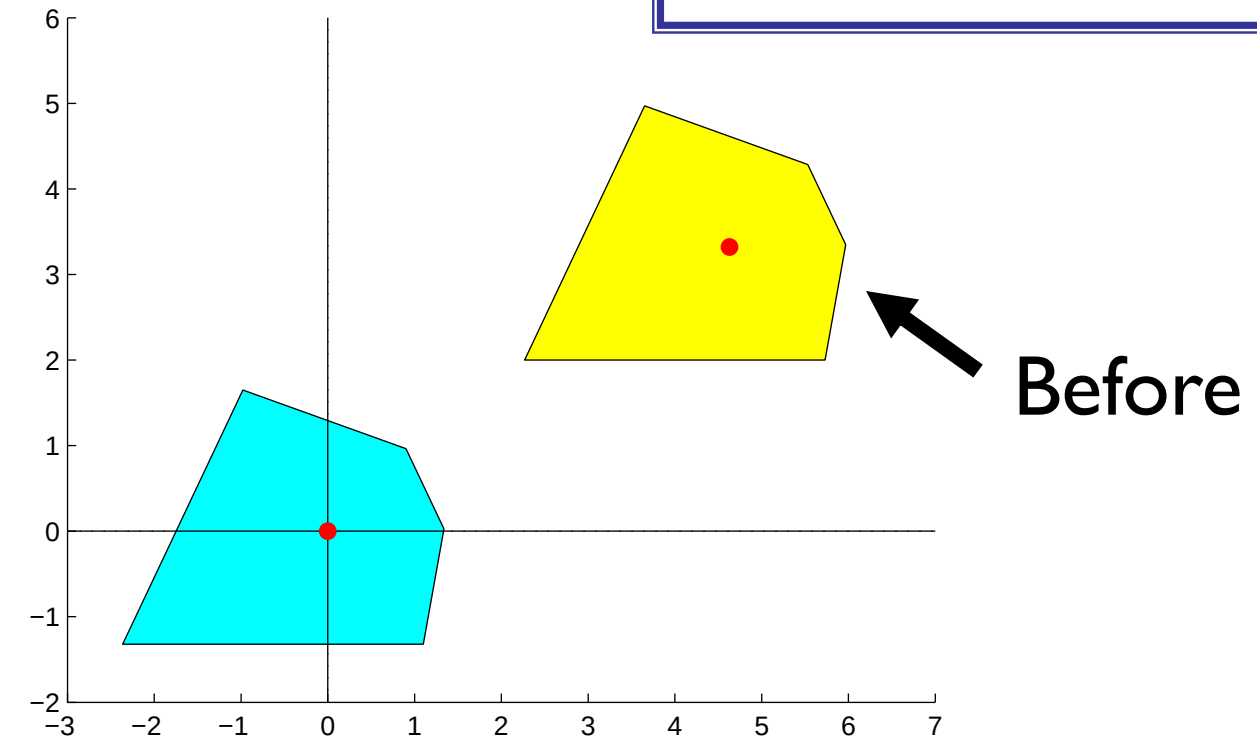


Can store the x-y coordinates in vectors x and y



First operation: centralize

Move a polygon so that the centroid of its vertices is at the origin



Before

After

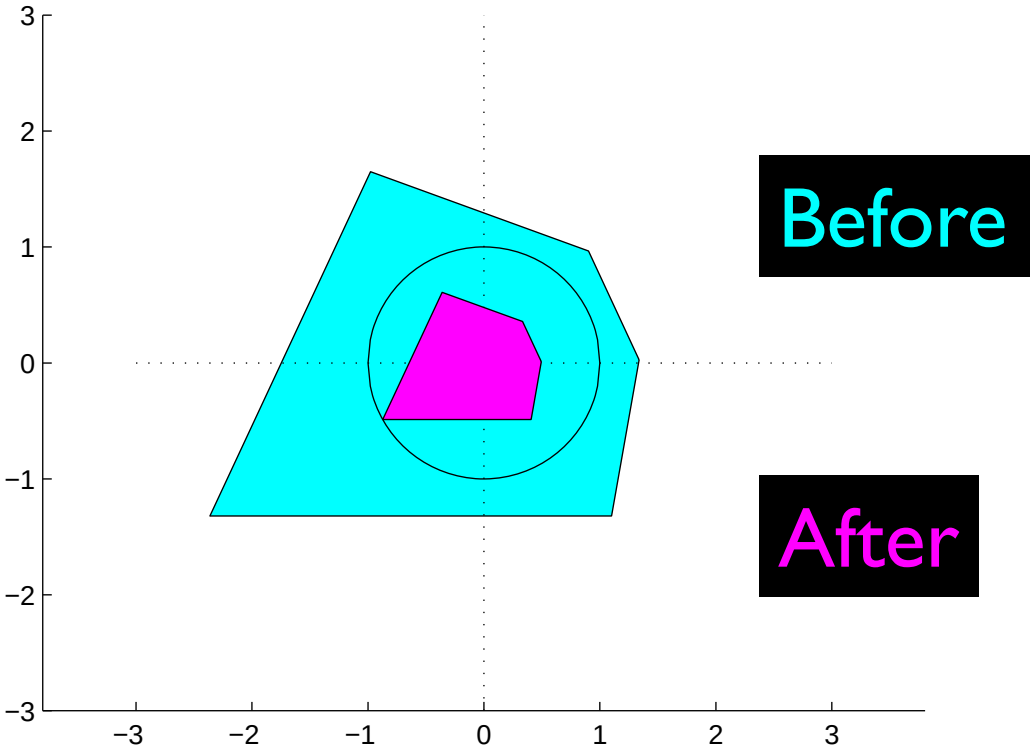
```
function [xNew,yNew] = Centralize(x,y)
% Translate polygon defined by vectors
% x,y such that the centroid is on the
% origin. New polygon defined by vectors
% xNew,yNew.
```

```
function [xNew,yNew] = Centralize(x,y)
% Translate polygon defined by vectors
% x,y such that the centroid is on the
% origin. New polygon defined by vectors
% xNew,yNew.

n = length(x);
xBar = sum(x)/n;    yBar = sum(y)/n;
xNew = zeros(n,1); yNew = zeros(n,1);
for k = 1:n
    xNew(k) = x(k)-xBar;
    yNew(k) = y(k)-yBar;
end
```

sum returns the sum of all values in the vector

Second operation: normalize

Shrink (enlarge) the polygon so that the vertex furthest from the (0,0) is on the unit circle



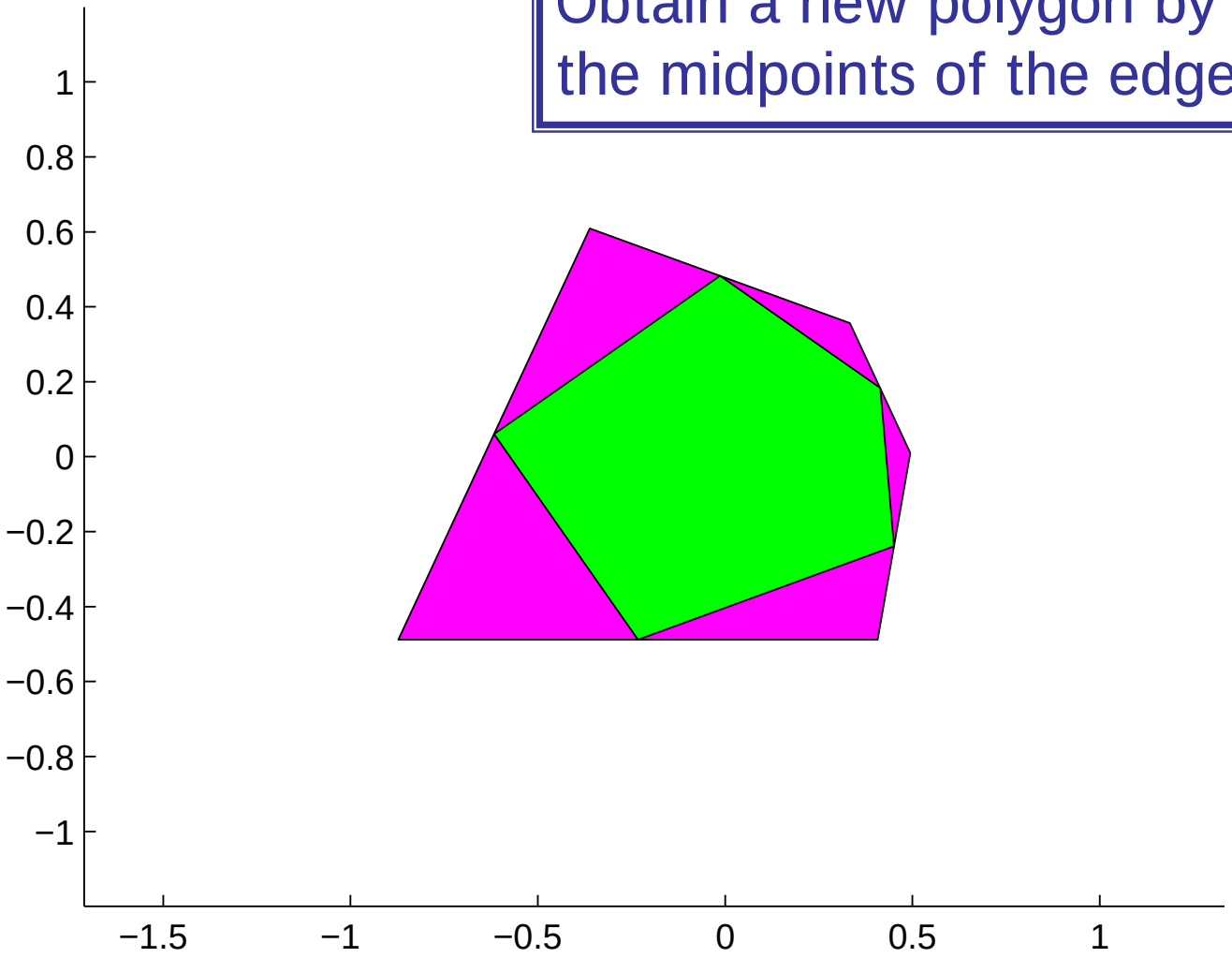
```
function [xNew,yNew] = Normalize(x,y)
% Resize polygon defined by vectors x,y
% such that distance of the vertex
% furthest from origin is 1
```

```
function [xNew,yNew] = Normalize(x,y)
% Resize polygon defined by vectors x,y
% such that distance of the vertex
% furthest from origin is 1

n = length(x);
for k = 1:n
   d(k) = sqrt(x(k)^2 + y(k)^2);
end
maxD = max(d);
xNew = zeros(n,1);  yNew = zeros(n,1);
for k = 1:n
   xNew(k)=x(k)/maxD; yNew(k)=y(k)/maxD;
end
```

Applied to a vector, max returns the largest value in the vector

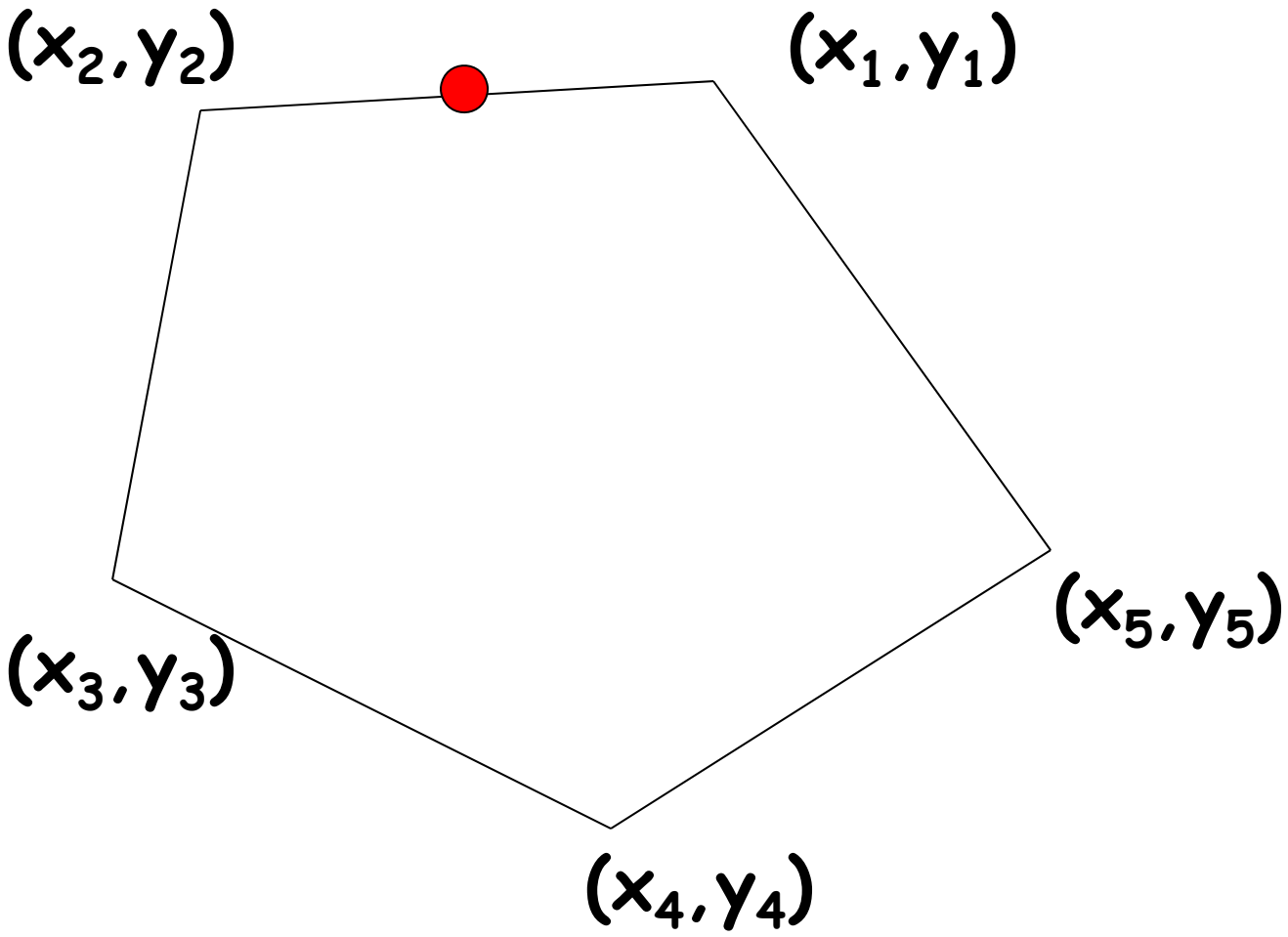
Third operation: smooth

Obtain a new polygon by connecting the midpoints of the edges

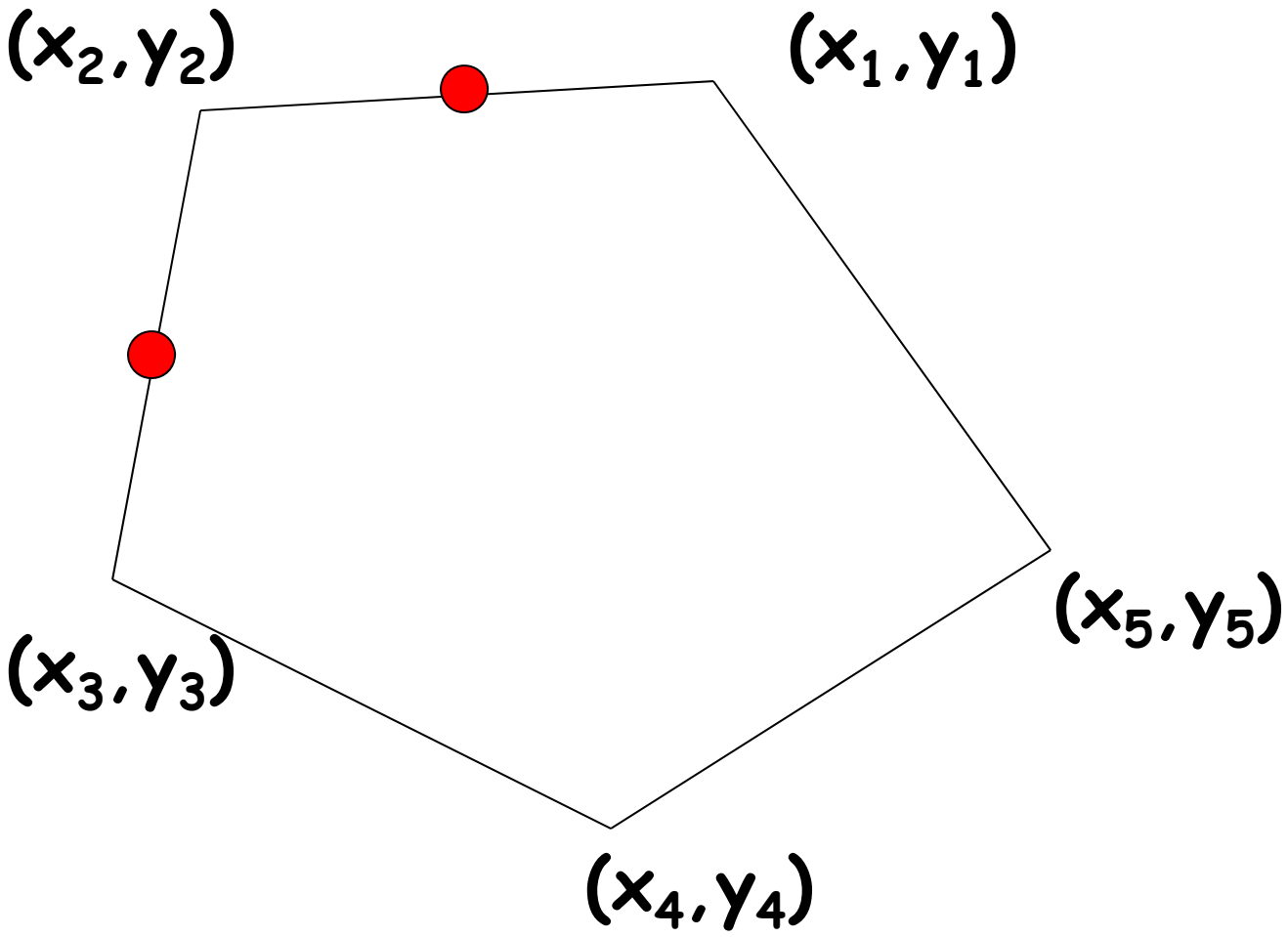


```
function [xNew,yNew] = Smooth(x,y)
% Smooth polygon defined by vectors x,y
% by connecting the midpoints of
% adjacent edges

n = length(x);
xNew = zeros(n,1);
yNew = zeros(n,1);
for i=1:n
  %Compute midpt of ith edge. Store in xNew(i), yNew(i)



end
```

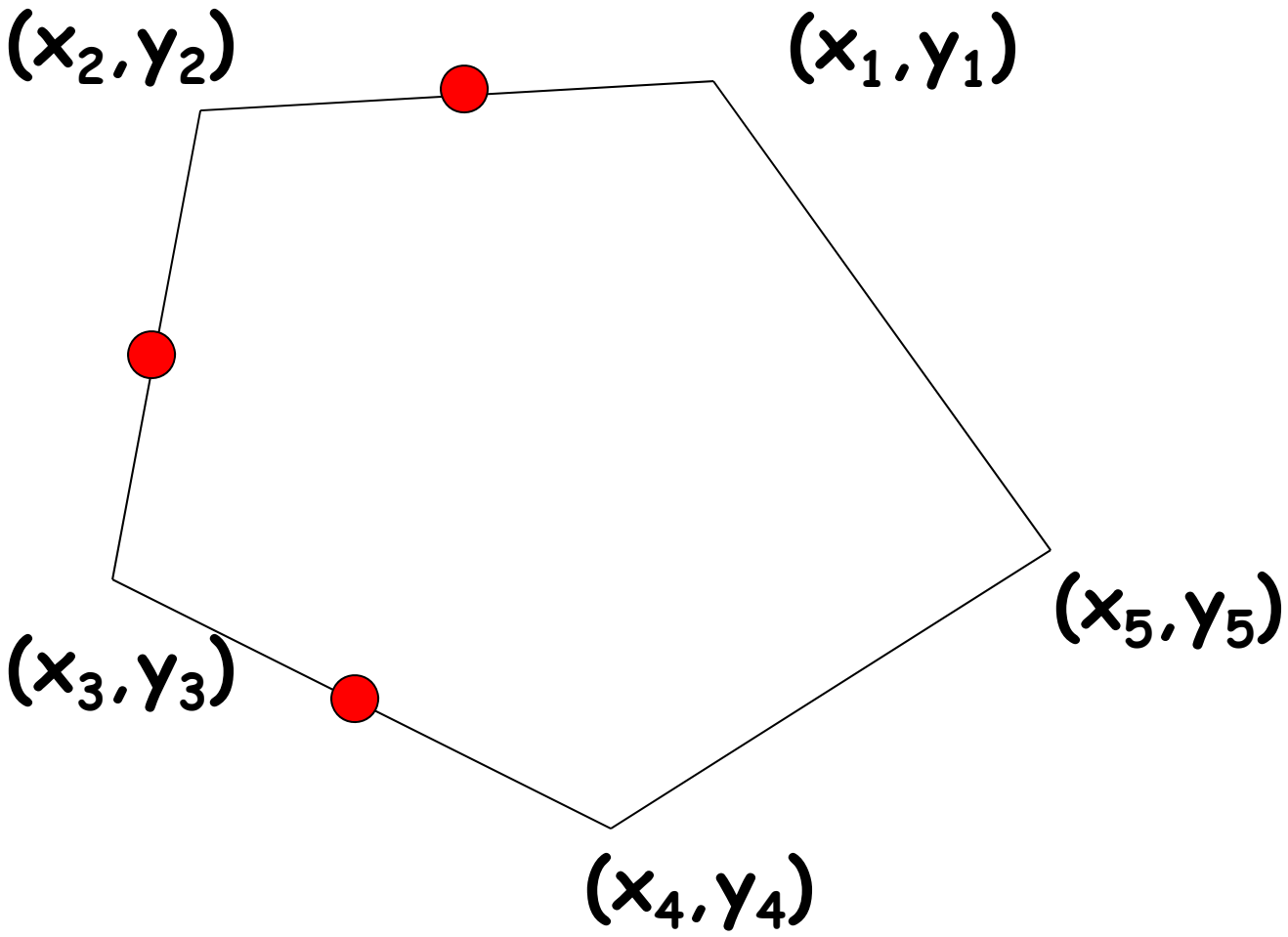
```
xNew(1) = (x(1)+x(2))/2
yNew(1) = (y(1)+y(2))/2
```



```
xNew(2) = (x(2)+x(3))/2
yNew(2) = (y(2)+y(3))/2
```



```
xNew(3) = (x(3)+x(4))/2
yNew(3) = (y(3)+y(4))/2
```



Polygon Smoothing

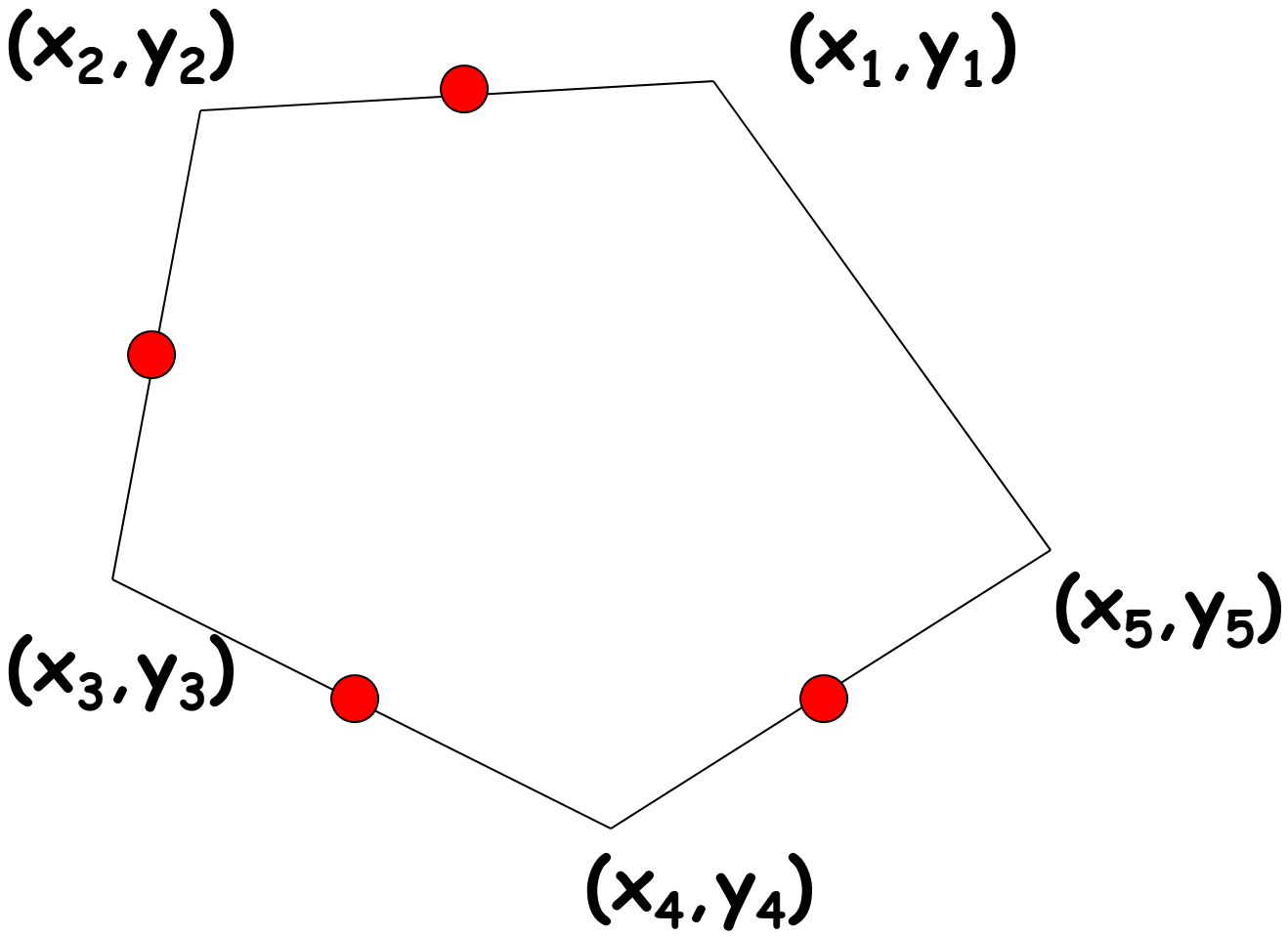
```
% Given n, x, y
for i=1:n
      xNew(i) = (x(i) + x(i+1))/2;
      yNew(i) = (y(i) + y(i+1))/2;
end
```

Does above fragment compute the new n-gon?

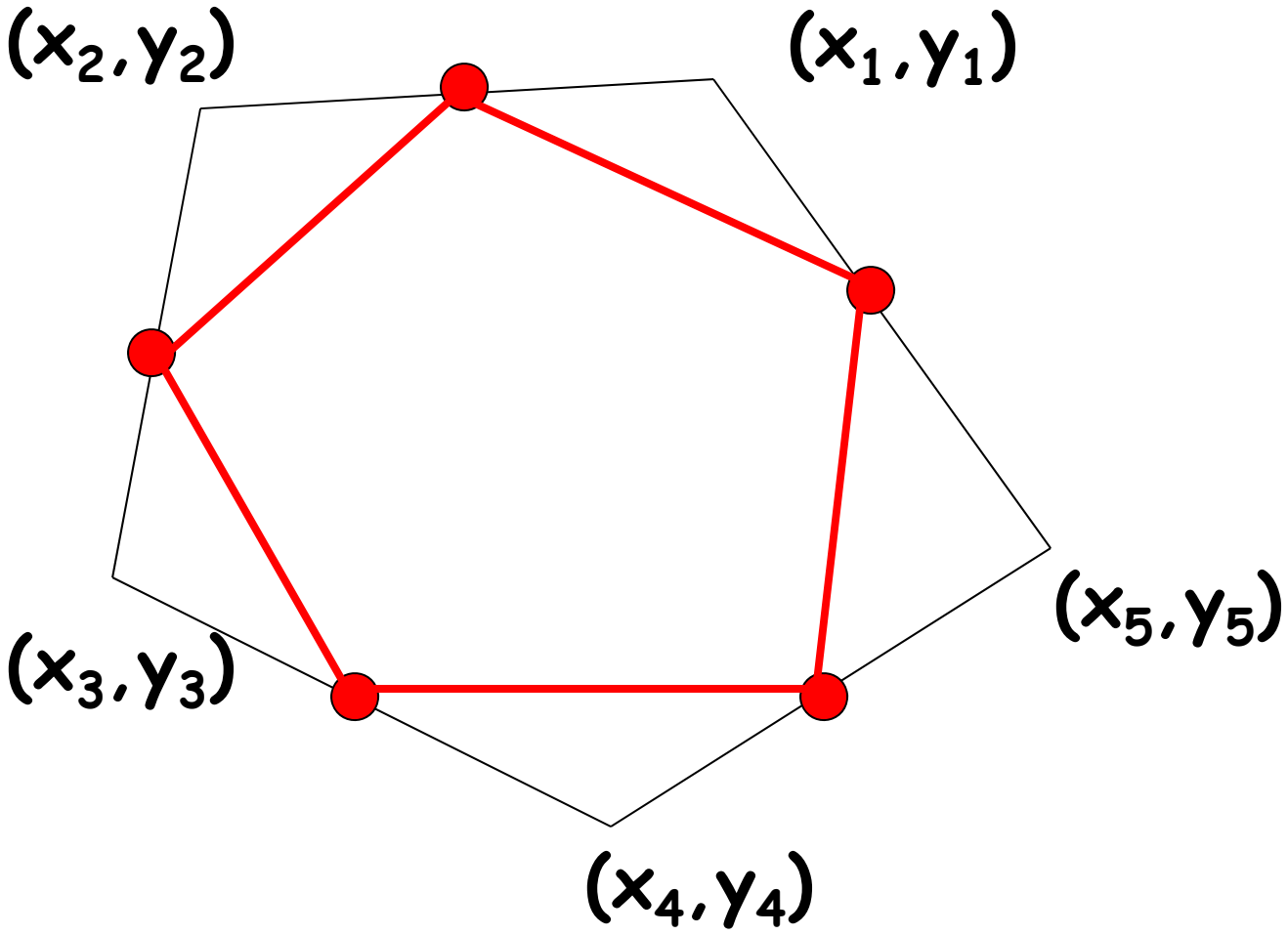
A: Yes

B: No

```
xNew(4) = (x(4)+x(5))/2
yNew(4) = (y(4)+y(5))/2
```



```
xNew(5) = (x(5)+x(1))/2
yNew(5) = (y(5)+y(1))/2
```



Smooth

```
for i=1:n
    xNew(i) = (x(i) + x(i+1))/2;
    yNew(i) = (y(i) + y(i+1))/2;
end
```

Will result in a subscript
out of bounds error when i is n.

Smooth

```
for i=1:n
    if i<n
      xNew(i) = (x(i) + x(i+1))/2;
      yNew(i) = (y(i) + y(i+1))/2;
    else
      xNew(n) = (x(n) + x(1))/2;
      yNew(n) = (y(n) + y(1))/2;
    end
end
```

Smooth

```
for i=1:n-1
      xNew(i) = (x(i) + x(i+1))/2;
      yNew(i) = (y(i) + y(i+1))/2;
end
xNew(n) = (x(n) + x(1))/2;
yNew(n) = (y(n) + y(1))/2;
```

Show a simulation of polygon smoothing

Create a polygon with randomly located vertices.

Repeat:

- Centralize
- Normalize
- Smooth

See **ShowSmooth.m**