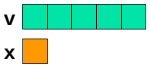
- Previous Lecture:
  - Recursion – partitioning a triangle
  - Insertion Sort
  - (Read about Bubble Sort in *Insight*)
- Today's Lecture:
  - "Divide and conquer" strategies
    - Binary search
    - Merge sort

Searching for an item in an unorganized collection?

- May need to look through the whole collection to find the target item
- E.g., find value x in vector v



- Linear search


```
% Linear Search
% f is index of first occurrence
%   of value x in vector v.
% f is -1 if x not found.
k= 1;
while  k<=length(v) && v(k)~=x
    k= k + 1;
end
if  k>length(v)
    f= -1; % signal for x not found
else
    f= k;
end
```

v | 12 | 35 | 33 | 15 | 42 | 45 |

x | 31 |


```
% Linear Search
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end
```

A. squared
B. doubled
C. the same
D. halved

Suppose another vector is twice as long as v. The expected "effort" required to do a linear search is …

```
% Linear Search
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% f is -1 if x not found.
k= 1;
while  k<=length(v) && v(k)~=x
    k= k + 1;
end
if  k>length(v)
    f= -1; % signal for x no
else
    f= k;
end
```

Searching in a sorted list should require less work

v | 12 | 15 | 33 | 35 | 42 | 45 |

x | 31 |

What if **v** is sorted?

An ordered (sorted) list

The Manhattan phone book has 1,000,000+ entries.

How is it possible to locate a name by examining just a tiny, tiny fraction of those entries?

Key idea of "phone book search": repeated halving

To find the page containing Pat Reed's number…

```
while (Phone book is longer than 1 page)
    Open to the middle page.
    if "Reed" comes before the first entry,
        Rip and throw away the 2nd half.
    else
        Rip and throw away the 1st half.
    end
end
```


What happens to the phone book length?

```
Original:      3000 pages
After 1 rip:   1500 pages
After 2 rips:   750 pages
After 3 rips:   375 pages
After 4 rips:   188 pages
After 5 rips:    94 pages
      :
After 12 rips:    1 page
```


Binary Search

Repeatedly halving the size of the "search space" is the main idea behind the method of binary search.

An item in a sorted array of length n can be located with just $\log_2 n$ comparisons.


```
% Linear Search
% f is index of first occurrence of value x in vector v.
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k= 1;
while  k<=length(v) && v(k)~=x
    k= k + 1;
end
if  k>length(v)
    f= -1; % signal for x not found
else
    f= k;
end
```

n comparisons against the target are needed in worst case, n=length(v).

Binary Search

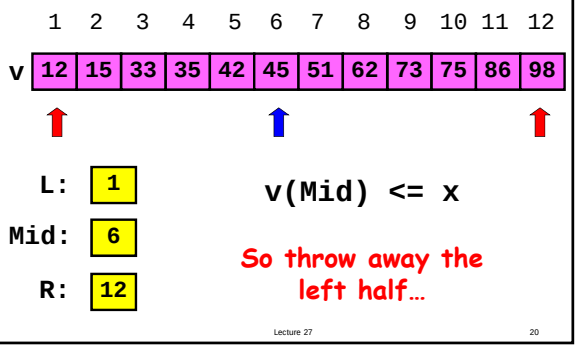
Repeatedly halving the size of the "search space" is the main idea behind the method of binary search.

An item in a sorted array of length n can be located with just $\log_2 n$ comparisons.

"Savings" is significant!

n	log2(n)
100	7
1000	10
10000	13

Binary search: target x = 70



Binary search: target x = 70

	1	2	3	4	5	6	7	8	9	10	11	12
v	12	15	33	35	42	45	51	62	73	75	86	98

L: 6
Mid: 9
R: 12

x < v(Mid)

So throw away the right half…


```
function L = binarySearch(x, v)
% Find position after which to insert x. v(1)<…<v(end).
% L is the index such that v(L) <= x < v(L+1);
% L=0 if x<v(1).  If x>v(end), L=length(v) but x~=v(L).

% Maintain a search window [L..R] such that v(L)<=x<v(R).
% Since x may not be in v, initially set ...
L=0;  R=length(v)+1;

% Keep halving [L..R] until R-L is 1,
%   always keeping  v(L) <= x < v(R)
while  R ~= L+1
    m= floor((L+R)/2);  % middle of search window
    if  v(m) <= x
        L= m;
    else
        R= m;
    end
end
```

20	30	40	46	50	52	68	70
0	1	2	3	4	5	6	7	8	9

Binary search is efficient, but we need to sort the vector in the first place so that we can use binary search

- Many different algorithms out there...
- We saw insertion sort (and read about bubble sort)
- Let's look at **merge sort**
- An example of the "divide and conquer" approach using recursion

Merge sort: Motivation

If I have two helpers, I'd…

- Give each helper half the array to sort
- Then I get back the sorted subarrays and merge them.

What if those two helpers each had two sub-helpers?

And the sub-helpers each had two sub-sub-helpers? And…

Subdivide the sorting task

H	E	M	G	B	K	A	Q	F	L	P	D	R	C	J	N

H	E	M	G	B	K	A	Q	F	L	P	D	R	C	J	N

Subdivide again

H	E	M	G	B	K	A	Q	F	L	P	D	R	C	J	N

H	E	M	G	B	K	A	Q	F	L	P	D	R	C	J	N

```
function y = mergeSort(x)
% x is a vector.  y is a vector
% consisting of the values in x
% sorted from smallest to largest.

n = length(x);
if n==1
      y = x;
else
      m  = floor(n/2);
      yL = mergeSort(x(1:m));
      yR = mergeSort(x(m+1:n));
      y  = merge(yL,yR);
end
```

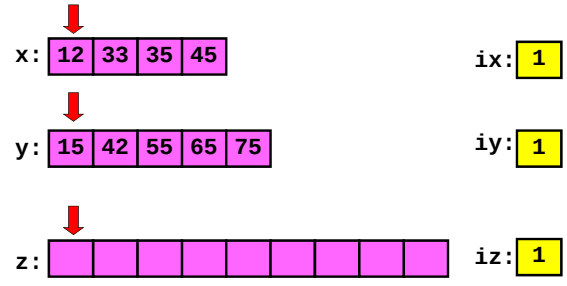

The central sub-problem is the merging of two sorted arrays into one single sorted array

12	33	35	45

15	42	55	65	75

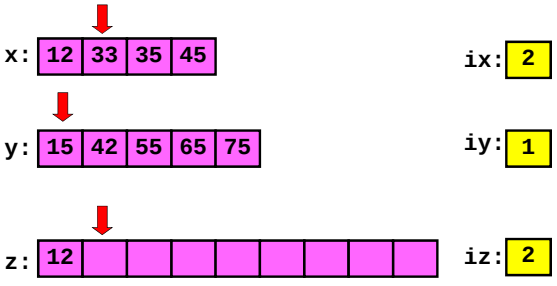
12	15	33	35	42	45	55	65	75

Merge



ix<=4 and iy<=5: x(ix) <= y(iy) ???

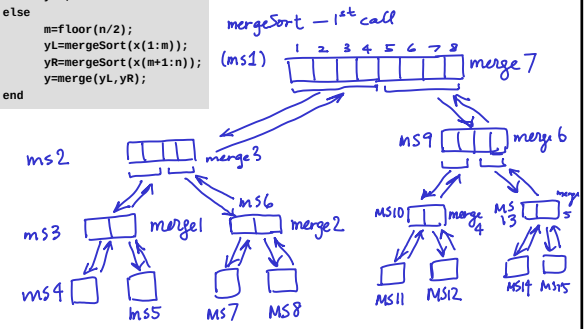
Merge



ix<=4 and iy<=5: x(ix) <= y(iy) ???

```
function z = merge(x,y)
nx = length(x); ny = length(y);
z = zeros(1, nx+ny);
ix = 1; iy = 1; iz = 1;
while ix<=nx && iy<=ny
    if  x(ix) <= y(iy)
        z(iz)= x(ix);  ix=ix+1;  iz=iz+1;
    else
        z(iz)= y(iy);  iy=iy+1;  iz=iz+1;
    end
end
while ix<=nx  % copy remaining x-values
  z(iz)= x(ix);  ix=ix+1;  iz=iz+1;
end
while iy<=ny  % copy remaining y-values
  z(iz)= y(iy);  iy=iy+1;  iz=iz+1;
end
```

```
function y=mergeSort(x)
n=length(x);
if n==1
      y=x;
else
      m=floor(n/2);
      yL=mergeSort(x(1:m));
      yR=mergeSort(x(m+1:n));
      y=merge(yL,yR);
end
```



How do merge sort, insertion sort, and bubble sort compare?

- Insertion sort and bubble sort are similar
  - Both involve a series of comparisons and swaps
  - Both involve nested loops
- Merge sort uses recursion


```
function x = insertSort(x)
% Sort vector x in ascending order with insertion sort

n = length(x);
for i= 1:n-1
   % Sort x(1:i+1) given that x(1:i) is sorted
   j= i;
   need2swap= x(j+1) < x(j);
   while  need2swap

      % swap x(j+1) and x(j)
      temp= x(j);
      x(j)= x(j+1);
      x(j+1)= temp;

      j= j-1;
      need2swap= j>0 && x(j+1)<x(j);
   end
end
```

Insertion sort is more efficient than bubble sort on average—fewer comparisons (see *Insight*)

How do merge sort and insertion sort compare?

- Insertion sort: (worst case) makes i comparisons to insert an element in a sorted array of i elements. For an array of length N:

  _____ for big N

- Merge sort: _____
- Insertion sort is done *in-place*; merge sort (recursion) requires much more memory

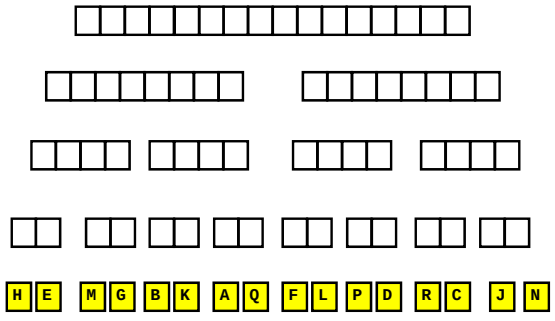

```
function y = mergeSort(x)
% x is a vector.  y is a vector
% consisting of the values in x
% sorted from smallest to largest.

n = length(x);
if n==1
      y = x;
else
      m  = floor(n/2);
      yL = mergeSort(x(1:m));
      yR = mergeSort(x(m+1:n));
      y  = merge(yL,yR);
end
```

All the comparisons between vector values are done in **merge**


```
function z = merge(x,y)
nx = length(x); ny = length(y);
z = zeros(1, nx+ny);
ix = 1; iy = 1; iz = 1;
while ix<=nx && iy<=ny
    if  x(ix) <= y(iy)
        z(iz)= x(ix);  ix=ix+1;  iz=iz+1;
    else
        z(iz)= y(iy);  iy=iy+1;  iz=iz+1;
    end
end
while ix<=nx  % copy remaining x-values
  z(iz)= x(ix);  ix=ix+1;  iz=iz+1;
end
while iy<=ny  % copy remaining y-values
  z(iz)= y(iy);  iy=iy+1;  iz=iz+1;
end
```


Merge sort: $\log_2(N)$ "levels"; N comparisons each level



How to choose??

- Depends on application
- Merge sort is especially good for sorting large data set (but watch out for memory usage)
- Insertion sort is "order N^2" at worst case, but what about an average case? If the application requires that you *maintain* a sorted array, insertion sort may be a good choice

Why not just use Matlab's sort function?

- Flexibility
- E.g., to maintain a sorted list, just write the code for insertion sort
- E.g., sort strings or other complicated structures
- Sort according to some criterion set out in a function file
  - Observe that we have the comparison `x(j+1)<x(j)`
  - The comparison can be a function that returns a boolean value
- Can combine different sort/search algorithms for specific problem

What we learned…

- Develop/implement algorithms for problems
- Develop programming skills
  - Design, implement, document, test, and debug
- Programming "tool bag"
  - Functions for reducing redundancy
  - Control flow (if-else; loops)
  - Recursion
  - Data structures
  - Graphics
  - File handling

What we learned… (cont'd)

- Applications and concepts
  - Image processing
  - Object-oriented programming
  - Sorting and searching—you should know the algorithms covered
  - Divide-and-conquer strategies
  - Approximation and error
  - Simulation
  - Computational effort and efficiency

Computing gives us *insight* into a problem

- Computing is not about getting one answer!
- We build models and write programs so that we can "play" with the models and programs, learning—gaining insights—as we vary the parameters and assumptions
- Good models require domain-specific knowledge (and experience)
- Good programs …
  - are modular and cleanly organized
  - are well-documented
  - use appropriate data structures and algorithms
  - are reasonably efficient in time and memory

Final Exam

- Friday 12/7, 9-11:30am, Barton West
- Covers entire course; some emphasis on material after Prelim 2
- Closed-book exam, no calculators
- Bring student ID card

- Check for announcements on webpage:
  - Study break office/consulting hours
  - Review session time and location
  - Review questions
  - List of potentially useful functions