

CS1112 Discussion Exercise 13

Efficient calculation of x^n where n is large

If you cannot use MATLAB's power operator \wedge how would you calculate x to the n -th power? One way is to use iteration—a loop that executes $n - 1$ times. Another strategy is recursion—repeated squaring in this case. The idea is illustrated with the following schematic that shows how to compute x^{21} :

$$\begin{array}{l} x^{21} = (x^{10})^2 \cdot x \\ \quad \downarrow \\ \quad x^{10} = (x^5)^2 \\ \qquad \quad \downarrow \\ \qquad \quad x^5 = (x^2)^2 \cdot x \\ \qquad \qquad \quad \downarrow \\ \qquad \qquad \quad x^2 = (x)^2 \end{array}$$

The recursive definition behind the scenes is given by

$$f(x, n) = \begin{cases} 1 & \text{if } n = 0 \\ f(x, n/2) \cdot f(x, n/2) & \text{if } n > 0 \text{ and } n \text{ is even} \\ f(x, (n-1)/2) \cdot f(x, (n-1)/2) \cdot x & \text{if } n > 0 \text{ and } n \text{ is odd} \end{cases}$$

Write the following function based on the *recursive* strategy. Do not use loops.

```
function y = Power(x, n)
% y = x^n where n is an integer >=0
```

Use any remaining time to work on Project 6 Part B!