Announcements

- P6 due today at 11pm
- Final exam:
  - Thurs, 5/12, 9am, Barton East (indoor field)
- Please fill out course evaluation on-line, see "Exercise 15"
- Regular office/consulting hours end tonight. Revised hours next week.
- Pick up papers during consulting hours at Carpenter
- Read announcements on course website!

- Previous Lecture:
  - Recursion review
  - Efficiency
- Today's Lecture:
  - Simulation—Google "page rank"
  - Optimization—the traveling salesperson problem

Quantifying Importance

How do you rank web pages for importance given that you know the link structure of the Web, i.e., the in-links and out-links for each web page?

A related question:

How does a deleted or added link on a webpage affect its "rank"?

Background

Index all the pages on the Web from 1 to n. (n is around ten billion.)

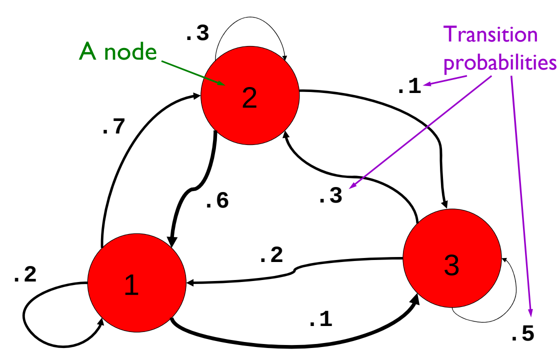
The PageRank algorithm orders these pages from "most important" to "least important."

It does this by analyzing links, not content.

Key ideas

- There is a random web surfer—a special random walk
- The surfer has some random "surfing" behavior—a transition probability matrix
- The transition probability matrix comes from the link structure of the web—a connectivity matrix
- Applying the transition probability matrix → Page Rank

A 3-node network with specified transition probabilities



A special random walk

Suppose there are a 1000 people on each node.

At the sound of a whistle they hop to another node in accordance with the "outbound" probabilities.

For now we assume we know these probabilities. Later we will see how to get them.

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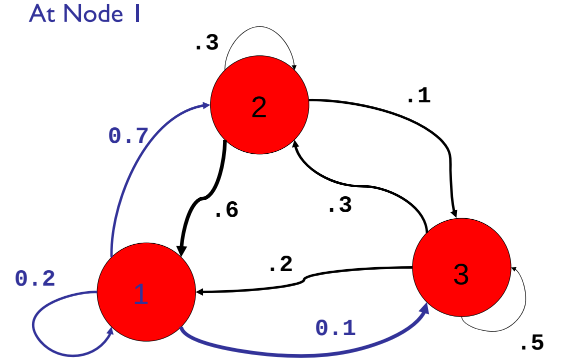
At Node 1



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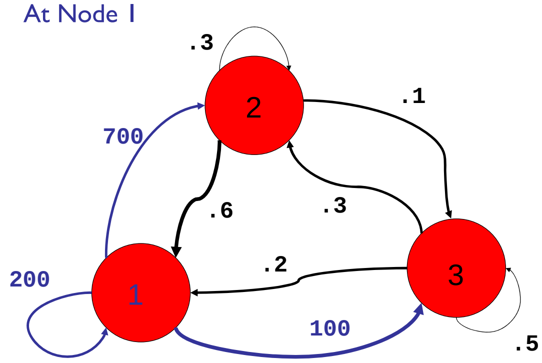
At Node 1



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State Vector:
describes the state at each node at a specific time

T=0	T=1	T=2
1000	1000	1120
1000	1300	1300
1000	700	580

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After 100 iterations

	T=99	T=100
Node 1	1142.85	1142.85
Node 2	1357.14	1357.14
Node 3	500.00	500.00

Appears to reach a steady state

Call this the stationary vector

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Formula for the new state vector

P

.2	.6	.2
.7	.3	.3
.1	.1	.5

`P(i,j)` is probability of hopping to node `i` from node `j`

```
w(1) = P(1,1)*v(1) + P(1,2)*v(2) + P(1,3)*v(3)
w(2) = P(2,1)*v(1) + P(2,2)*v(2) + P(2,3)*v(3)
w(3) = P(3,1)*v(1) + P(3,2)*v(2) + P(3,3)*v(3)
```

`v` is the old state vector
`w` is the updated state vector

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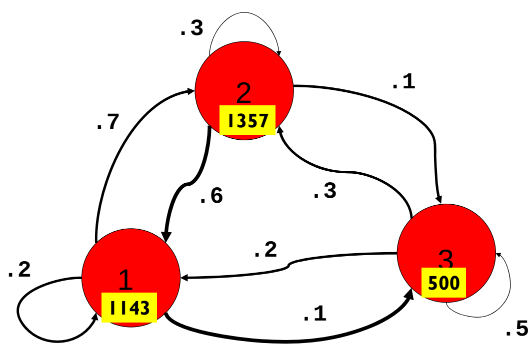
The general case

```
function w = Update(P,v)
% Update state vector v based on transition
% probability matrix P to give state vector w
n = length(v);
w = zeros(n,1);
for i=1:n
   for j=1:n
      w(i) = w(i) + P(i,j)*v(j);
   end
end
```


To obtain the stationary vector…

```
function [w,err]= StatVec(P,v,tol,kMax)
% Iterate to get stationary vector w
w = Update(P,v);
err = max(abs(w-v));
k = 1;
while k<kMax && err>tol
       v = w;
       w = Update(P,v);
       err = max(abs(w-v));
       k = k+1;
end
```


Stationary vector indicates importance: 2 1 3



A random walk on the web

Repeat:
You are on a webpage.
There are m outlinks,
so choose one at random.
Click on the link.

Random island hopping

Repeat:
You are on an island.
According to the transitional probabilities,
go to another island.

Use the link structure of the web to figure out the transitional probabilities!

(Assume no dead ends for now; we deal with them later.)

Connectivity Matrix

G

0	0	0	0	0	0	1	1
1	0	0	1	0	0	0	0
1	0	1	0	0	1	0	1
0	0	0	0	1	0	0	0
1	0	1	0	0	0	0	1
0	0	1	0	0	0	0	1
0	0	1	0	0	0	0	0
0	1	0	1	0	0	0	0

`G(i,j)` is **1** if there is a link on page `j` to page `i`.

(I.e., you can get to `i` from `j`.)

Connectivity Matrix

G

0	0	0	0	0	0	1	1
1	0	0	1	0	0	0	0
1	0	1	0	0	1	0	1
0	0	0	0	1	0	0	0
1	0	1	0	0	0	0	1
0	0	1	0	0	0	0	1
0	0	1	0	0	0	0	0
0	1	0	1	0	0	0	0

Transition Probability Matrix

derived from Connectivity Matrix

P

0	0	0	0	0	0	?	?
?	0	0	?	0	0	0	0
?	0	?	0	0	?	0	?
0	0	0	0	?	0	0	0
?	0	?	0	0	0	0	?
0	0	?	0	0	0	0	?
0	0	?	0	0	0	0	0
0	?	0	?	0	0	0	0

Connectivity Matrix

G

0	0	0	0	0	0	1	1
1	0	0	1	0	0	0	0
1	0	1	0	0	1	0	1
0	0	0	0	1	0	0	0
1	0	1	0	0	0	0	1
0	0	1	0	0	0	0	1
0	0	1	0	0	0	0	0
0	1	0	1	0	0	0	0

Transition Probability

A. 0
B. 1/8
C. 1/3
D. 1
E. rand(1)

P

0	0	0	0	0	0	?	?
?	0	0	?	0	0	0	0
?	0	?	0	0	?	0	?
0	0	0	0	?	0	0	0
?	0	?	0	0	0	0	?
0	0	?	0	0	0	0	?
0	0	?	0	0	0	0	0
0	?	0	?	0	0	0	0

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Connectivity (G) → Transition Probability (P)

```
[n,n] = size(G);
P = zeros(n,n);
for j=1:n
   P(:,j) = G(:,j)/sum(G(:,j));
end
```

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To obtain the stationary vector…

```
function [w,err]= StatVec(P,v,tol,kMax)
% Iterate to get stationary vector w
w = Update(P,v);
err = max(abs(w-v));
k = 1;
while k<kMax && err>tol
      v = w;
      w = Update(P,v);
      err = max(abs(w-v));
      k = k+1;
end
```

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Stationary vector represents how "popular" the pages are → PageRank

statVec	sorted	idx	pR
0.5723	0.8911	6	4
0.8206	0.8206	2	2
0.7876	0.7876	3	3
0.2609	0.5723	1	6
0.2064	0.4100	8	8
0.8911	0.2609	4	1
0.2429	0.2429	7	7
0.4100	0.2064	5	5

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```
[sorted, idx] = sort(-statVec);
for k= 1:length(statVec)
   j = idx(k);  % index of kth largest
   pR(j) = k;
end
```

statVec	sorted	idx	pR
0.5723	-0.8911	6	4
0.8206	-0.8206	2	2
0.7876	-0.7876	3	3
0.2609	-0.5723	1	6
0.2064	-0.4100	8	8
0.8911	-0.2609	4	1
0.2429	-0.2429	7	7
0.4100	-0.2064	5	5

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The random walk idea gets the transitional probabilities from connectivity. So how to deal with dead ends?

Repeat:

You are on a webpage.
There are m outlinks.
Choose one at random.
Click on the link.

What if there are no outlinks?

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The random walk idea gets transitional probabilities from connectivity. Can modify the random walk to deal with dead ends.

```
Repeat:
    You are on a webpage.
    If there are no outlinks
        Pick a random page and go there.
    else
        Flip an unfair coin.
        if heads
            Click on a random outlink and go there.
        else
            Pick a random page and go there.
        end
    end
```

In practice, an unfair coin with prob .85 heads works well.

This results in a different transitional probability matrix.

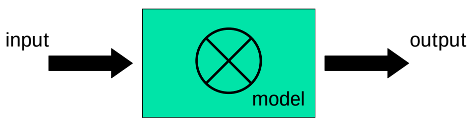
Quantifying Importance

How do you rank web pages for importance given that you know the link structure of the Web, i.e., the in-links and out-links for each web page?

A related question:

How does a deleted or added link on a webpage affect its "rank"?

Simulation vs. optimization



- Simulation: Specify input, run it through the model, and get result.
- Optimization is the reverse problem: Specify the result that you want—your objectives—and try to find what "starting values" or actions are needed

Optimization

- Find the "best" of something
  - the shortest path
  - the most cost efficient production line
  - the lowest-risk investment strategy
- There is a search (solution) space
- There is some kind of objective function
- There are usually constraints
- Usually willing to accept suboptimal solution if it is "good enough" and is cheap to compute

The Traveling Salesperson Problem (TSP)

A salesperson must travel to a set of cities exactly once and then return to the starting city. What is the shortest path?

Enumerate all the possibilities and pick the shortest

What is the best itinerary to visit Boston, Miami, LA, Dallas?

1	2	3	4
1	2	4	3
1	3	2	4
1	3	4	2
1	4	2	3
1	4	3	2

3! = 6 possibilities

Enumerate all the possibilities and pick the shortest

What is the best itinerary to visit Boston, Miami, LA, Dallas?

3! = 6 possibilities

Add Seattle, NYC Austin, Denver

7! = 5040

If a computer can process 1 billion itineraries a second, how long does it take to solve a 100-city problem?

A heuristic is a computational rule-of-thumb that points us towards optimality but without any guarantee that optimality will actually be achieved.

- A heuristic for the TSP:

  From the current location, choose to visit the *nearest unvisited* city

---

Organization of the TSP program

```
% Visit n cities, starting from city 1
Put cities 2:n in unvisited list
for k= 2:n
   Find nearest unvisited city, c
   Put city c in the tour path
   Remove city c from unvisited list
end
Return to city 1
```

What we learned…

- Develop/implement algorithms for problems
- Develop programming skills
  - Design, implement, document, test, and debug
- Programming "tool bag"
  - Functions for reducing redundancy
  - Control flow (if-else; loops)
  - Recursion
  - Data structures
  - Graphics
  - File handling

What we learned… (cont'd)

- Applications and concepts
  - Image and sound
  - Sorting and searching—you should know the algorithms covered
  - Divide-and-conquer strategies
  - Approximation and error
  - Simulation
  - Computational effort and efficiency

Final Exam

- Thurs 5/12, 9-11:30am, Barton East
- Covers entire course; some emphasis on material after Prelim 3
- Closed-book exam, no calculators
- Bring student ID card

- Check for announcements on webpage:
  - Study break office/consulting hours
  - Review session time and location
  - Review questions
  - List of potentially useful functions