- Previous Lecture:
  - Linear Search
  - Bubble Sort, Insertion Sort

- Today's Lecture:
  - "Divide and conquer" strategies
    - Binary search
    - Merge sort
    - Recursion

- Announcements:
  - Discussion this week in classrooms
  - Prelim 3 will be returned at end of lecture. If your paper isn't here, pick it up from CS1112 consultants in ACCEL during consulting hrs (Sunday to Thrusdays 5-10pm)
  - Project 6 due May 5th. Part 1 posted; Part 2 to be posted later.

Other efficiency considerations

- Worst case, best case, average case
- Use of subfunction incurs an "overhead"
- Memory use and access

- Example: Rather than directing the *insert* process to a subfunction, have it done "in-line."
- Also, Insertion sort can be done "in-place," i.e., using "only" the memory space of the original vector.

An ordered (sorted) list

The Manhattan phone book has 1,000,000+ entries.

How is it possible to locate a name by examining just a tiny, tiny fraction of those entries?

wide at SuperPages.com 195 Car C

Cartage New England Inc
26 Allen Ln Ipswich 01938 978 356-9960
Cartagema Lydia
18 Jewett Ros 02131 617 323-7639
Cartagena Avith
9 Bancroft Rox 02119 617 442-9780
B Hyd 02136 617 361-5253
Jessica 50 Decatur Cha 02129 617 241-0152
Lucilla 174 Harvard Cam 02139 617 491-5621
M 95 Rowe Ros 02131 617 323-9713
Melvin 501 Green Cam 02139 617 576-1061
Carte Nicholas
18 Appleton Boston 02116 617 695-6996
Cartegena O 4 Milford Bos 02118 617 338-8219
Carten Thos J Sr & Claire
1 Paradise Rd Mil 02186 617 698-6163
Thomas & Kathleen
50 Thompson Ln Mil 02186 617 696-6919
Carter A Ros 02131 617 327-2257
A Roxbury 617 442-5230
A 31 Bethune Wy Roxbury 02119 617 442-1219
A 260 Putnam Av Cambridge 02139 617 492-4174
A M 255 Maschsts Av Bos 02115 617 266-7153
Adams 361 Centre St Mil 02186 617 698-9074
Alice 108 Kilmarnock Bos 02215 617 425-0193
Alice 45 Market Cambridge 02139 617 945-2711
Andrew F 62 Vinal Av Som 02143 617 625-7623
Carter Anne MD
1101 Beacon Bro 02446 617 739-1022
Carter Athens
272 Newbury Boston 02116 617 536-6329
B E 68 Gladeside Av Mat 02126 617 296-6911
Carter Barbara L MD
Tufts-New England Medical Center Bos 02111
Call 617 636-0051
Carter Becky Bos 02114 617 523-4368
Bernard J
112 Gladstone E Bos 02128 617 567-3430
Bithiah 25 Medway Dor 02124 617 298-8713
Blake 26 Mt Vernon Bos 02108 617 367-9931
Carter Broadcasting Co
20 Park Plz Bos 02116 617 423-0210
Carter & Burgess Consultants Inc
23 East St Cam 02141 617 225-0200
Carter C 2000 Commwlth Av Bri 02135 617 782-2118
C 228 Faywood Av East Boston 02128 617 569-1545
C 359 Harvard Cam 02138 617 491-4822
C 610 Walk Hill Mat 02126 617 296-6392
C & M 43 Burroughs Jam 02130 617 524-9558
Carter F 24 Hillock Ros 02131 617 327-1105
Faye & Ricky
357 Columbus Av Bos 02116 617 437-7331
Francis S 134 Temple W Rox 02132 617 323-6781
Franklin & Anne
221 Mt Auburn Cam 02138 617 354-0798
Fred 42 Haverford Jam 02130 617 524-3078
Fred 96 Hinckley Rd Mil 02186 617 698-1343
G & R 8 Verdun Dor 02124 617 436-8906
G T 27 Franklin Av Som 02145 617 623-7121
Gayle 25 Frontenac Dor 02124 617 825-0322
Geo S 115 Moss Hill Rd Jam 02130 617 522-3215
George 125 Nashua Bos 02114 617 367-9548
Carter Halliday Associate
107 S Street Bos 02111 617 456-1689
Carter Harry F
26 Running Brk Rd W Rox 02132 617 325-5465
Carter Hide Co Inc
146 Summer Bos 02110 617 542-7987
Carter Hilary 61 Harvey Cam 02140 617 876-2750
Horace
241 Walnut Av Roxbury 02119 617 442-5307
Howard Jr 26 Notre Dme Rox 02119 617 445-5552
J Cam 617 354-2688
J 15 Chatham Bro 02446 617 232-7990
J 518 Harvard Bro 02446 617 730-9483
J 775 Yfw Pkwy West Roxbury 02132 617 323-5574
Carter J Jacques MD
1 Brookline Pl Bro 02446 617 735-8787
Carter J M
1410 Columbia Rd S Bos 02127 617 464-1040
Carter J M Ornamental Ironworks
Call Pembroke TelNo 617 436-5353
Carter J Veal Co
48 Newmarket Sq Rox 02118 617 442-1775
Carter James
1573 Cambridge St Cam 02138 617 492-1214
James 182 Fisher Av Roxbury 02120 617 739-2193
James
37 Gold Star Rd Cambridge 02140 617 876-8841
Jas L 14 Roseberry Rd Mat 02126 617 361-0773
Jane 114 Adena Rd Newton 02465 617 964-0435
Jeffrey 41 Warren Av Bos 02116 617 426-5994
John 11 Mansfield Bri 02134 617 987-2163
John 327 Summer Bos 02210 617 423-4334
John 40 Westwind Rd Dor 02125 617 282-1235
June O 329 A Summit Av Bri 02135 617 734-6109
K 38 Browning Av Dorchester 02124 617 265-8456
K 17 Esmond Dorchester 02121 617 282-1593
Carter Nella E
333 Maschsts Av Bos 02115 617 267-6483
Nicholas S F
115 Randolph Av Mil 02186 617 698-5307
Nick 21 Fairfield Bos 02116 617 267-5222
Nick & Debbi
196 Herrick Rd Newton 02459 617 527-0480
Nicole 617 698-0713
Norman G
38 Chickatawbut Dor 02122 617 822-1203
P 94 Crestwood Pk Rox 02121 617 427-4754
P E 501 E Sixth S Bos 02127 617 268-4213
P L 44 Hutchings Rox 02121 617 427-9170
P R 91 Bynner Jam 02130 617 983-8692
Paul & Constance
114 Anawan Av W Rox 02132 617 325-2036
Paul E 501 E Sixth St S Bos 02127 617 268-4546
Paul M 27 Union Bri 02135 617 787-2115
Carter Pile Driving Inc 17 Beaver Ct
Framingham 01702 Wellesley TelNo 781 235-8488
Carter Prudence
46 Franklin Watertown 02172 617 393-3782
Prudence
46 Franklin Watertown 02172 617 926-7063
Reginald
106 Brunswick Dorchester 02121 617 541-2843
Renee & Andrew
10 Walnut Bos 02108 617 720-3765
Carter Rice Dowd
Bulkley Dunton Publishing 163 Main Wilmington 01887
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Cust Svc-Industrial Prod 613 Main Wilmington
Toll Free-Dial '1' & Then 800 619-7447
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Toll Free-Dial '1' & Then 800 648-7447
Headquarters 613 Main Wilmington 01887
Call 978 988-7447
Ingalls Cronin 163 Main Wilmington 01887
Toll Free-Dial '1' & Then 800 638-1673
Carter Richard
1079 Commwlth Av Brighton 02215 617 987-0836
Richard A 97 Mt Vernon Bos 02108 617 566-7293
Carter Richard A MD
170 Commwlth Av Bos 02116 617 267-0710
Carter Richard K
15 Mercer S Bos 02127 617 268-0448
Robert L 175 Richdale Av Cam 02140 617 864-1535
Roger 150 St Botolph Bos 02115 617 424-6148
Roy 44 Concord Av Cam 02138 617 491-6115
Royce 18 Seminary Cha 02129 617 241-0418

```
% Linear Search
% f is index of first occurrence of value x in vector v.
% f is -1 if x not found.
k= 1;
while  k<=length(v) && v(k)~=x
    k= k + 1;
end
if  k>length(v)
    f= -1; % signal for x not found
else
    f= k;
end
```

Searching in an unsorted list

v	12	35	33	15	42	45

x	31

Key idea of "phone book search": repeated halving

To find the page containing Pat Reed's number…

```
while  (Phone book is longer than 1 page)
        Open to the middle page.
        if  "Reed" comes before the first entry,
                Rip and throw away the 2nd half.
        else
                Rip and throw away the 1st half.
        end
end
```

What happens to the phone book length?

```
Original:       3000 pages
After 1 rip:    1500 pages
After 2 rips:    750 pages
After 3 rips:    375 pages
After 4 rips:    188 pages
After 5 rips:     94 pages
       :
After 12 rips:     1 page
```

Binary Search

Repeatedly halving the size of the "search space" is the main idea behind the method of binary search.

An item in a sorted array of length n can be located with just $\log_2 n$ comparisons.

```
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% f is index of first occurrence of value x in vector v.
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k= 1;
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    k= k + 1;
end
if  k>length(v)
    f= -1; % signal for x not found
else
    f= k;
end
```
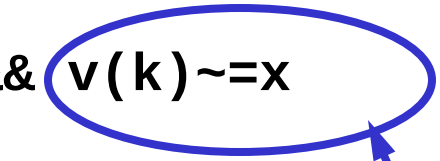



n comparisons against the target are needed in worst case, n=length(v).

Binary Search

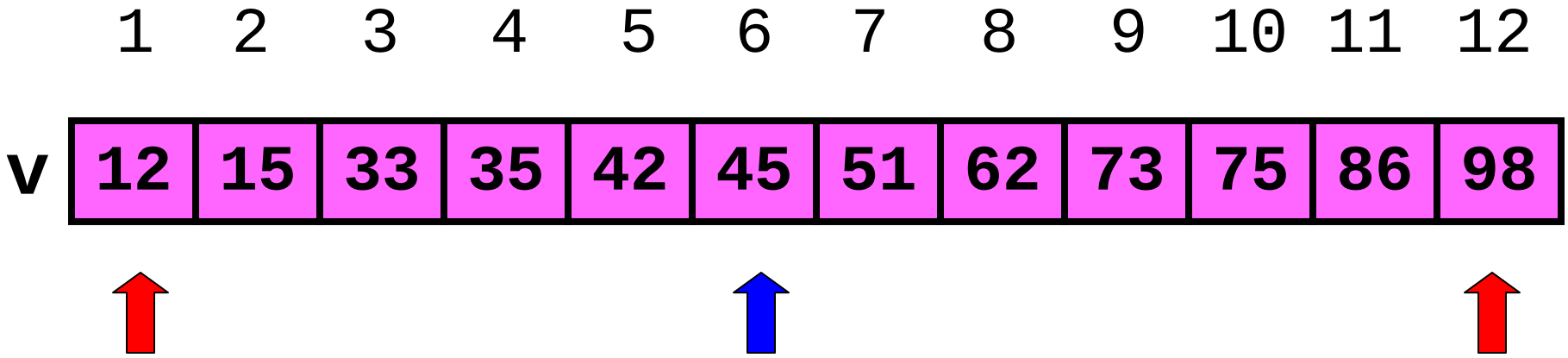
Repeatedly halving the size of the "search space" is the main idea behind the method of binary search.

An item in a sorted array of length n can be located with just $\log_2 n$ comparisons.

"Savings" is significant!

n	log2(n)
100	7
1000	10
10000	13

Binary search: target x = 70



L: 1

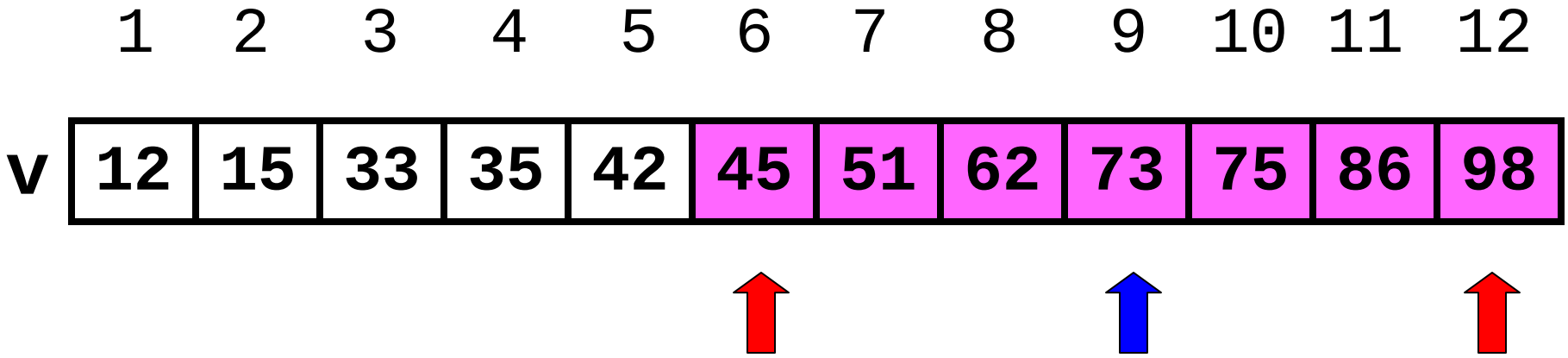
Mid: 6

R: 12

v(Mid) <= x

So throw away the left half…

Binary search: target x = 70



L: 6

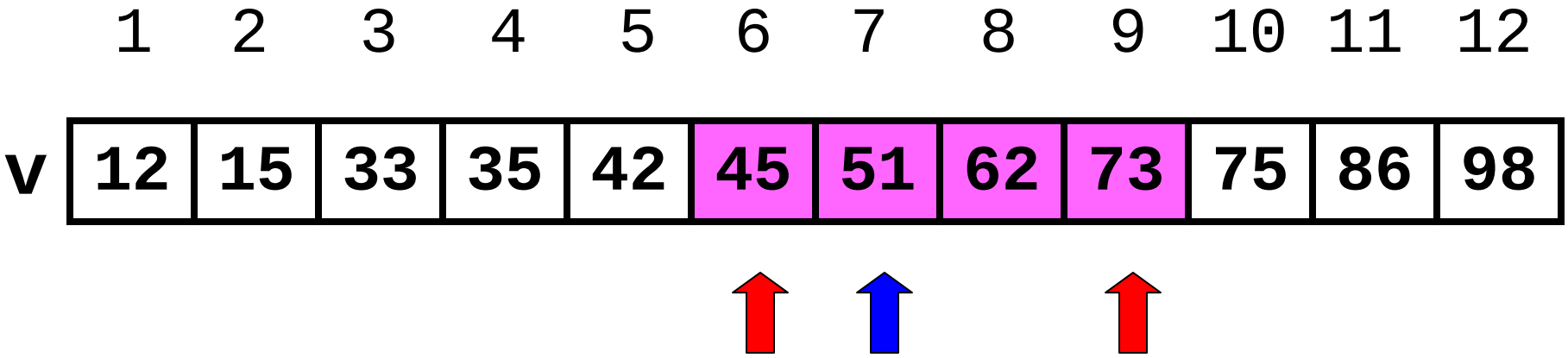
Mid: 9

R: 12

x < v(Mid)

So throw away the right half…

Binary search: target x = 70



L: 6

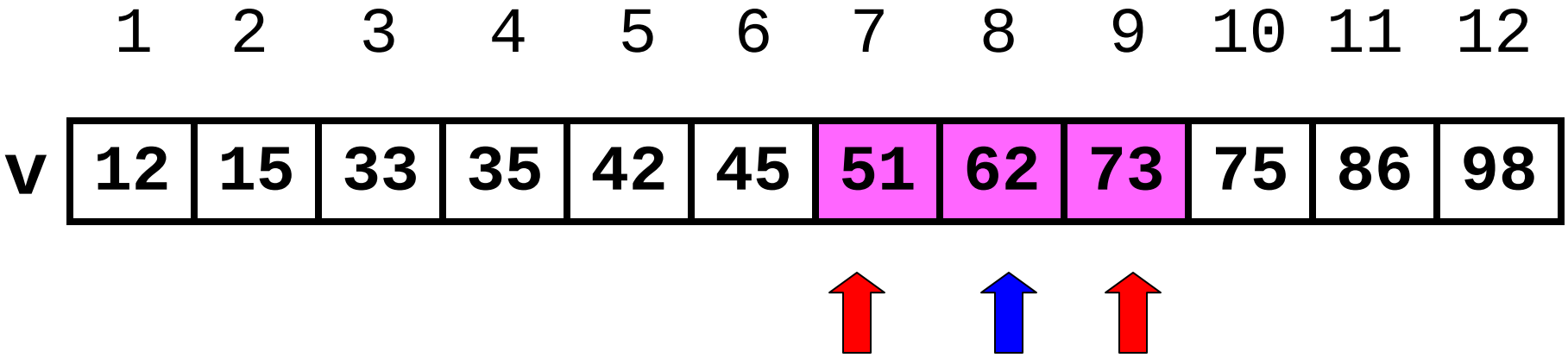
Mid: 7

R: 9

v(Mid) <= x

So throw away the left half…

Binary search: target x = 70



L: 7

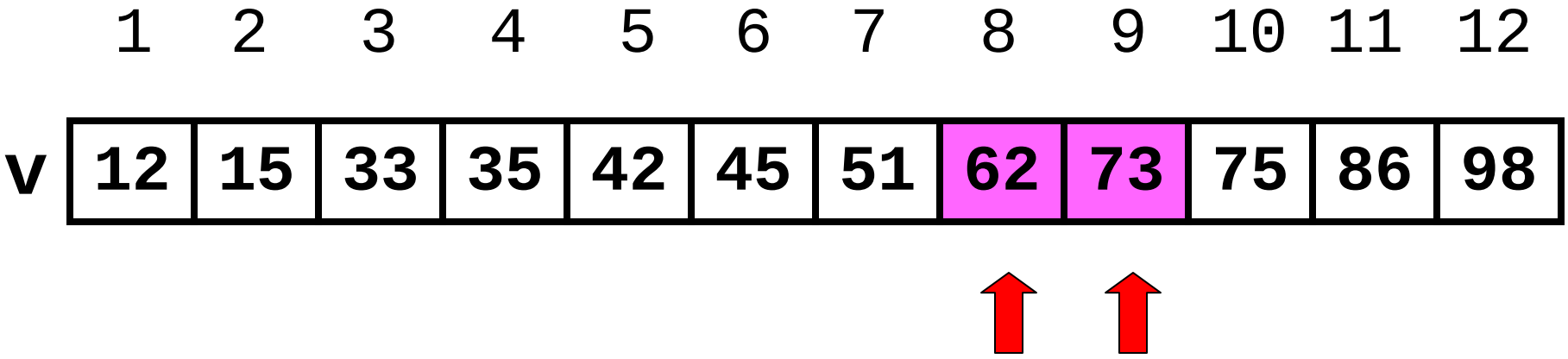
Mid: 8

R: 9

v(Mid) <= x

So throw away the left half…

Binary search: target x = 70



L: 8

Mid: 8

R: 9

Done because

R-L = 1

```
function L = binarySearch(x, v)
% Find position after which to insert x. v(1)<…<v(end).
% L is the index such that v(L) <= x < v(L+1);
% L=0 if x<v(1).  If x>v(end), L=length(v) but x~=v(L).

% Maintain a search window [L..R] such that v(L)<=x<v(R).
% Since x may not be in v, initially set ...
L=0;  R=length(v)+1;

% Keep halving [L..R] until R-L is 1,
%   always keeping  v(L) <= x < v(R)
while  R ~= L+1
    m= floor((L+R)/2);  % middle of search window
    if



    else



    end
end
```

```
function L = binarySearch(x, v)
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% Keep halving [L..R] until R-L is 1,
%   always keeping  v(L) <= x < v(R)
while  R ~= L+1
    m= floor((L+R)/2);  % middle of search window
    if  v(m) <= x

        L= m;
    else

        R= m;
    end
end
```

This version is different from that in *Insight*

```
function L = binarySearch(x, v)
% Find position after which to insert x. v(1)<…<v(end).
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while  R ~= L+1
    m= floor((L+R)/2);  % middle of search window
    if


    else


    end
end
```
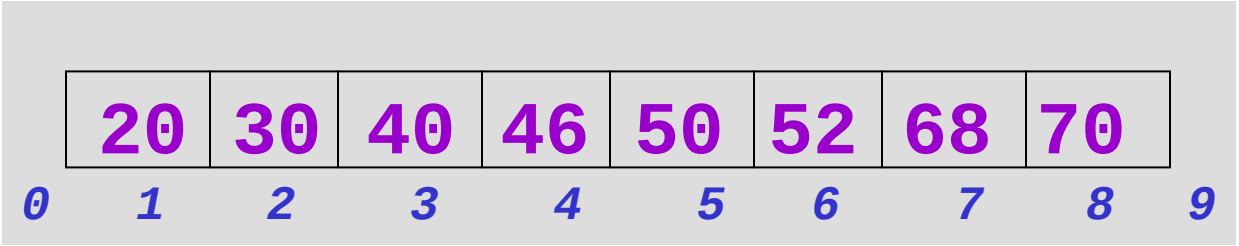



Binary search is efficient, but how do we sort a vector in the first place so that we can use binary search?

- Many different algorithms out there...
- We saw bubble sort and insertion sort
- Let's look at **merge sort**
- An example of the "divide and conquer" approach
- We'll compare their efficiency later

Which task is "easier," sort a length 1000 array or merge* two length 500 sorted arrays into one?

A. Sort

B. Merge

*Merge two sorted arrays so that the resultant array is sorted

Merge sort: Motivation

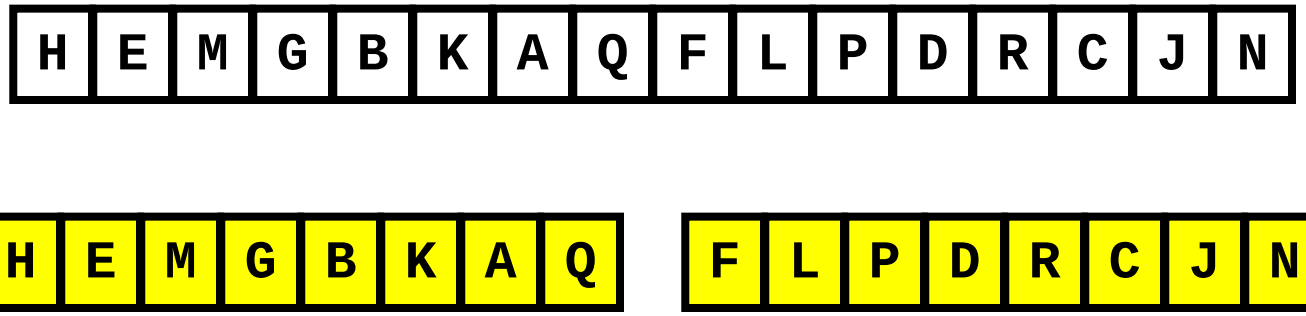
If I have two helpers, I'd…

- Give each helper half the array to sort
- Then I get back the sorted subarrays and merge them.

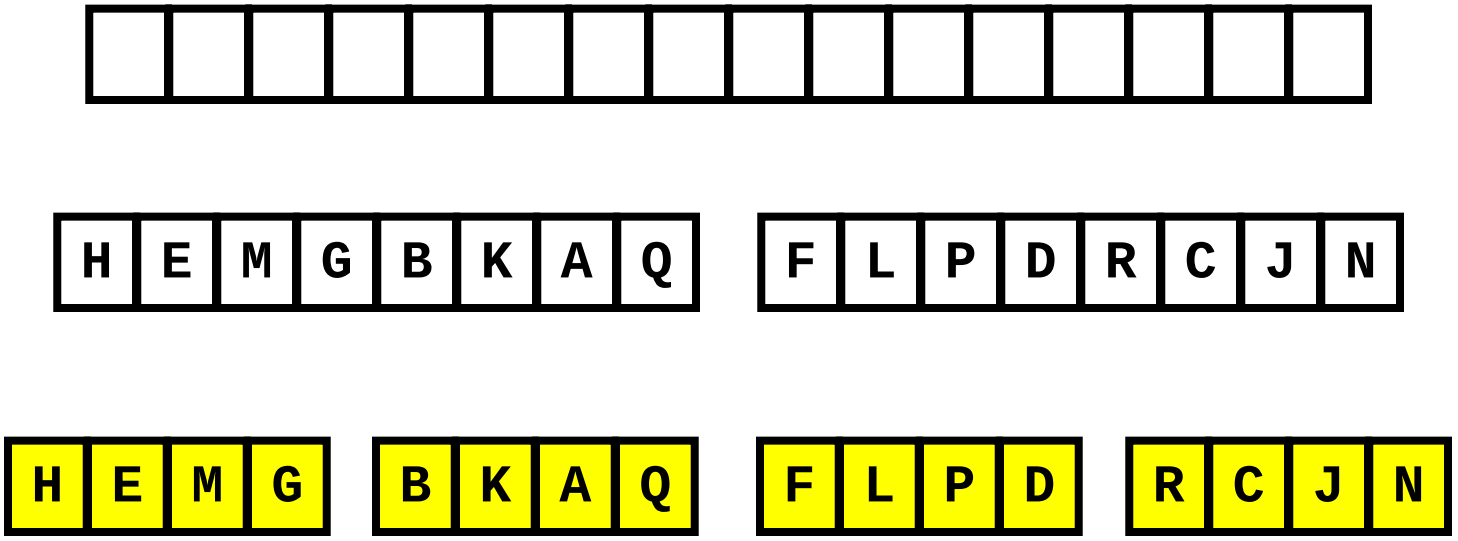
What if those two helpers each had two sub-helpers?

And the sub-helpers each had two sub-sub-helpers? And…

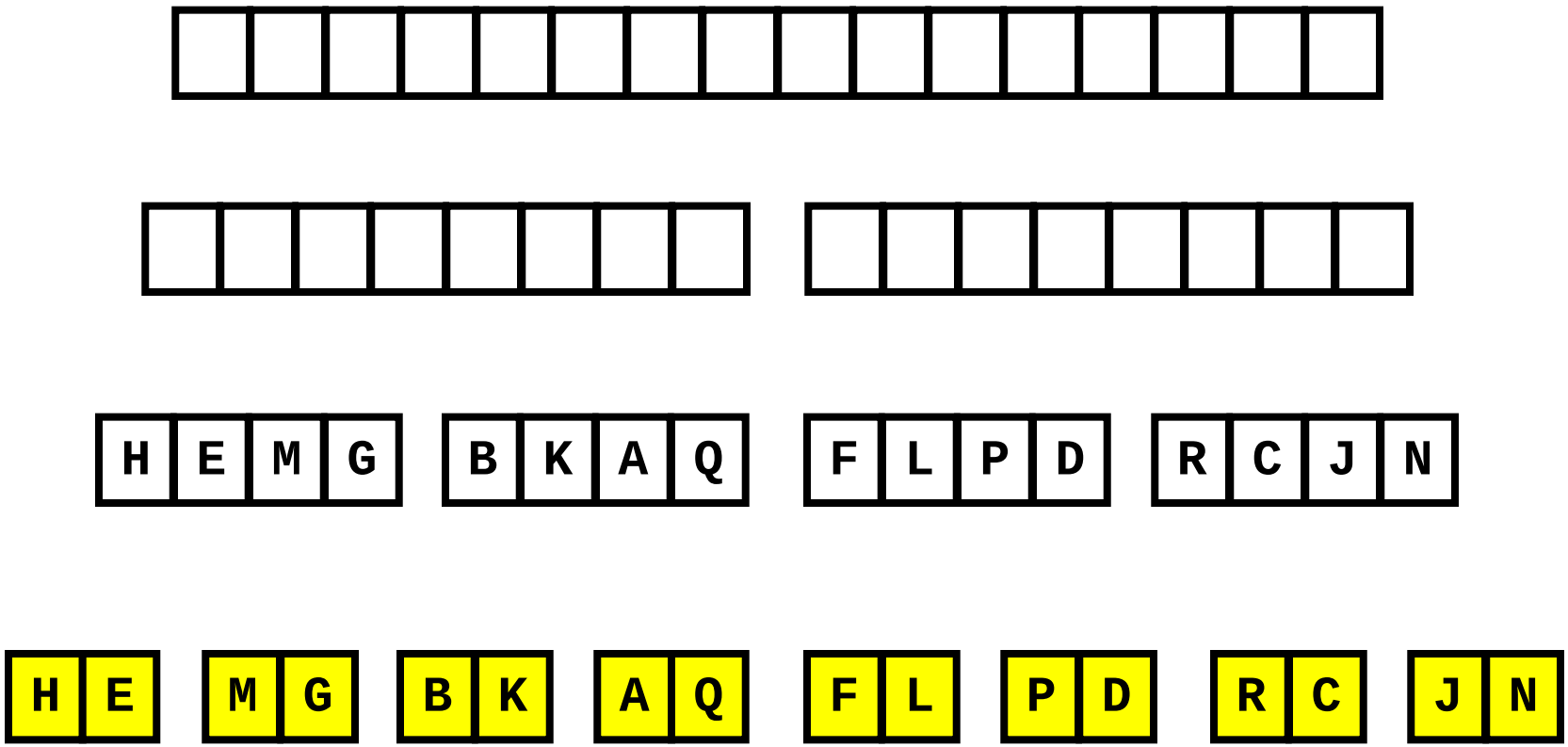
Subdivide the sorting task



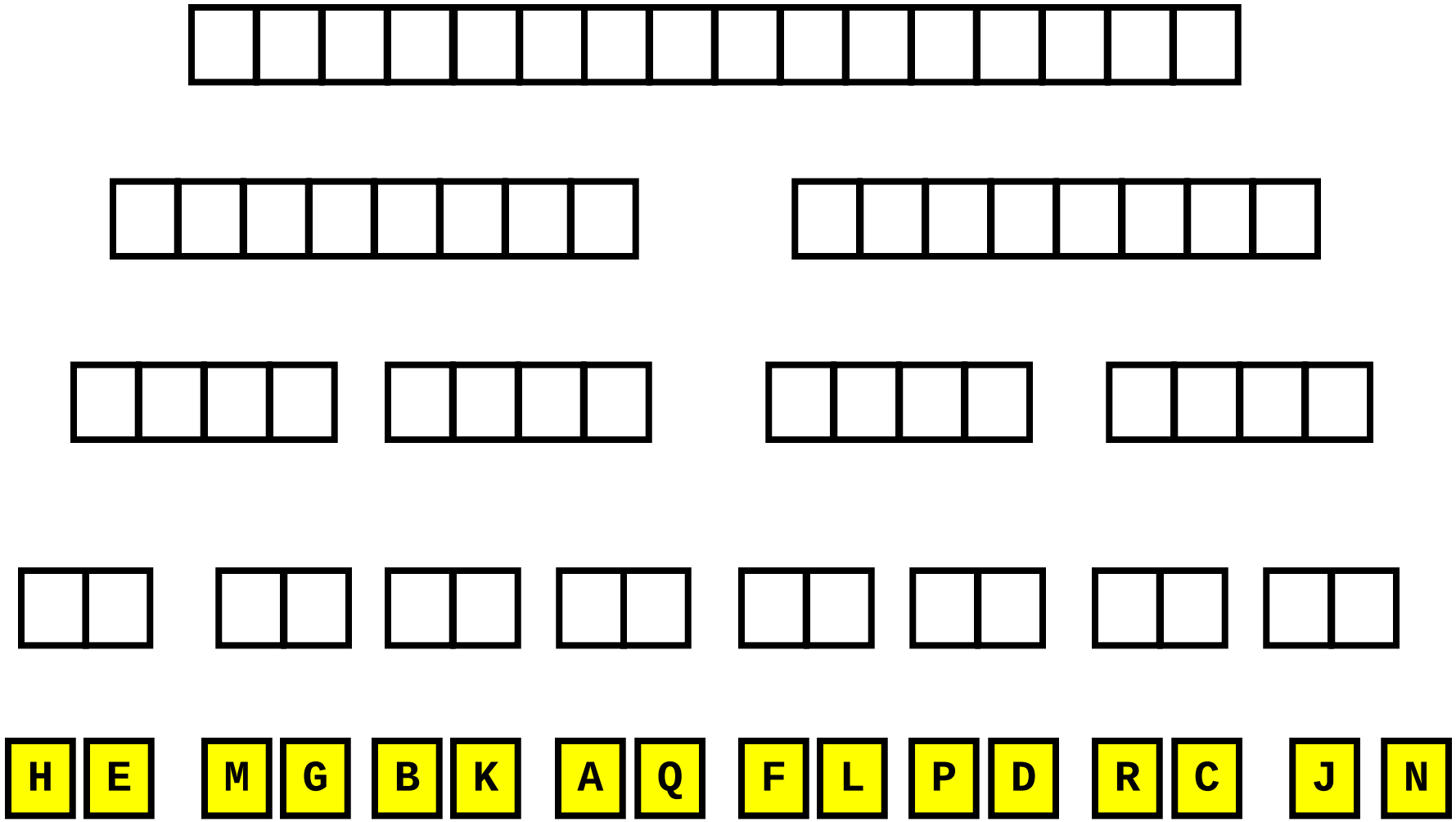
Subdivide again



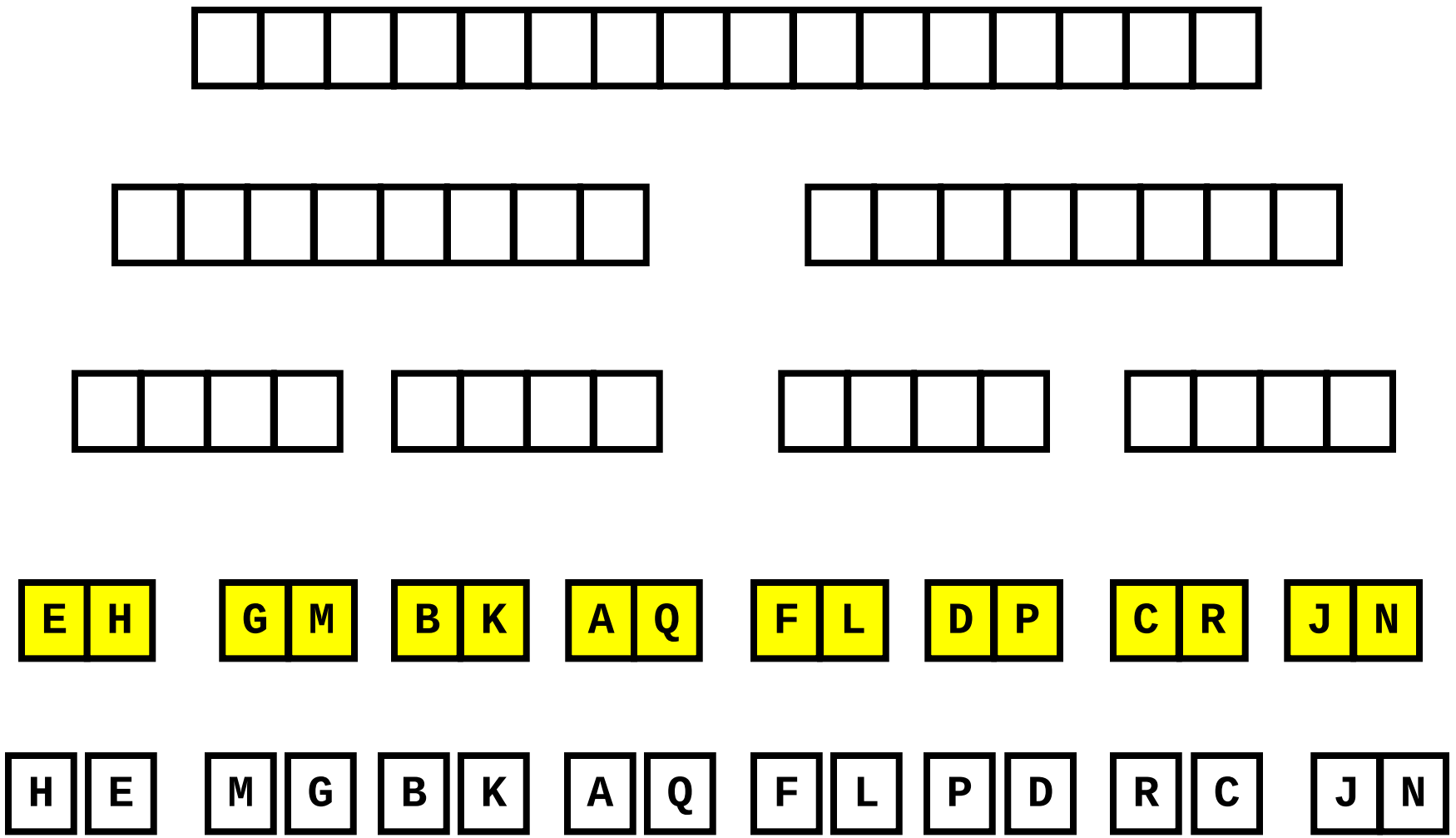
And again



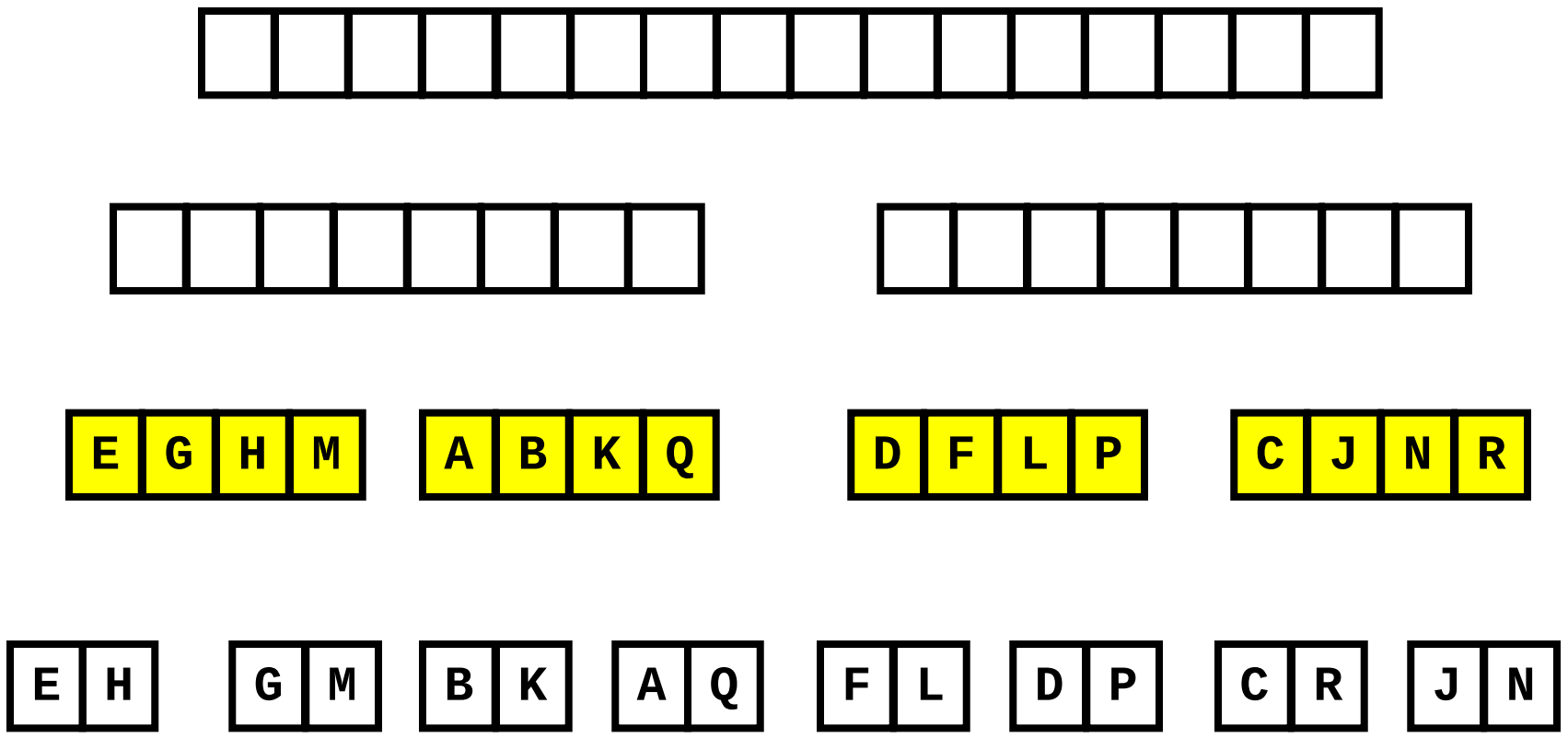
And one last time



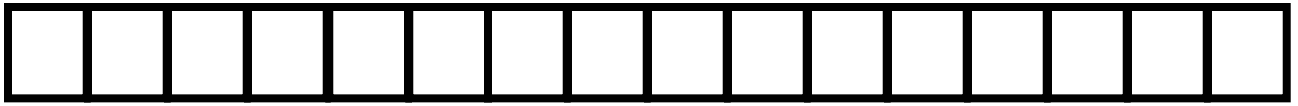
Now merge



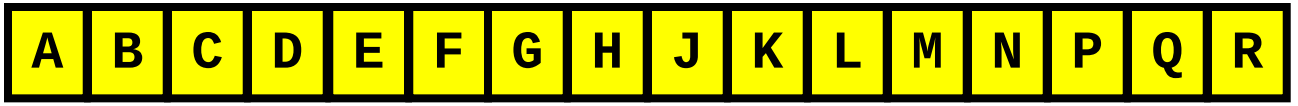
And merge again



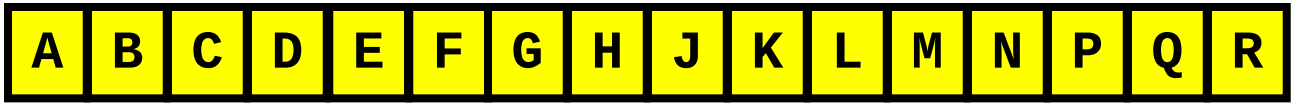
And again



And one last time

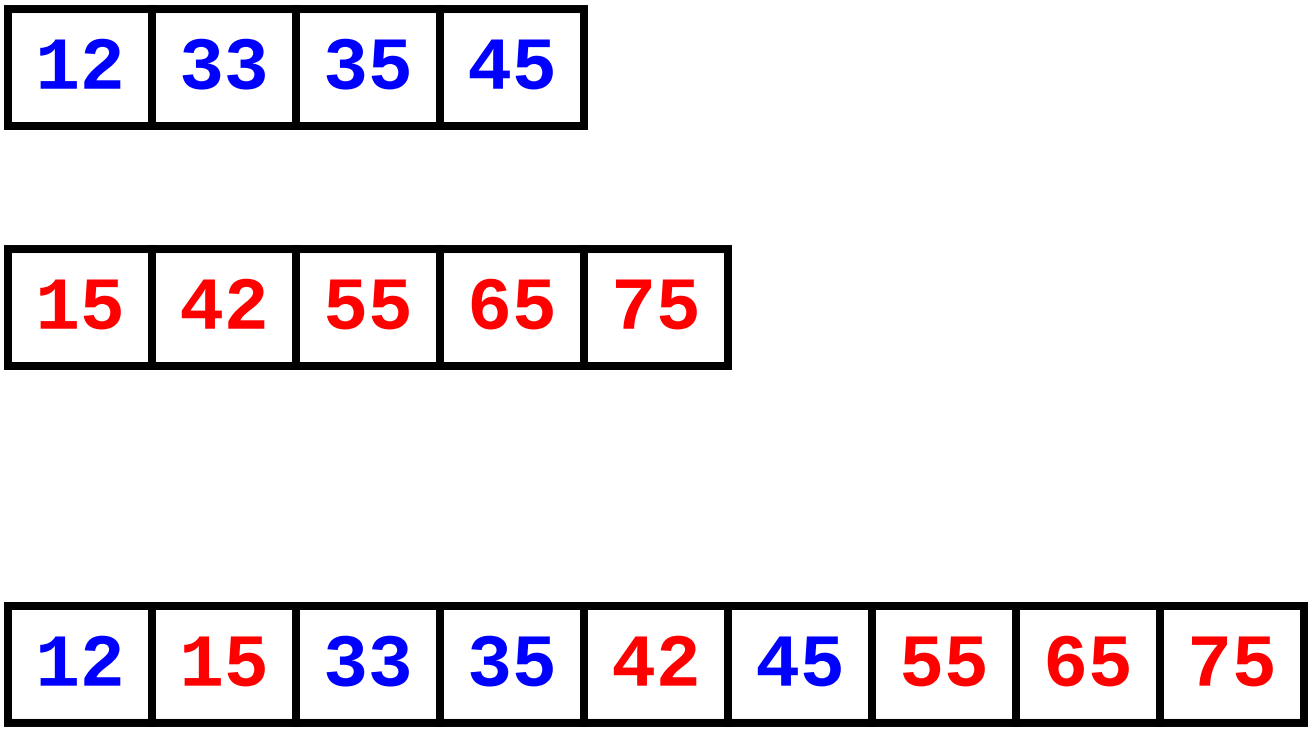


Done!

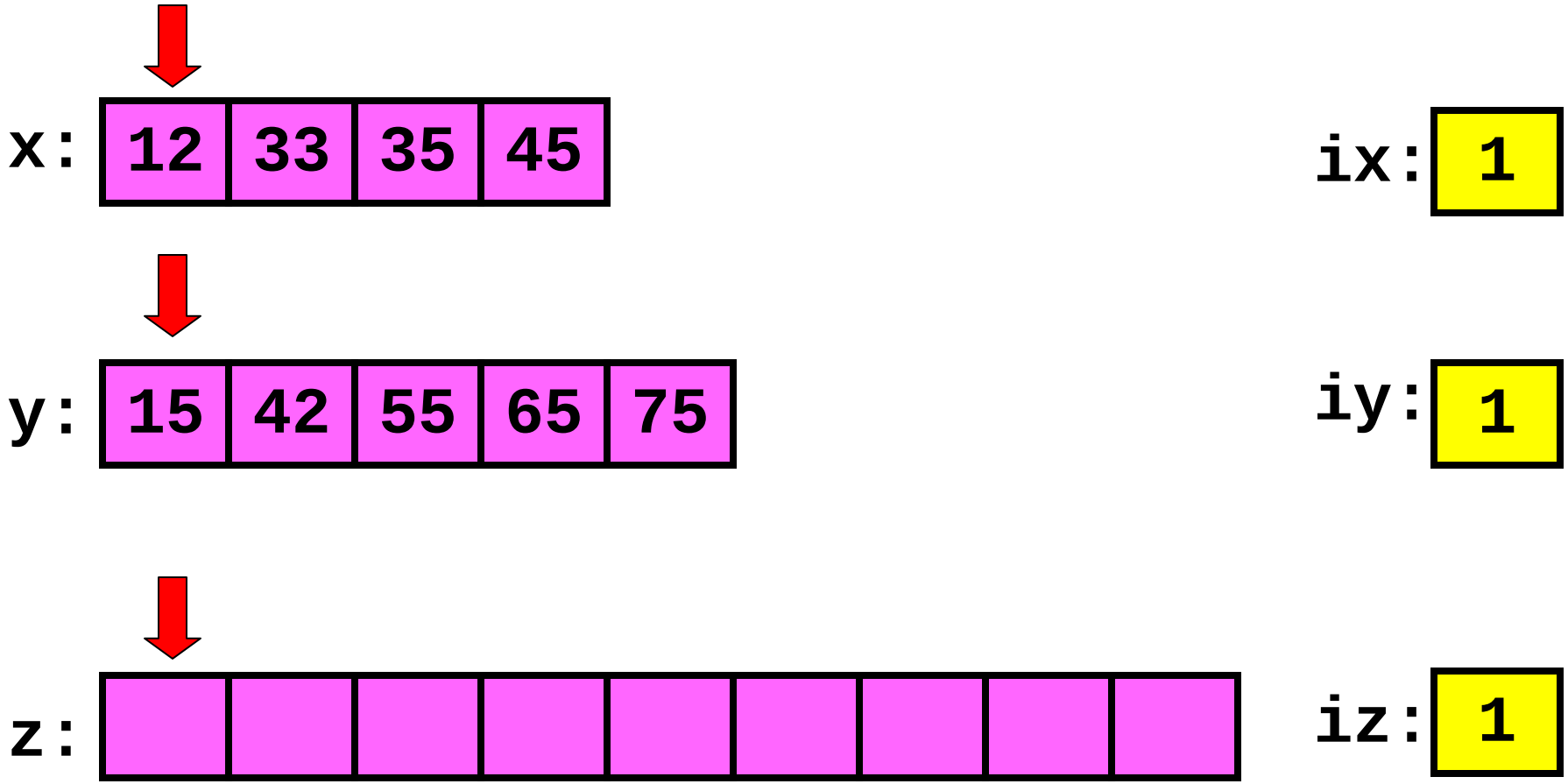


```
function y = mergeSort(x)
% x is a vector.  y is a vector
% consisting of the values in x
% sorted from smallest to largest.

n = length(x);
if n==1
        y = x;
else
        m  = floor(n/2);
        yL = mergeSortL(x(1:m));
        yR = mergeSortR(x(m+1:n));
        y  = merge(yL,yR);
end
```

The central sub-problem is the merging of two sorted arrays into one single sorted array

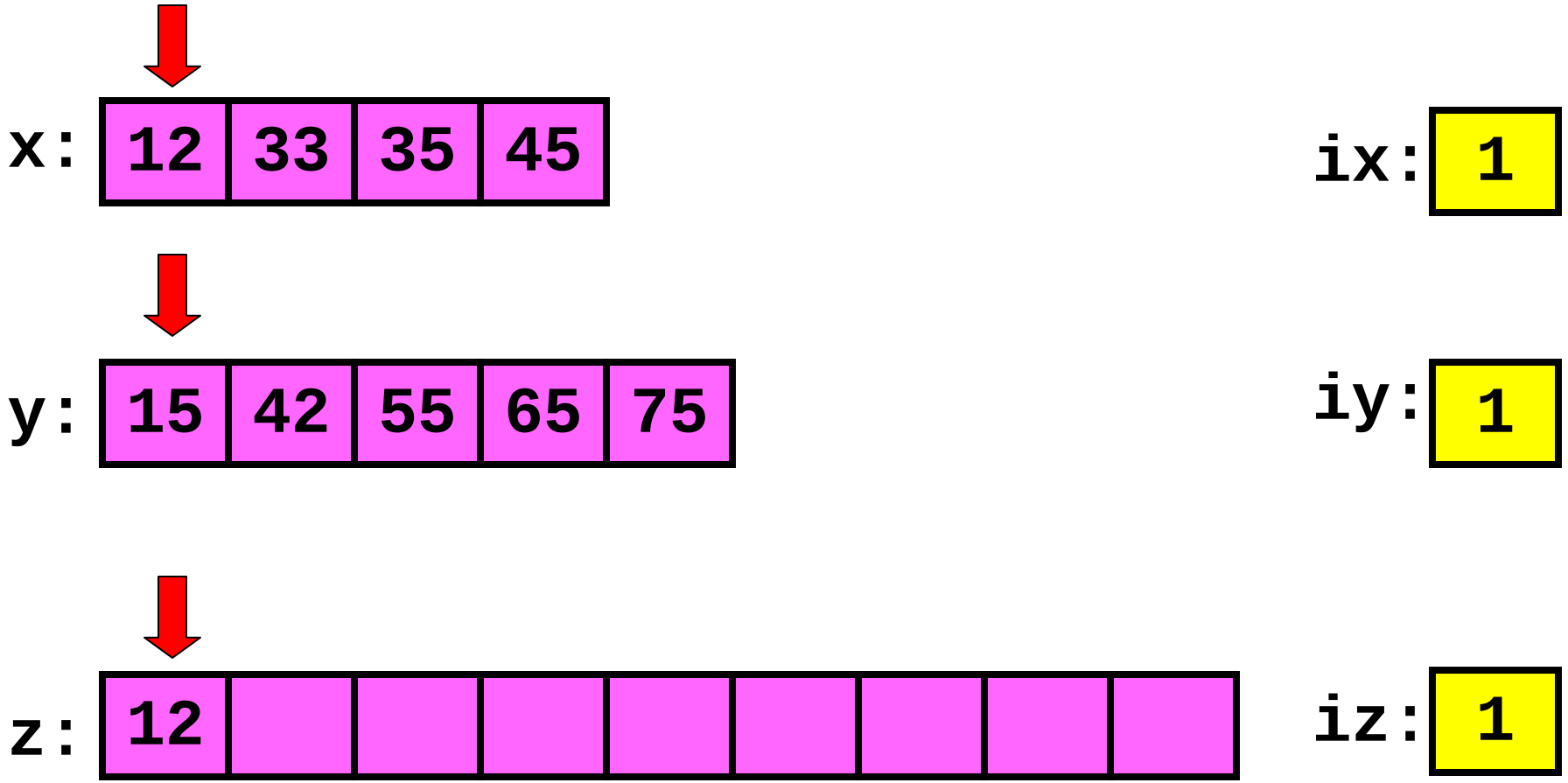


Merge



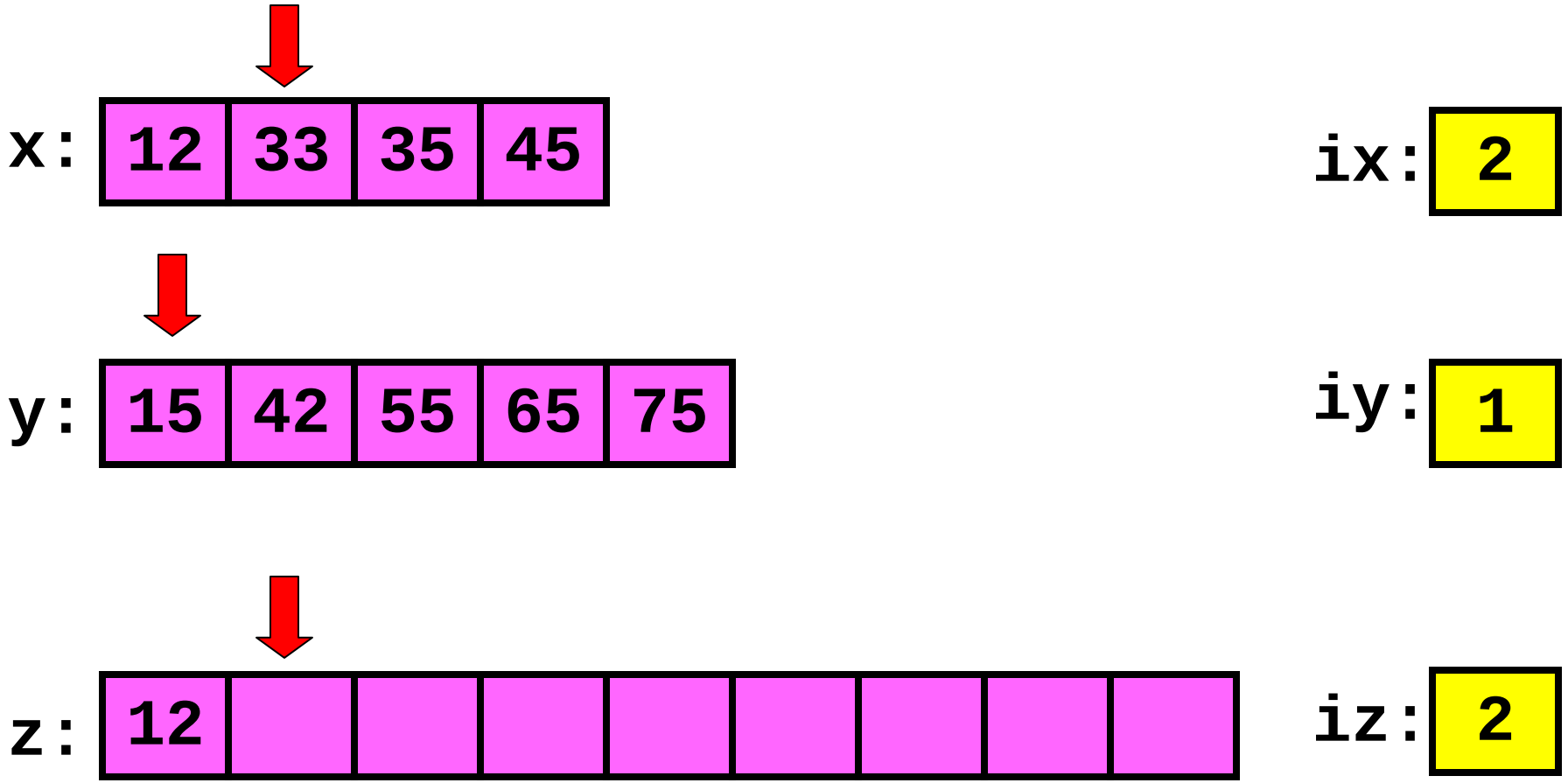

```
ix<=4 and iy<=5:  x(ix) <= y(iy)  ???
```

Merge



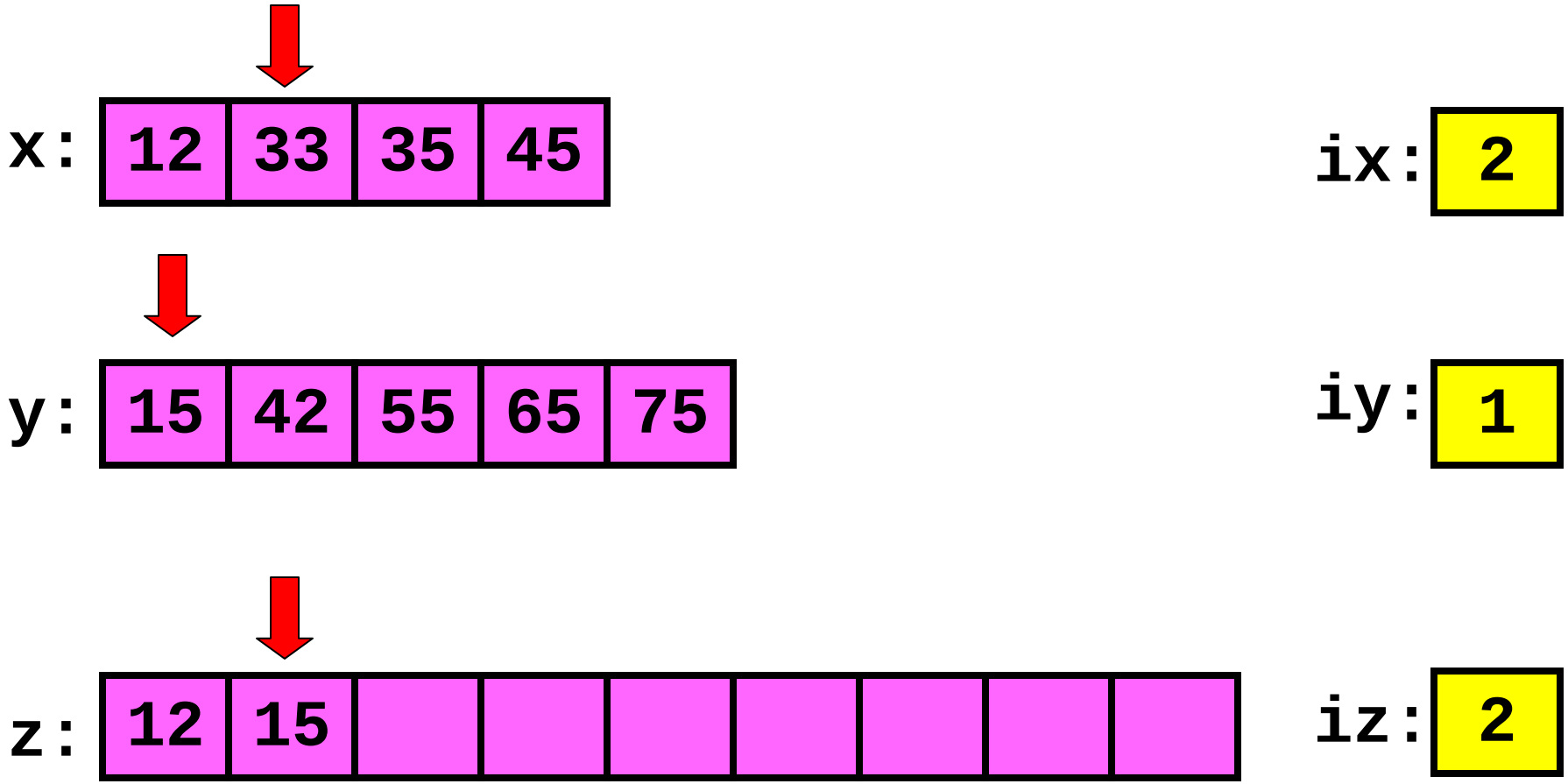
ix<=4 and iy<=5: x(ix) <= y(iy) YES

Merge



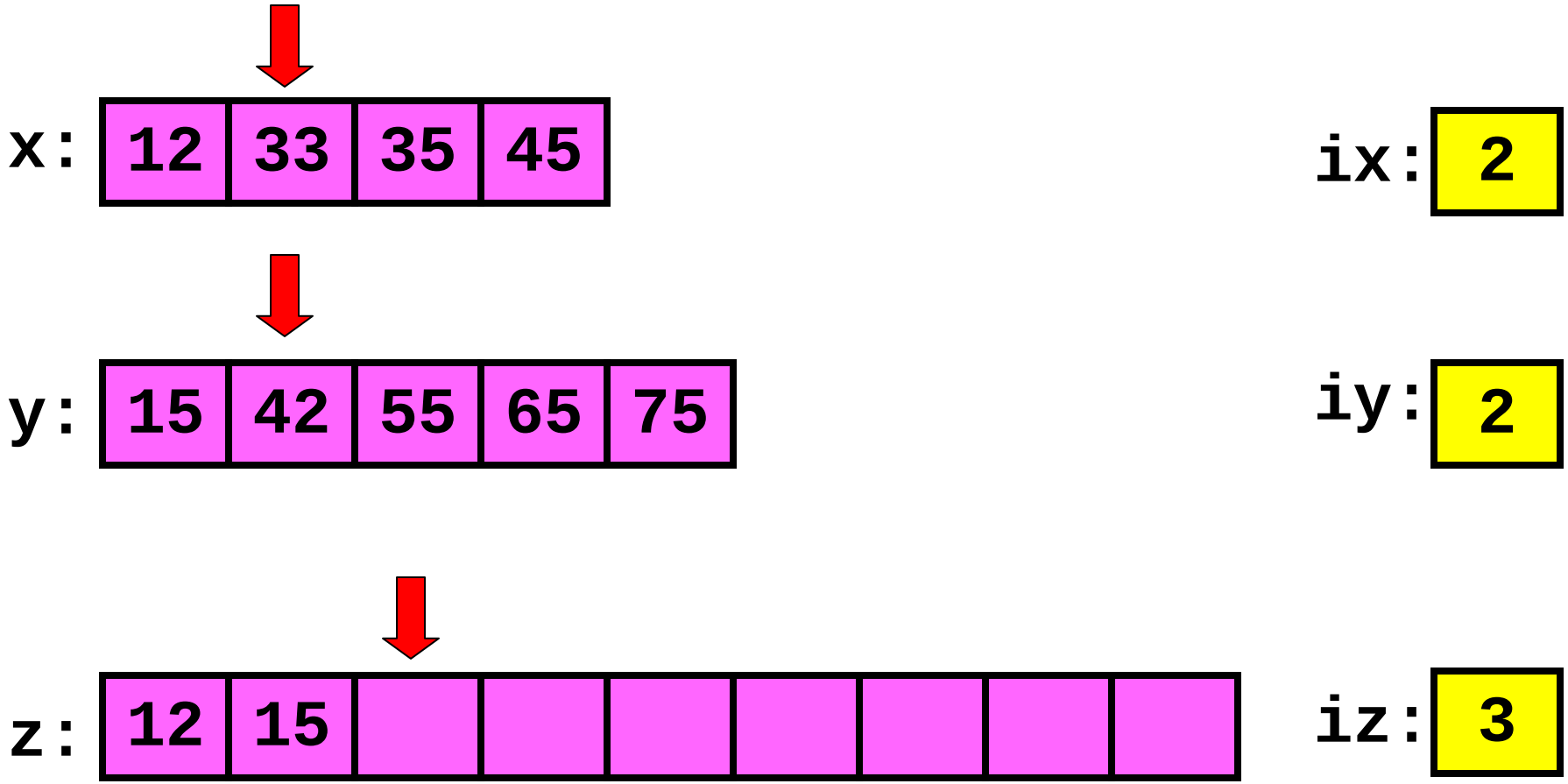

```
ix<=4 and iy<=5:  x(ix) <= y(iy)  ???
```

Merge



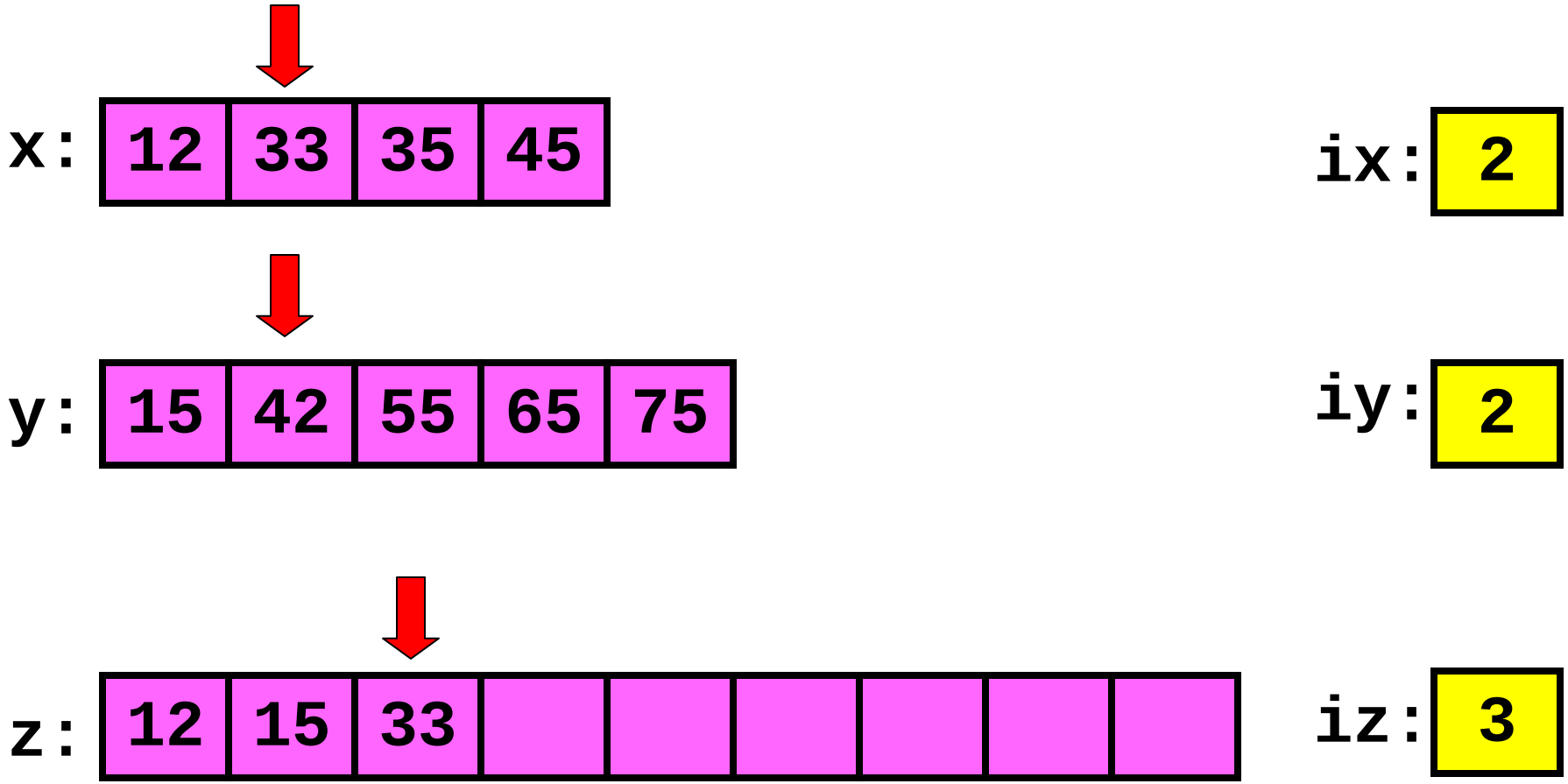
ix<=4 and iy<=5: x(ix) <= y(iy) NO

Merge



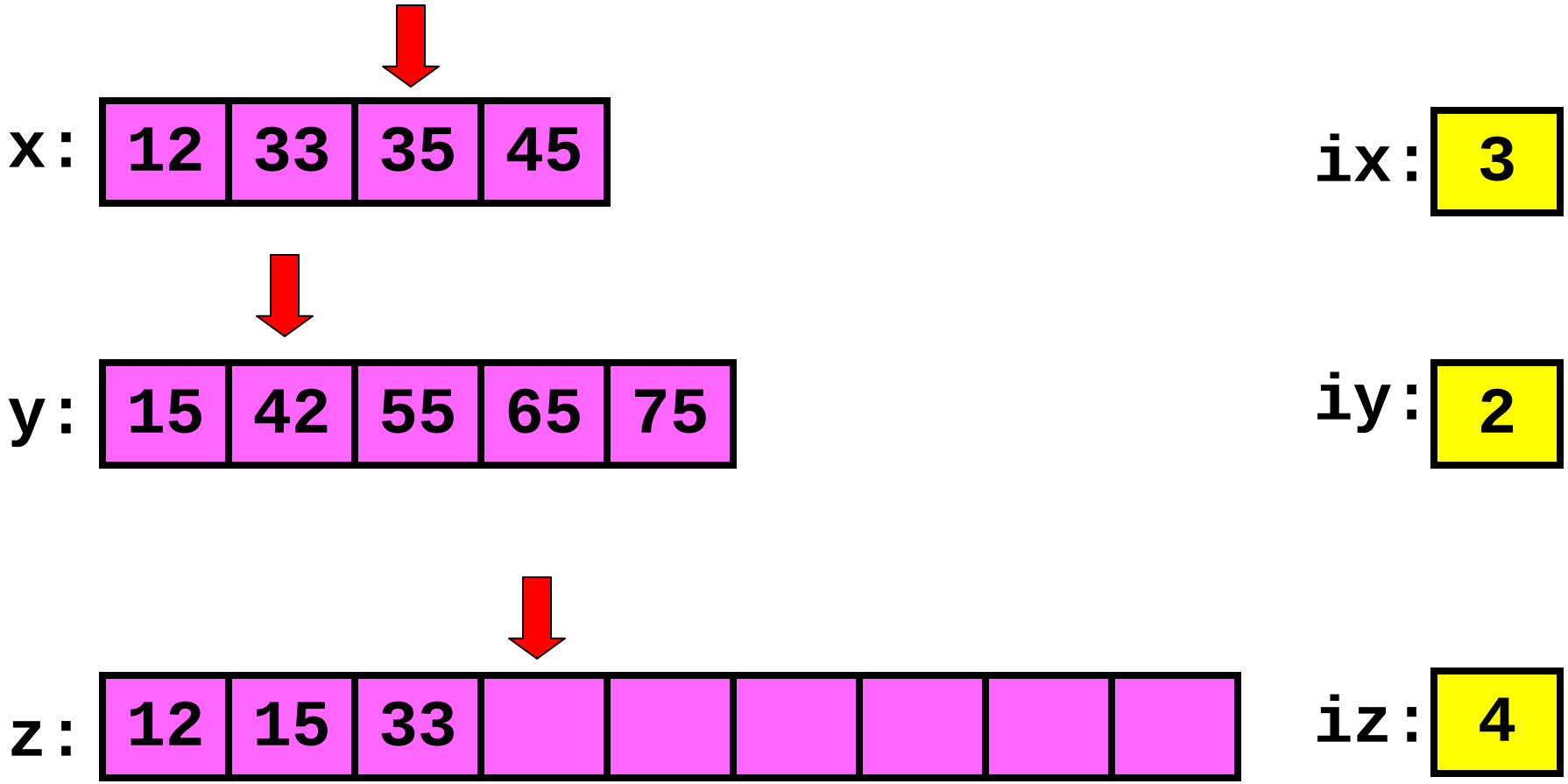

```
ix<=4 and iy<=5:  x(ix) <= y(iy)  ???
```

Merge



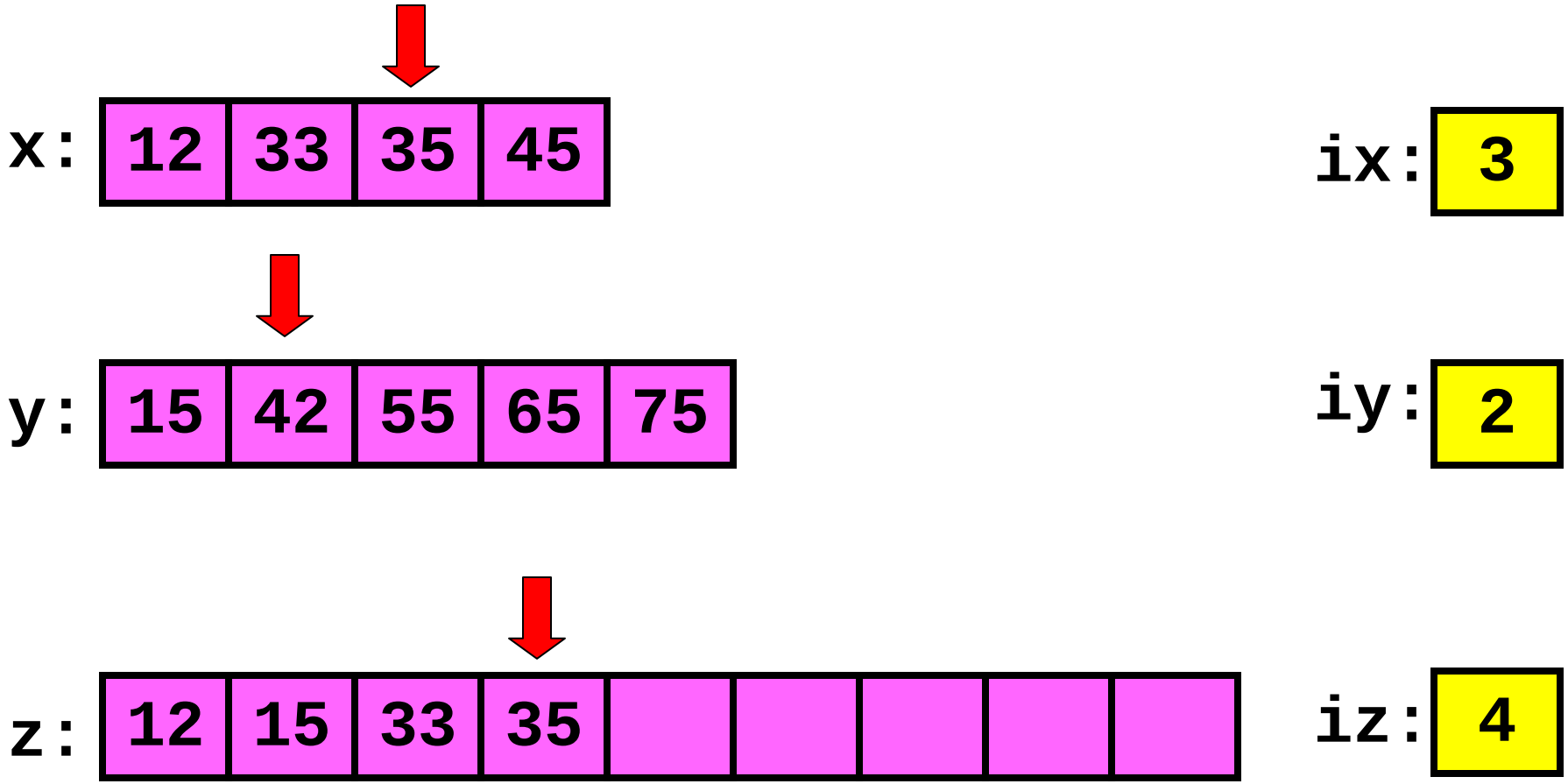
`ix<=4 and iy<=5:  x(ix) <= y(iy)` **YES**

Merge



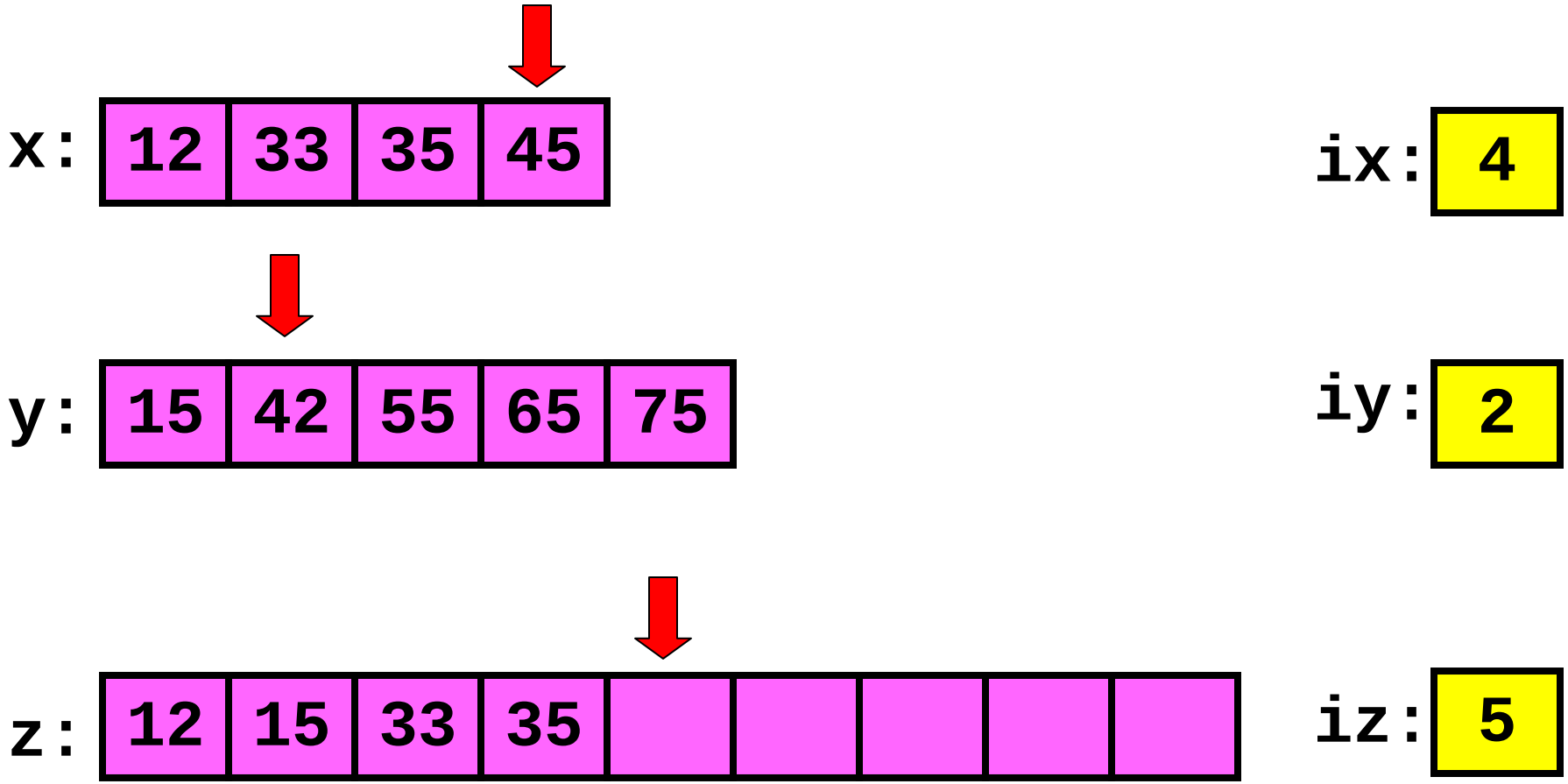
ix<=4 and iy<=5: x(ix) <= y(iy) ???

Merge



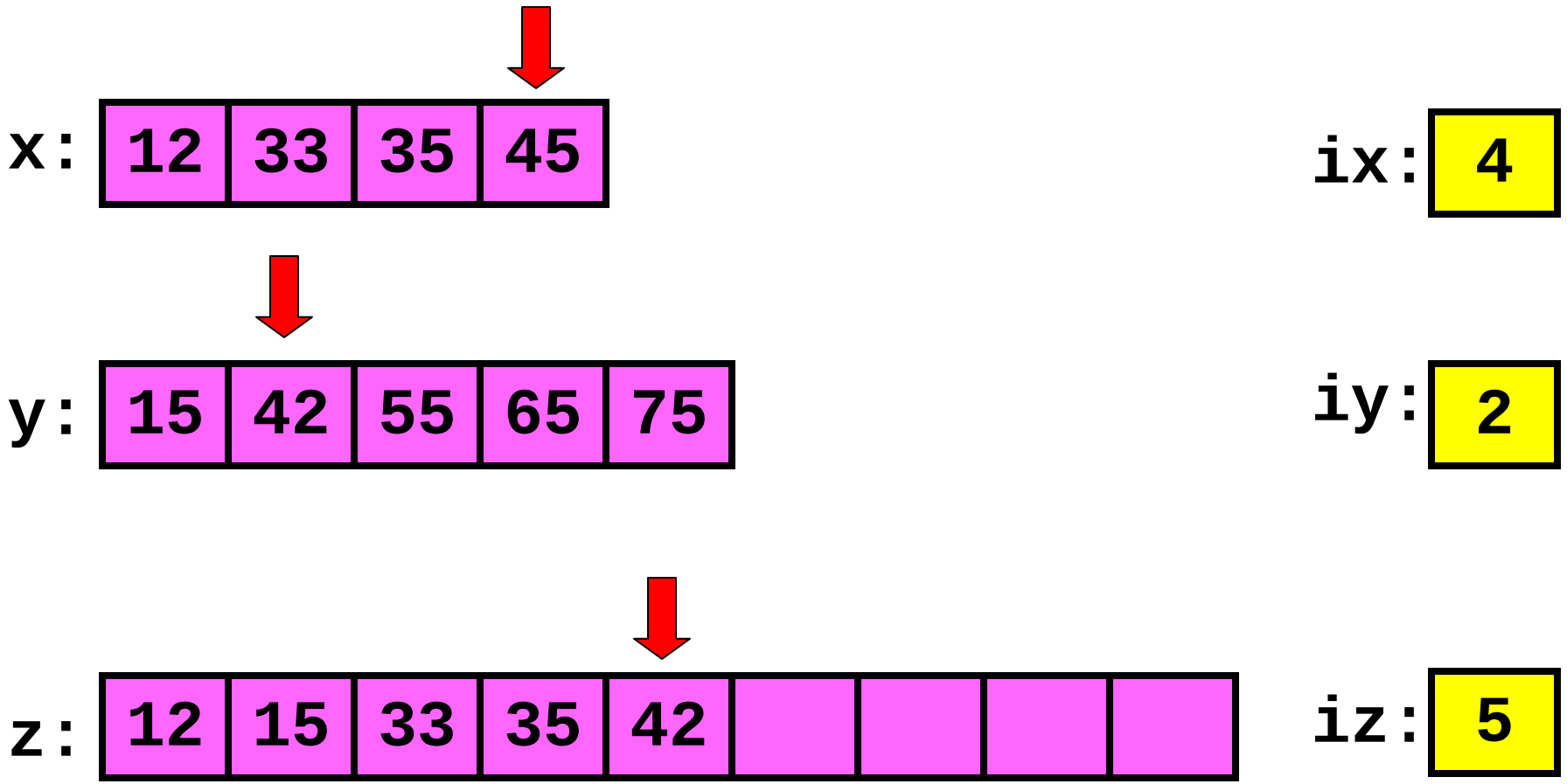
`ix<=4 and iy<=5:  x(ix) <= y(iy)` **YES**

Merge



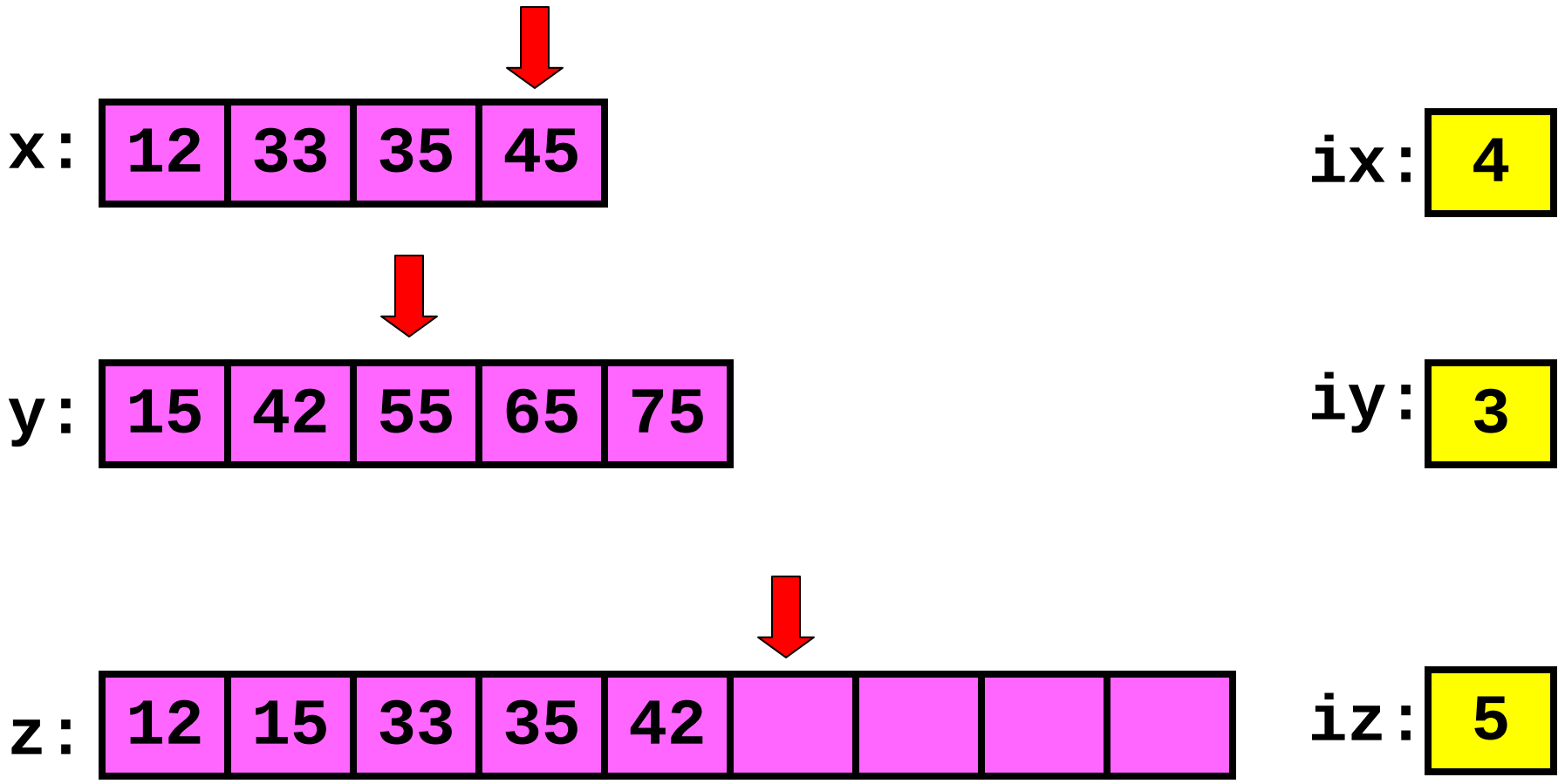
ix<=4 and iy<=5: x(ix) <= y(iy) ???

Merge



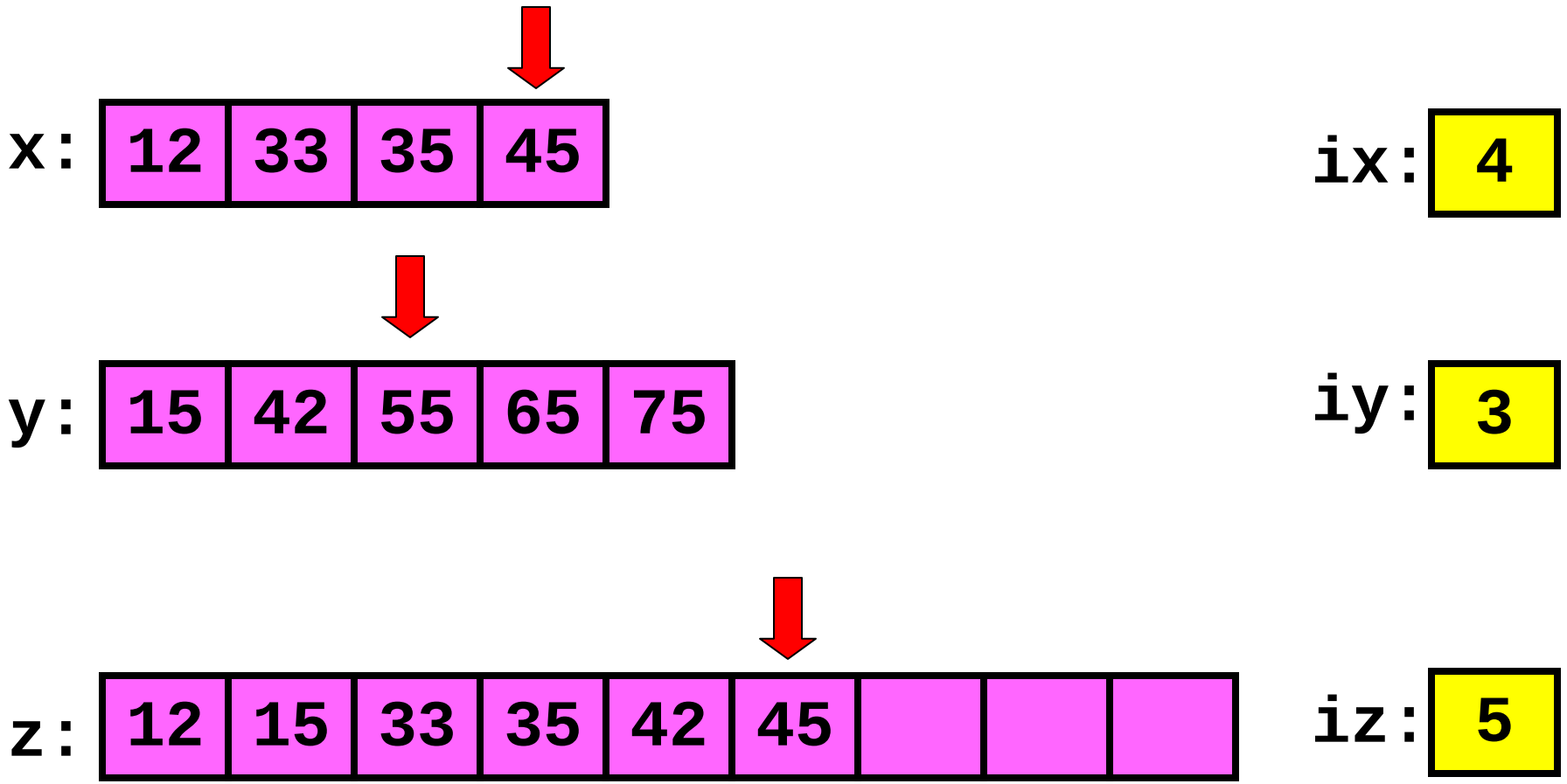
ix<=4 and iy<=5: x(ix) <= y(iy) NO

Merge



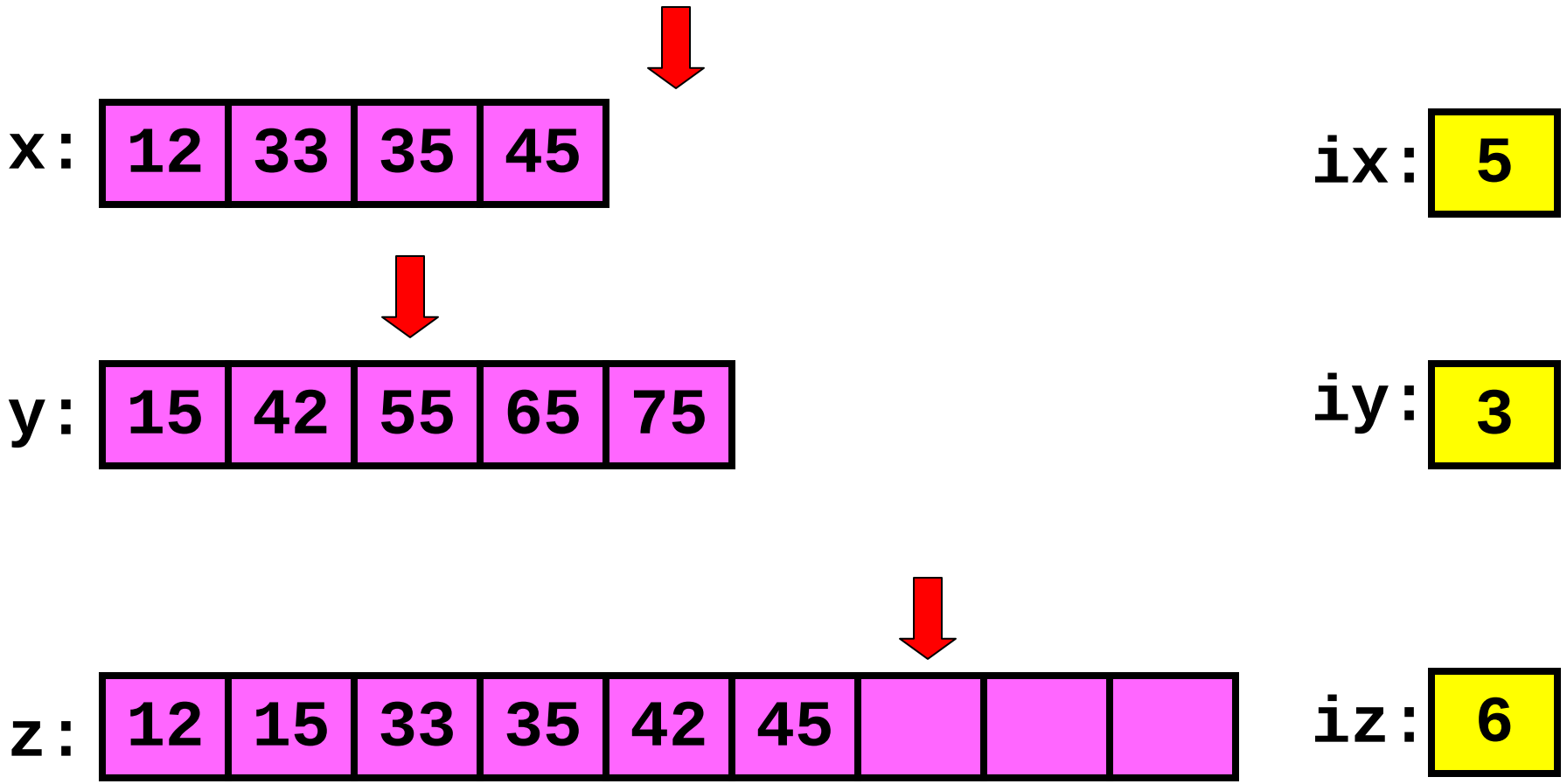
ix<=4 and iy<=5: x(ix) <= y(iy) ???

Merge



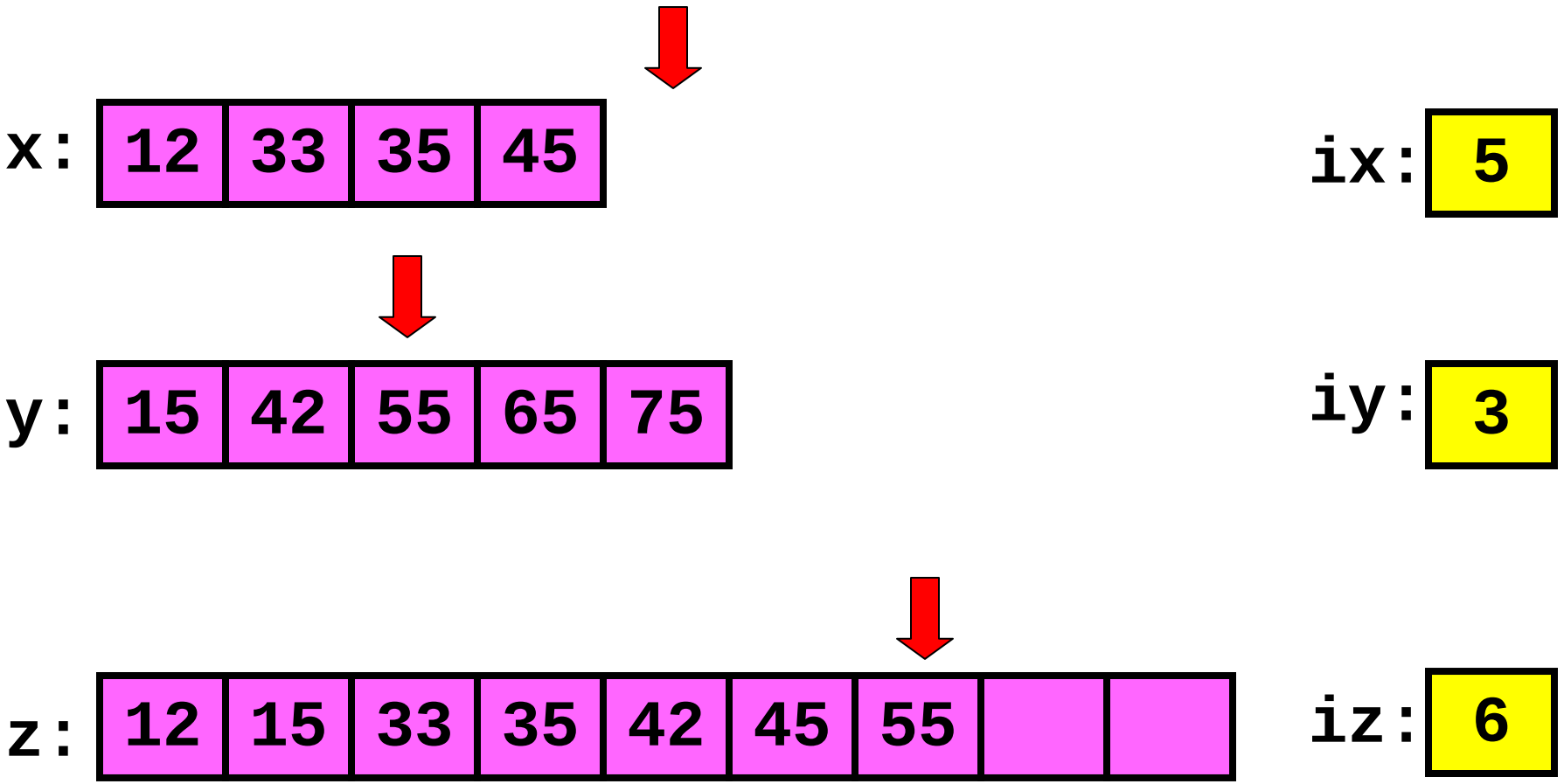
ix<=4 and iy<=5: x(ix) <= y(iy) YES

Merge



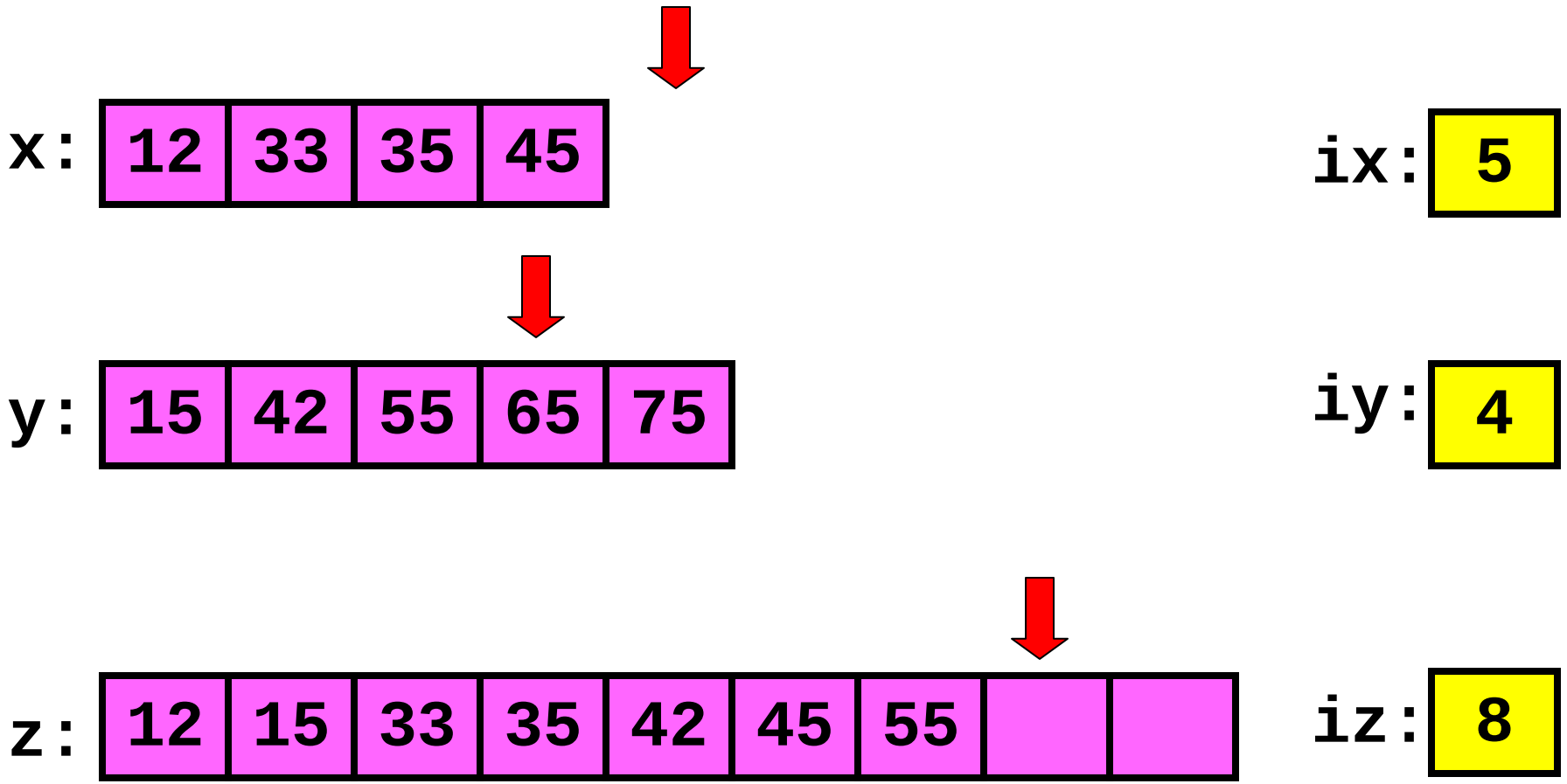
ix > 4

Merge



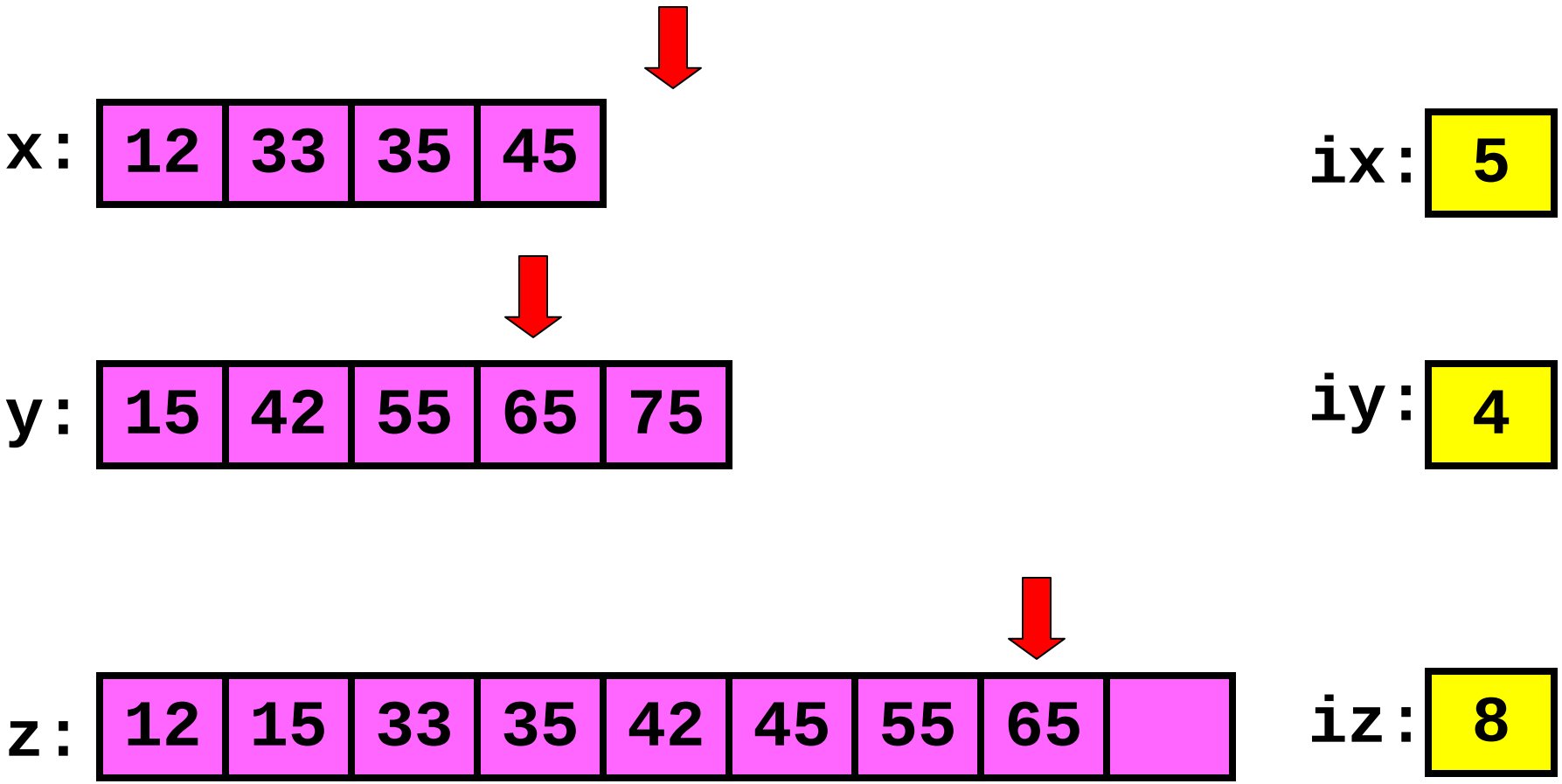
ix > 4: take y(iy)

Merge



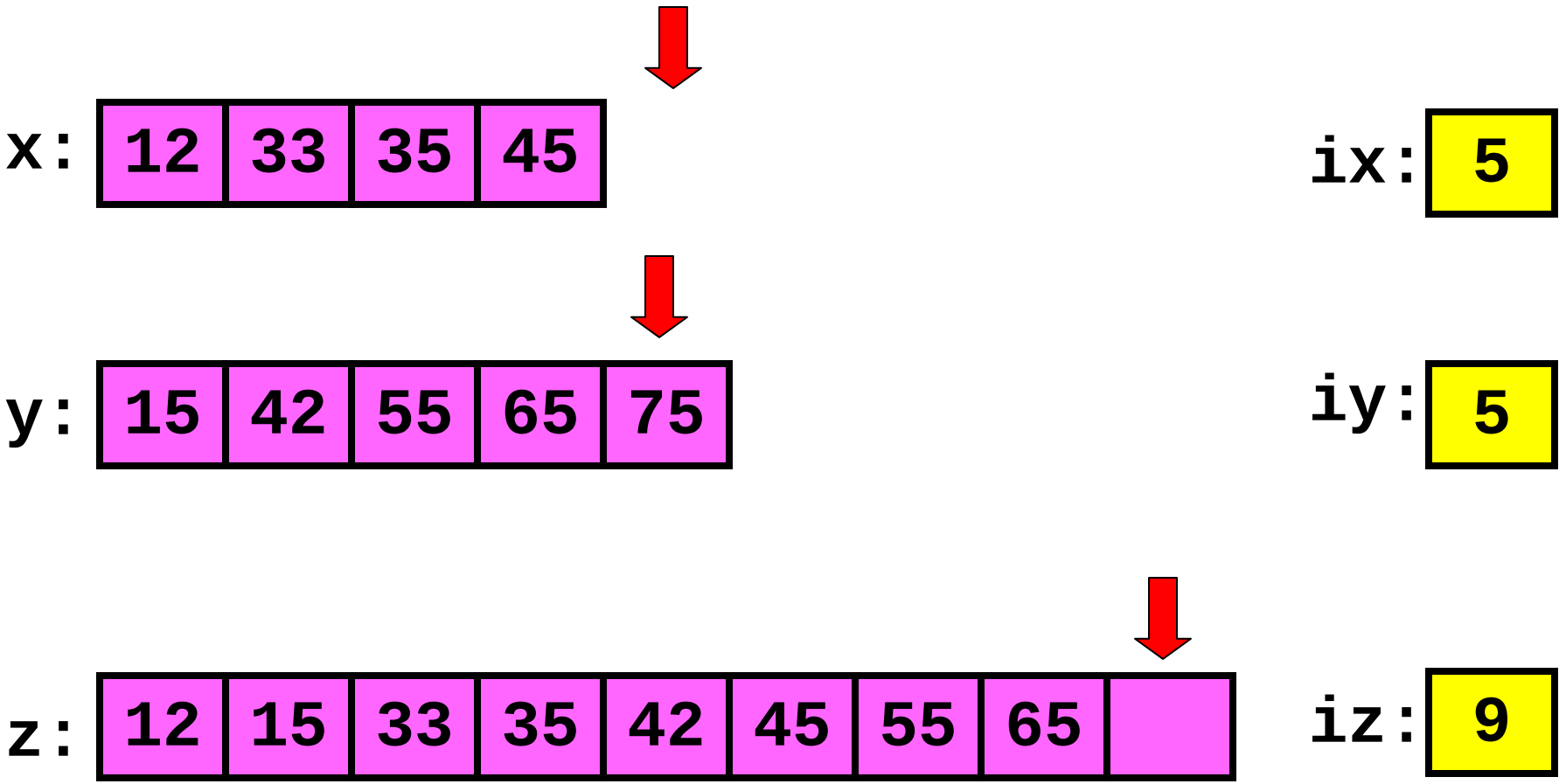

```
iy <= 5
```

Merge



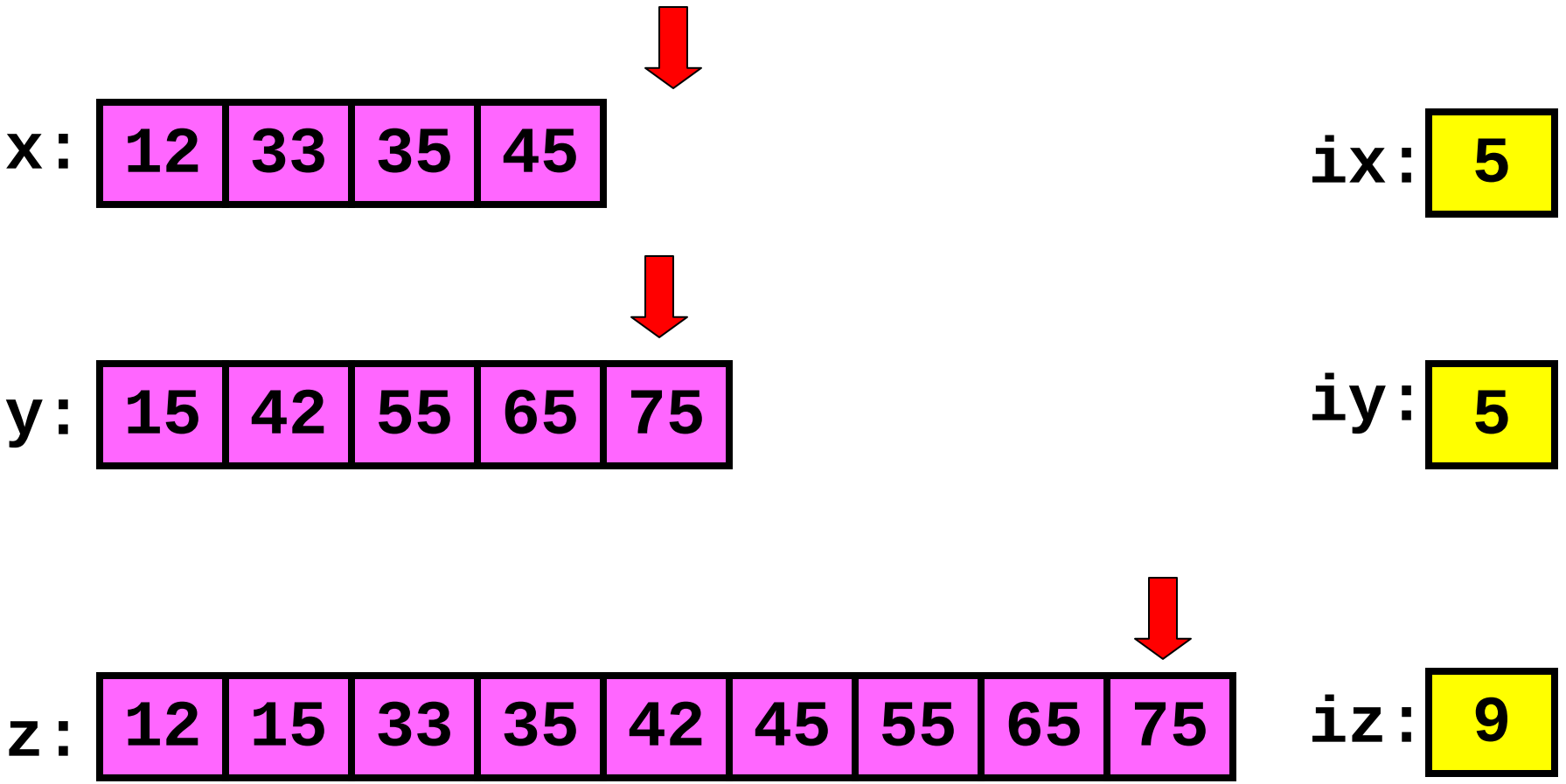

```
iy <= 5
```

Merge




```
iy <= 5
```

Merge




```
iy <= 5
```

```
function z = merge(x,y)
nx = length(x); ny = length(y);
z = zeros(1, nx+ny);
ix = 1; iy = 1; iz = 1;
```

```
function z = merge(x,y)
nx = length(x); ny = length(y);
z = zeros(1, nx+ny);
ix = 1; iy = 1; iz = 1;
while ix<=nx && iy<=ny









end
% Deal with remaining values in x or y
```

```
function z = merge(x,y)
nx = length(x); ny = length(y);
z = zeros(1, nx+ny);
ix = 1; iy = 1; iz = 1;
while ix<=nx && iy<=ny
    if  x(ix) <= y(iy)
        z(iz)= x(ix);  ix=ix+1;  iz=iz+1;
    else
        z(iz)= y(iy);  iy=iy+1;  iz=iz+1;
    end
end
% Deal with remaining values in x or y
```

```
function z = merge(x,y)
nx = length(x); ny = length(y);
z = zeros(1, nx+ny);
ix = 1; iy = 1; iz = 1;
while ix<=nx && iy<=ny
    if  x(ix) <= y(iy)
        z(iz)= x(ix);  ix=ix+1;  iz=iz+1;
    else
        z(iz)= y(iy);  iy=iy+1;  iz=iz+1;
    end
end
while ix<=nx  % copy remaining x-values
  z(iz)= x(ix);  ix=ix+1;  iz=iz+1;
end
while iy<=ny  % copy remaining y-values
  z(iz)= y(iy);  iy=iy+1;  iz=iz+1;
end
```

```
function y = mergeSort(x)
% x is a vector.  y is a vector
% consisting of the values in x
% sorted from smallest to largest.

n = length(x);
if n==1
        y = x;
else
        m  = floor(n/2);
        yL = mergeSortL(x(1:m));
        yR = mergeSortR(x(m+1:n));
        y  = merge(yL,yR);
end
```

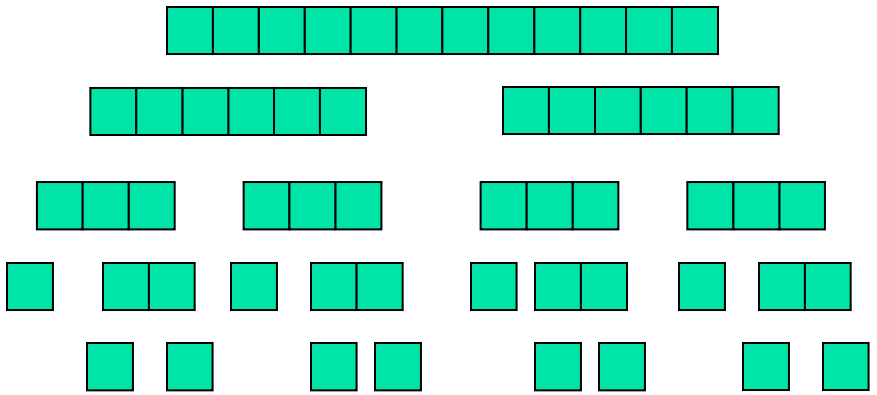
```
function y = mergeSortL(x)
% x is a vector.  y is a vector
% consisting of the values in x
% sorted from smallest to largest.

n = length(x);
if n==1
        y = x;
else
        m  = floor(n/2);
        yL = mergeSortL_L(x(1:m));
        yR = mergeSortL_R(x(m+1:n));
        y  = merge(yL,yR);
end
```

```
function y = mergeSortL_L(x)
% x is a vector.  y is a vector
% consisting of the values in x
% sorted from smallest to largest.

n = length(x);
if n==1
        y = x;
else
        m  = floor(n/2);
        yL = mergeSortL_L_L(x(1:m));
        yR = mergeSortL_L_R(x(m+1:n));
        y  = merge(yL,yR);
end
```

```
function y = mergeSort(x)
% x is a vector.  y is a vector
% consisting of the values in x
% sorted from smallest to largest.

n = length(x);
if n==1
        y = x;
else
        m  = floor(n/2);
        yL = mergeSort(x(1:m));
        yR = mergeSort(x(m+1:n));
        y  = merge(yL,yR);
end
```

```
function y=mergeSort(x)
n=length(x);
if n==1
      y=x;
else
      m=floor(n/2);
      yL=mergeSort(x(1:m));
      yR=mergeSort(x(m+1:n));
      y=merge(yL,yR);
end
```

```
function y=mergeSort(x)
n=length(x);
if n==1
      y=x;
else
      m=floor(n/2);
      yL=mergeSort(x(1:m));
      yR=mergeSort(x(m+1:n));
      y=merge(yL,yR);
end
```



```
function y = mergeSort(x)
% x is a vector.  y is a vector
% consisting of the values in x
% sorted from smallest to largest.

n = length(x);
if n==1
        y = x;
else
        m  = floor(n/2);
        y1 = mergeSort(x(1:m));
        y2 = mergeSort(x(m+1:n));
        y  = merge(y1,y2);
end
```