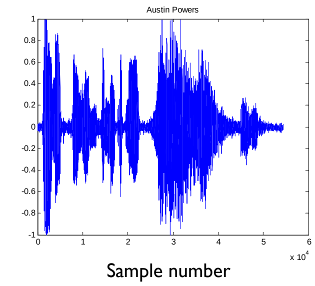
- Previous Lecture:
  - Working with sound files
- Today's Lecture:
  - Frequency computation
  - Touchtone phone
- Announcement:
  - Discussion in the computer lab this week. Bring headphones.
  - Prelim 3 tonight at 7:30pm, Statler Auditorium
    - Lastnames A-O: main seating area
    - Lastnames P-Z: balcony

We looked at the time domain

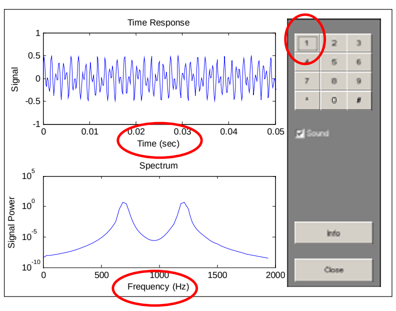


Sample number

→ Time

What about the frequency domain?

>> phone

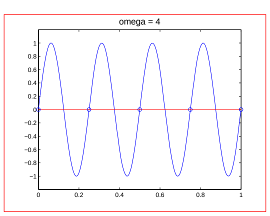


A "pure-tone" sound is a sinusoidal function

$$y(t) = \sin(2\pi\omega t)$$

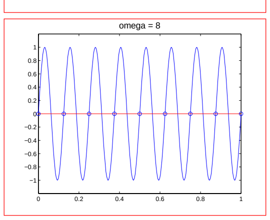
ω = the frequency

Higher frequency means that y(t) changes more rapidly with time.



Still looking at the time domain

$$y(t) = \sin(2\pi \cdot 4t)$$



$$y(t) = \sin(2\pi \cdot 8t)$$

Digitize for Graphics	Digitize for Sound

<pre>% Sample "Rate"
  n = 200
% Sample times
  tFinal = 1;
  t = 0:(1/n):tFinal
% Digitized Plot…
  omega = 8;
  y= sin(2*pi*omega*t)
  plot(t,y)</pre>
	<pre>% Sample Rate
	  Fs = 32768
	% Sample times
	  tFinal = 1;
	  t = 0:(1/Fs):tFinal
	% Digitized sound…
	  omega = 800;
	  y= sin(2*pi*omega*t);
	  sound(y,Fs)</pre>

Equal-Tempered Tuning

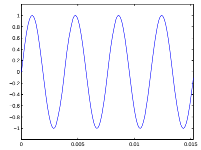
0 A	55.00	110.00	220.00	440.00	880.00	1760.00
1 A#	58.27	116.54	233.08	466.16	932.33	1864.66
2 B	61.74	123.47	246.94	493.88	987.77	1975.53
3 C	65.41	130.81	261.63	523.25	1046.50	2093.01
4 C#	69.30	138.59	277.18	554.37	1108.73	2217.46
5 D	73.42	146.83	293.67	587.33	1174.66	2349.32
6 D#	77.78	155.56	311.13	622.25	1244.51	2489.02
7 E	82.41	164.81	329.63	659.26	1318.51	2637.02
8 F	87.31	174.61	349.23	698.46	1396.91	2793.83
9 F#	92.50	185.00	369.99	739.99	1479.98	2959.95
10 G	98.00	196.00	391.99	783.99	1567.98	3135.96
11 G#	103.83	207.65	415.31	830.61	1661.22	3322.44
12 A	110.00	220.00	440.00	880.00	1760.00	3520.00

Entries are frequencies. Each column is an octave.
Magic factor = 2^(1/12). C3 = 261.63, A4 = 440.00

Lecture 23 8

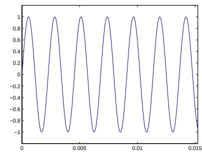
"Adding" Sinusoids

Middle *C*:
$\omega = 262$



\+

A above middle *C*:
$\omega = 440$



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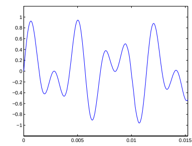


`playTwoNotes.m`

Lecture 23 9

"Adding" Sinusoids → *averaging* the sine values

```
Fs = 32768; tFinal = 1;
t  = 0:(1/Fs):tFinal;

C3  = 261.62;
yC3 = sin(2*pi*C3*t);
A4  = 440.00;
yA4 = sin(2*pi*A4*t);
y   = (yC3 + yA4)/2;

sound(y,Fs)
```

Lecture 23 10

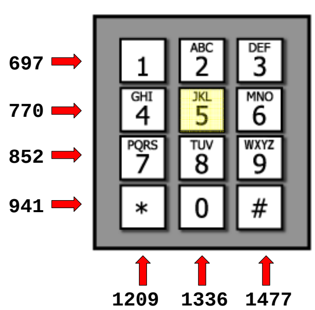
Application: touchtone telephones

Make a signal by combining two sinusoids

Lecture 23 11

A frequency is associated with each row & column.

So two frequencies are associated with each button.



The "5"-Button corresponds to (770,1336)

Each button has its own 2-frequency "fingerprint"!

Lecture 23 12

Signal for button 5:

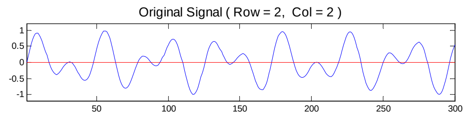
```
Fs = 32768;
tFinal = .25;
t  = 0:(1/Fs):tFinal;

yR = sin(2*pi*770*t);
yC = sin(2*pi*1336*t)
y  = (yR + yC)/2;

sound(y,Fs)
```

`MakeShowPlay.m`

Lecture 23 13



This is the "fingerprint" of button '5'

playAllButtons.m

To Minimize Ambiguity…

- No frequency is a multiple of another
- The difference between any two frequencies does not equal any of the frequencies
- The sum of any two frequencies does not equal any of the frequencies

Why is this important?

I dial a number (send signal). The receiver of the signals get a "noisy" version of the real signal. How will the noisy data be interpreted?

SendNoisy.m

How to compare two signals (vectors)?

Given two vectors x and y of the same length, the cosine of the angle between the two vectors is a measure of the correlation between vectors x and y:

$$\cos_{xy} = \frac{\left|\sum_{i=1}^{n} x_i y_i\right|}{\sqrt{\sum_{i=1}^{n} x_i^2} \cdot \sqrt{\sum_{i=1}^{n} y_i^2}}$$

Small cosine → low correlation
High cosine → highly correlated

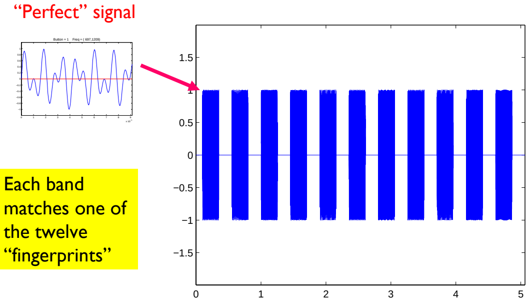
cos_xy.m
ShowCosines.m

Sending and deciphering noisy signals

- Randomly choose a button
  - Choose random row and column numbers
- Construct the real signal (**MakeShowPlay**)
- Add noise to the signal (**SendNoisy**)
- Compute cosines to decipher the signals (**ShowCosines**)
- See **Eg13_2**

What does the signal look like for a multi-digit call?

"Perfect" signal

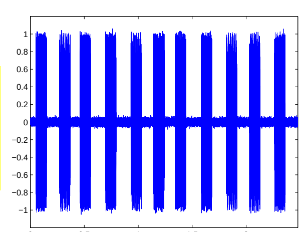


Each band matches one of the twelve "fingerprints"

Buttons pushed at equal time intervals

One of the most difficult problems is how to segment the multi-button signal!

"Noisy" signal



Each band approximately matches one of the twelve "fingerprints." There is noise between the button pushes.

Buttons pushed at unequal time intervals